Empirical Modeling of the Retrograde Free Core Nutation (Technical Note)

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1 Introduction

This note presents the derivation of an empirical model for the mantle oscillation associated with the retrograde free core nutation (RFCN). It is a generalization of an earlier work ofter referred to as Lambert's FCN model (Lambert, 2007). The earlier work was devoted to fitting the RFCN to the IERS reference series only while this one proposes adjustments to all available nutation series.

2 Least-Squares Model

I use the daily combined series IERS EOP 14C04 data set (Bizouard et al, 2018) computed by the International Earth rotation and Reference systems Service (IERS) as well as all available nutation offset series made available through the IERS Earth Orientation Center. The series provide values for celestial pole offsets dX and dY referred to the MHB expansion (Mathews et al, 2002). I fix the period of the free motion to the estimated value in the MHB work, let -430.21 days in a spaced-fixed frame of reference, and I consider any variation of the apparent period as included in the phase. Moreover, the space motion of the figure axis due to the RFCN is considered as circular, ignoring any possible asymmetry in the distribution of mass in the core.

The computation is based on a weighted least-squares fit of a circular term plus a constant to the complex-valued quantity $dX + \mathrm{i}\, dY$. The model is expressed as

$$dX + \mathrm{i}\, dY = A\mathrm{e}^{\mathrm{i}\,\sigma t} + X_0 + \mathrm{i}\, Y_0, \tag{1}$$

where A is the complex amplitude, σ the FCN frequency, and t is the time measured from J2000.0. When the nutation series do not provide the non-diagonal covariance information, the data is simply weighted by the inverse of the squared standard error as provided in the data file. Otherwise, the full covariance information is used. This leads to

$$\begin{aligned} dX &= A_c \cos\sigma t - A_s \sin\sigma t + X_0, \\ dY &= A_c \sin\sigma t + A_s \cos\sigma t + Y_0, \end{aligned} \tag{2}$$

allowing the estimation of four parameters: A_c and A_s and the constant offsets X_0 and Y_0. The offsets account for the long-term variations appearing in the nutation residuals and are not physically related to the core nutation. Apart a slight correction to the precession, the offsets include reference frame biases and contributions to the 18.6-yr nutation and other prominent terms mismodeled in MHB. The contribution of the FCN only to the celestial pole offsets is given by

$$\begin{aligned} X_{\mathrm{FCN}} &= X_s \sin\sigma t + X_c \cos\sigma t, \\ Y_{\mathrm{FCN}} &= Y_s \sin\sigma t + Y_c \cos\sigma t, \end{aligned} \tag{3}$$

where

$$X_s = Y_c = A_s, \quad X_c = -Y_s = A_c. \tag{4}$$

To account for the time variability of the amplitude and the phase, the estimates are done over a sliding window. The tabulated epoch for each window is the middle date of the window. The window width must be sufficiently large to separate the FCN from the retrograde annual oscillation expected to show up at about 0.1 mas. The demodulation period is 6.7 years. I chose 7 years which is conservative. It must be noted that such a large window implies that consecutive yearly amplitudes are not independent but that this is a compromise between frequency and time resolution. A smaller window, thus providing a higher time resolution, would capture a part of the retrograde annual oscillation and provoque an artificial 6.7-yr beating of the FCN amplitude.

3 Results and Availability

Adjusted coefficients can be found at http://ivsopar.obspm.fr/fcn in the file table-asc-XXX.txt together with a FCN time series table-ser-XXX.txt computed for the nutation epochs and a FORTRAN subroutine fcnnut.f that is able to compute the FCN amplitude at any epoch. This subroutine takes as argument the yearly amplitudes and uncertainties adjusted above and that can be found in the file table-asc-XXX.txt.

It can be noticed that the formal error on these amplitudes varies between 10 μas in the early years down to less than 1 μas for the most recent years. As already mentioned, the reader must keep in mind that a more realistic error estimated through statistical tests might replace these formal errors.

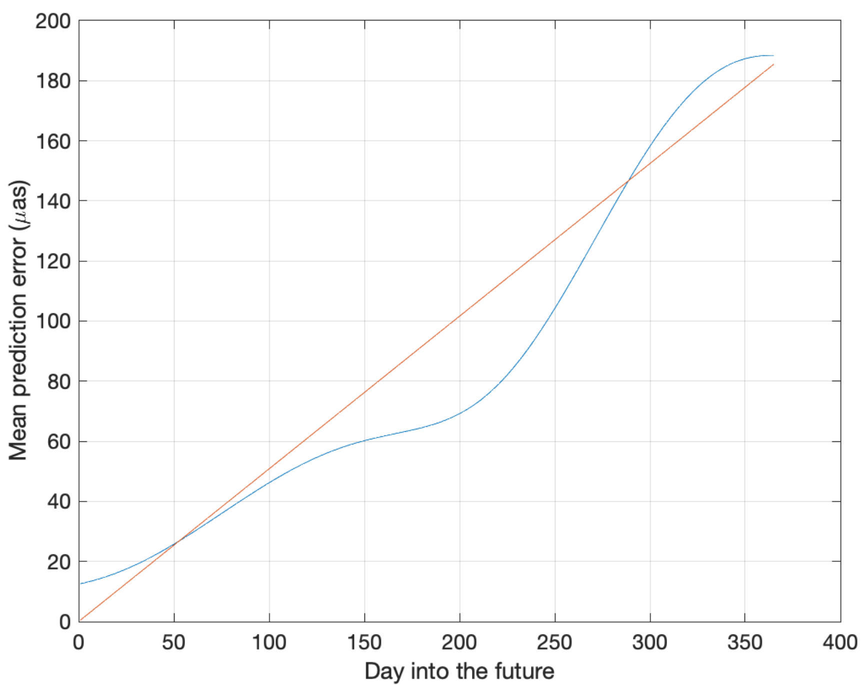

Figure 1: Blue curve: the mean prediction error as a function of the prediction horizon into the future. Red line: first-order polynomial fitted to the mean prediction error.

A mean prediction error has been introduced in the FORTRAN routine to account for the degradation of the uncertainty in predictive mode (forward or backward). It is estimated through the average of a thousand predictions over past time intervals with a method similar to Lambert (2007): one thousand predictions are launched starting at random epochs after 2010 over one-year into the future based on the previously fitted C04 FCN coefficients truncated at the integer year preceding the prediction epoch. Then, the predicted signal is compared against the one modeled with the full table. The standard deviation of the thousand differences is computed (Fig. 1), varying roughly as a second order polynomial. However, the prediction error is fitted with a first-order polynomial whose slope gives the degradation in forward or backward predictive mode starting from any given epoch. The found slope is 0.28 μas/day. For the routine implementation, I adopt the conservative value of 0.3 μas/day.

The data is updated regularly, typically every week. Note that unless the nutation data set is strongly modified (e.g., due a complete reanalysis after changing the IERS combination strategy or introducing a new ITRF), the amplitudes for past years will remain the same or very close. Only the coefficient relative to the present year can be affected significantly.

References

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