

Simultaneous comparisons of optical and atomic fountain clocks using broadband TWSTFT¹



Franziska Riedel

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Outline

Introduction

Frequency comparison with satellites: **Two-Way Satellite Time and Frequency Transfer (TWSTFT)**

Planning and execution of the clock comparison campaign

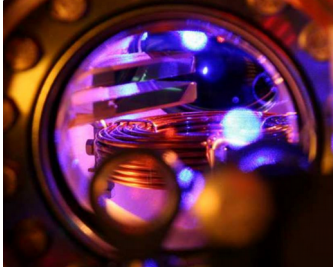
Data analysis

Discussion of results

Summary and outlook

Introduction

EMRP-project ITOC: “International Timescales with Optical Clocks”



Contents: local comparisons between optical clocks, absolute frequency measurements, assess information about implementing optical clocks in timescales ...
... remote comparisons!

Optical fibers



limited in availability

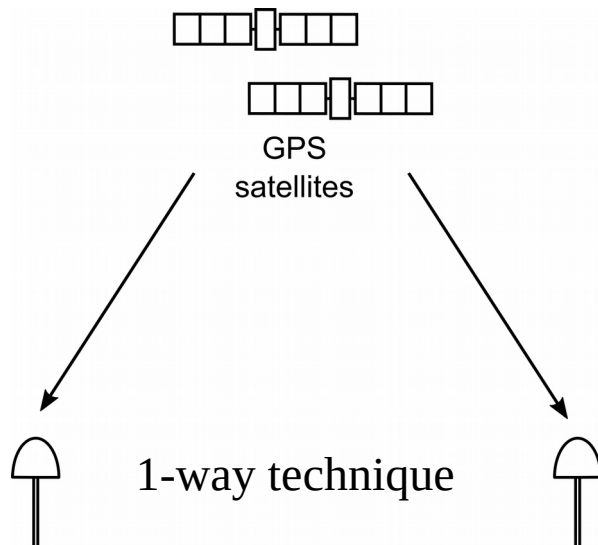
Satellite-based techniques



limited in uncertainty

Remote comparisons: using satellites

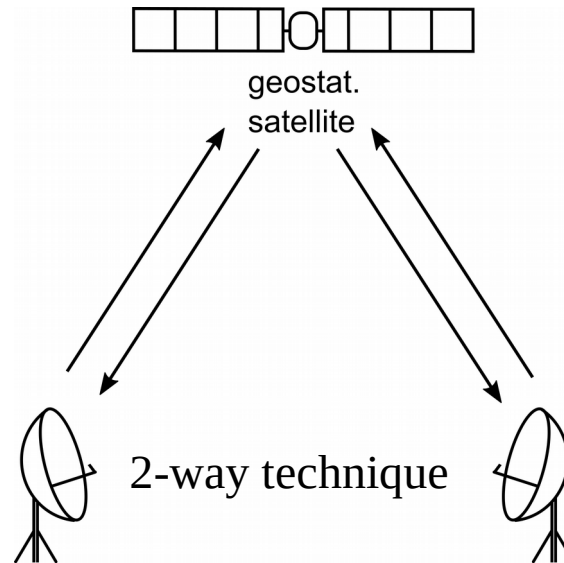
Navigation satellites:



Precise Point Positioning (PPP): best openly available technique

→ no significant reduction of instability possible by changes in the setup

Geostationary satellites:



Instability depends significantly on signal properties set by the station (signal power, modulation bandwidth)

→ try out highest modulation bandwidth possible with equipment

Introduction

Frequency comparison with satellites: **Two-Way Satellite Time and Frequency Transfer (TWSTFT)**

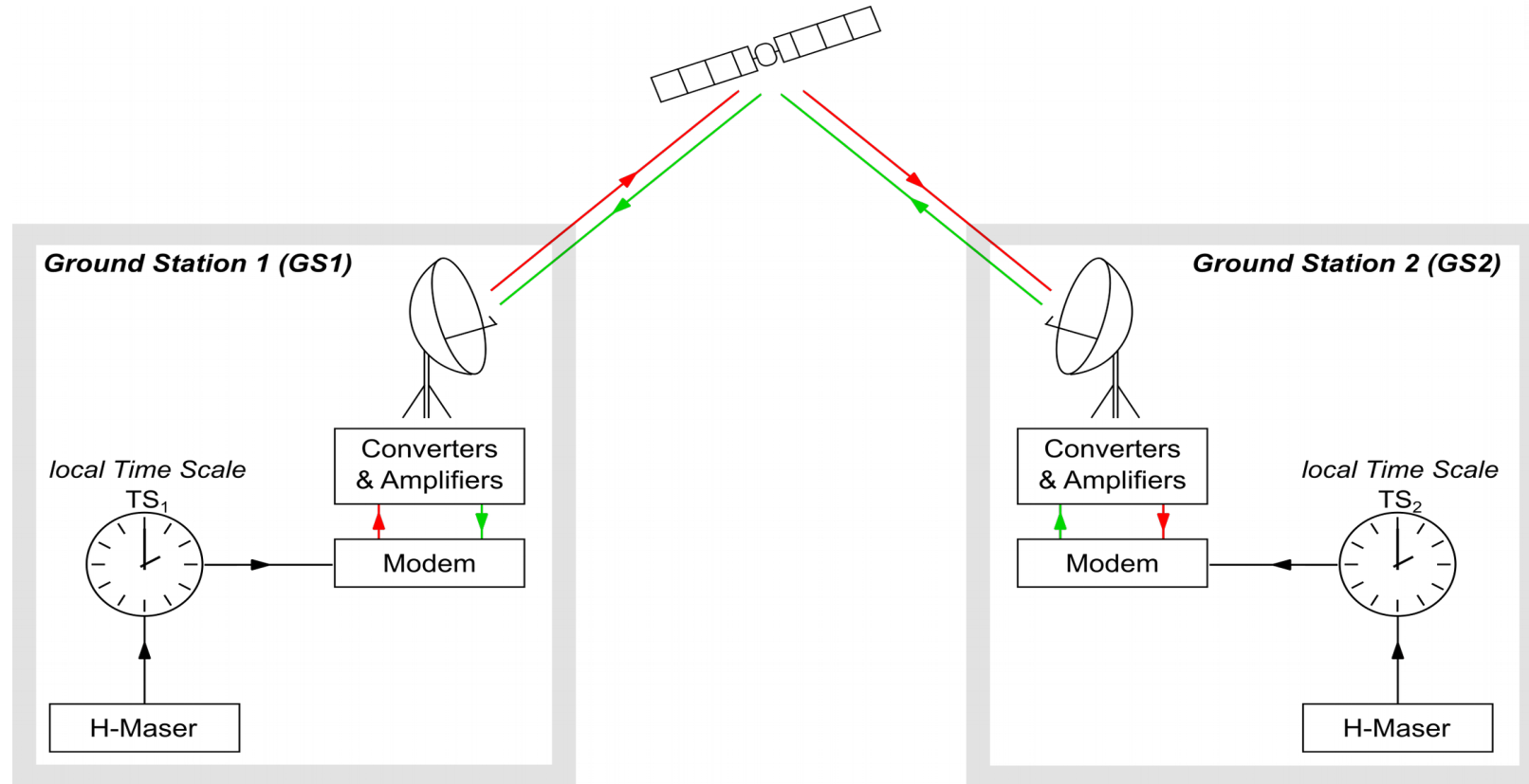
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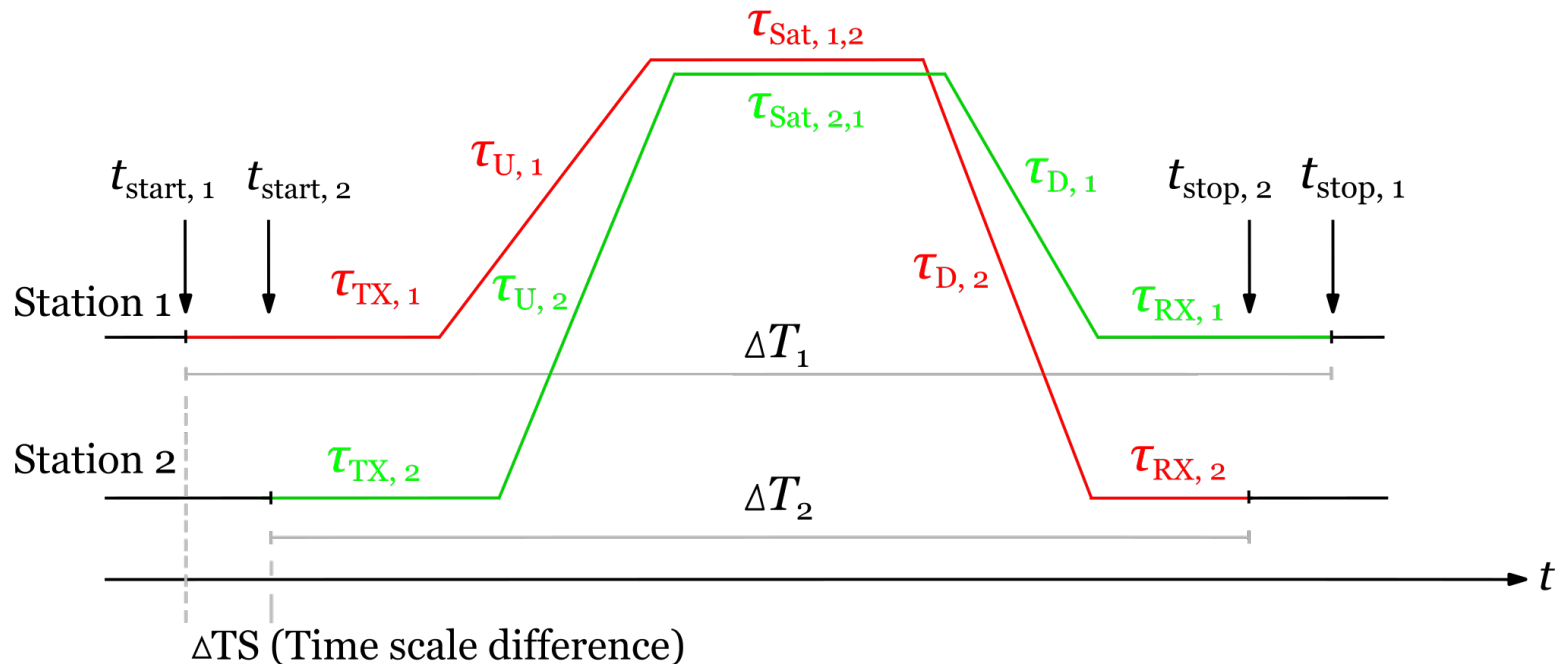
Summary and outlook

Two-Way Satellite Time and Frequency Transfer



Two-Way Satellite Time and Frequency Transfer

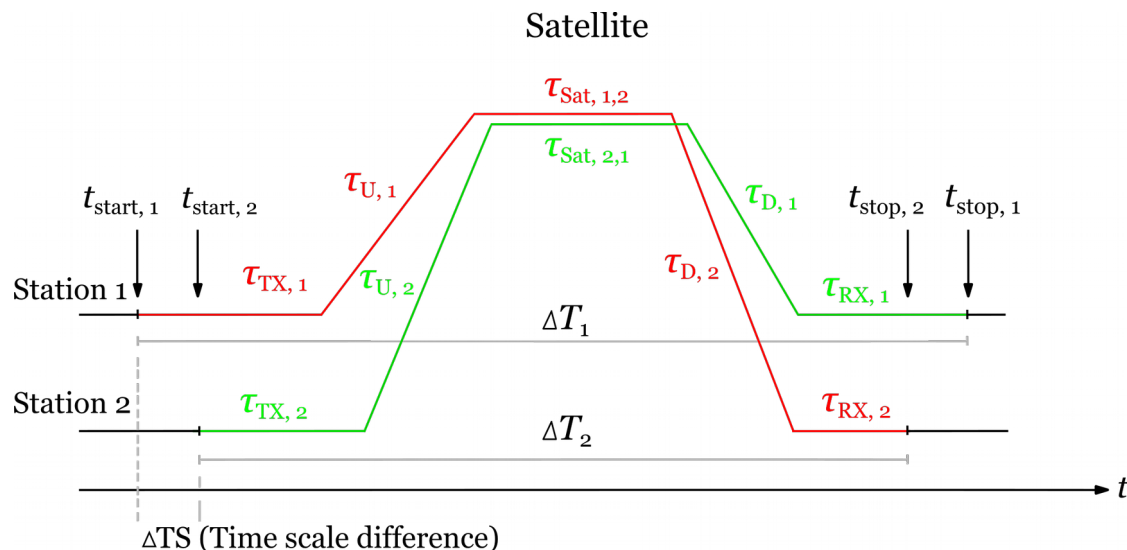
Satellite



$$\Delta T_1 = \Delta \text{TS} + \tau_{\text{TX}, 2} + \tau_{U, 2} + \tau_{21} + \tau_{D, 1} + \tau_{\text{RX}, 1}$$

$$\Delta T_2 = -\Delta \text{TS} + \tau_{\text{TX}, 1} + \tau_{U, 1} + \tau_{12} + \tau_{D, 2} + \tau_{\text{RX}, 2}$$

Two-Way Satellite Time and Frequency Transfer



$$\Delta T_1 = \Delta \text{TS} + \tau_{\text{TX},2} + \tau_{\text{U},2} + \tau_{\text{D},1} + \tau_{\text{RX},1}$$

$$\Delta T_2 = -\Delta \text{TS} + \tau_{\text{TX},1} + \tau_{\text{U},1} + \tau_{\text{D},2} + \tau_{\text{RX},2}$$

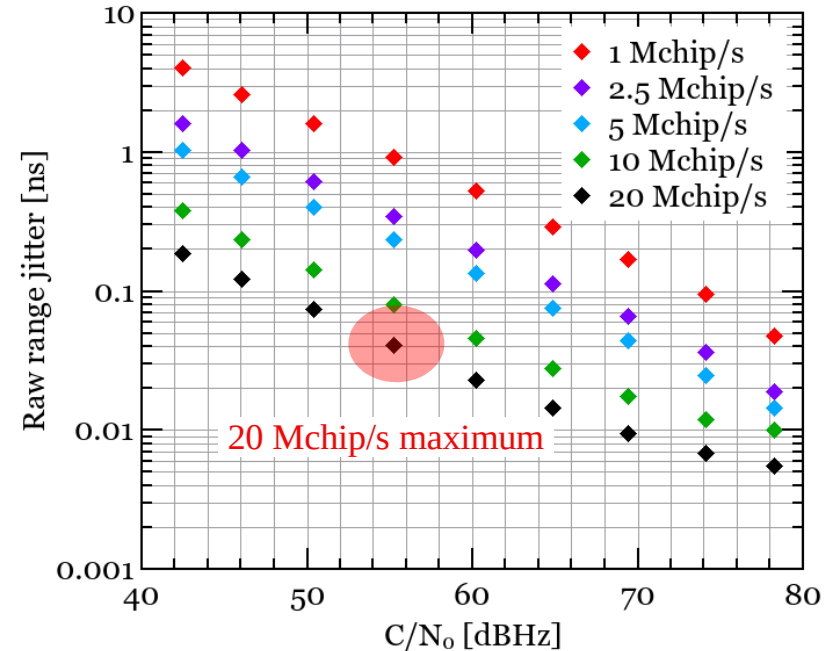
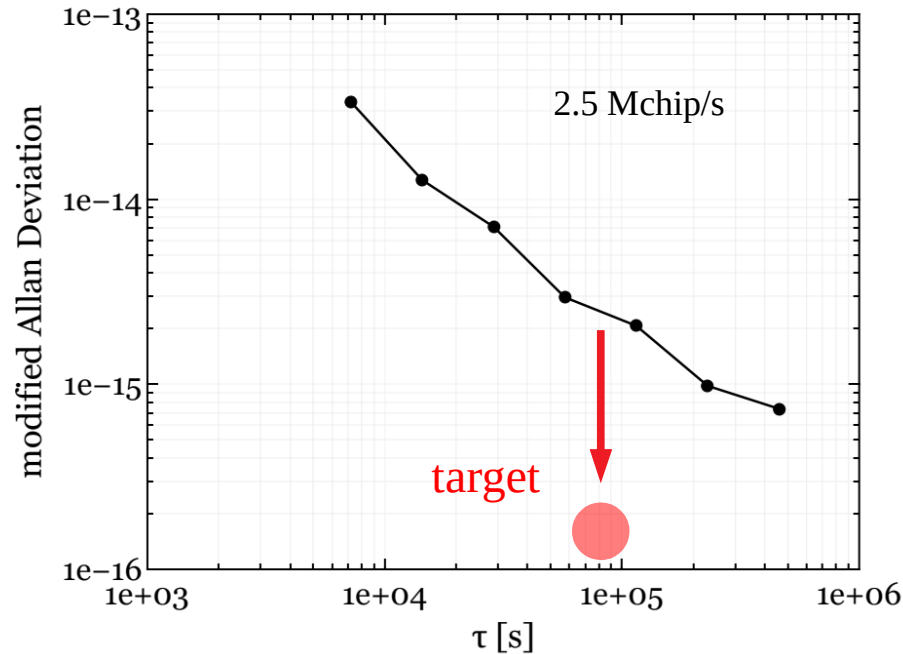
$$\begin{aligned} \Delta \text{TS} = & \frac{1}{2} [\Delta T_1 - \Delta T_2 \\ & - (\tau_{\text{sat},2,1} - \tau_{\text{sat},1,2}) - (\tau_{\text{U},2} - \tau_{\text{D},2}) - (\tau_{\text{D},1} - \tau_{\text{U},1}) - (\tau_{\text{TX},2} + \tau_{\text{RX},1} - \tau_{\text{TX},1} - \tau_{\text{RX},2})] \\ & \underbrace{\hspace{10em}}_{= 0 \text{ for reciprocal path}} \quad \underbrace{\hspace{10em}}_{= \text{const. (calibrated)}} \end{aligned}$$

Frequency comparison: only $d(\Delta \text{TS})/dt \rightarrow d\tau_x/dt$

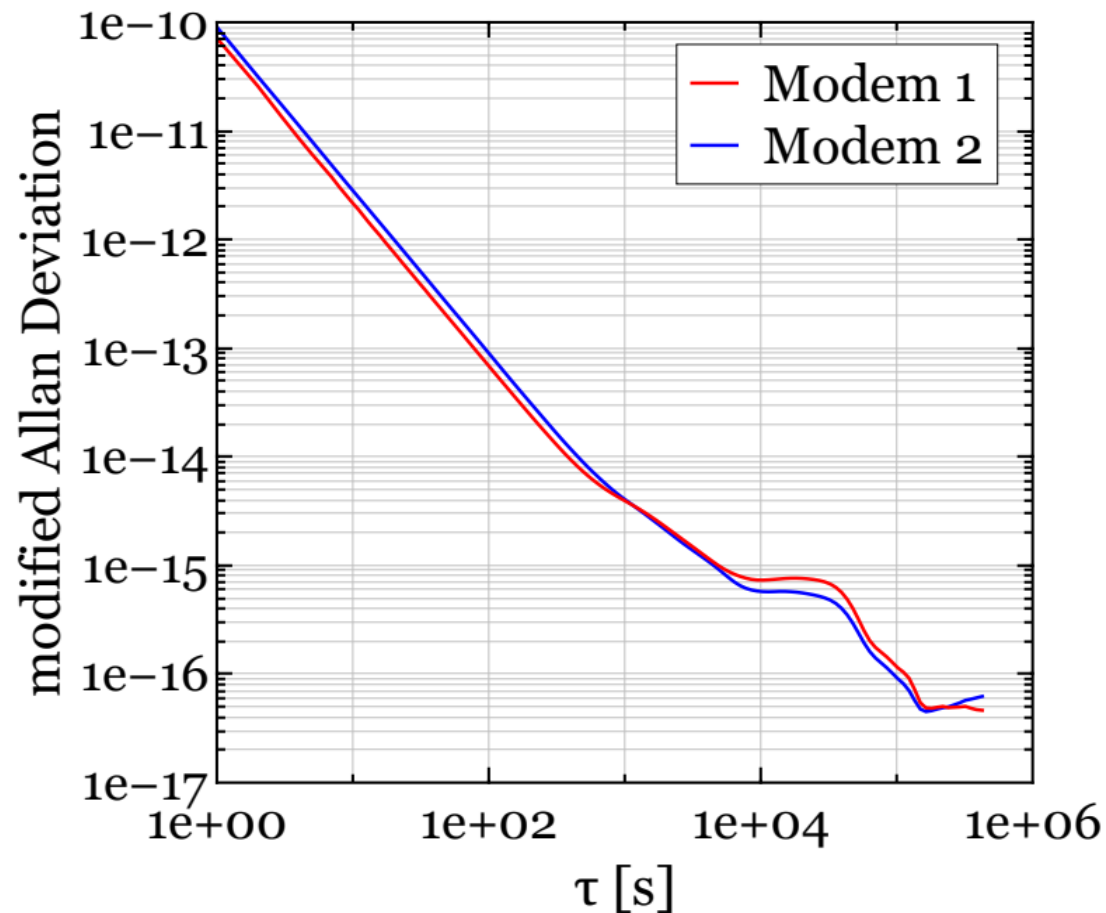
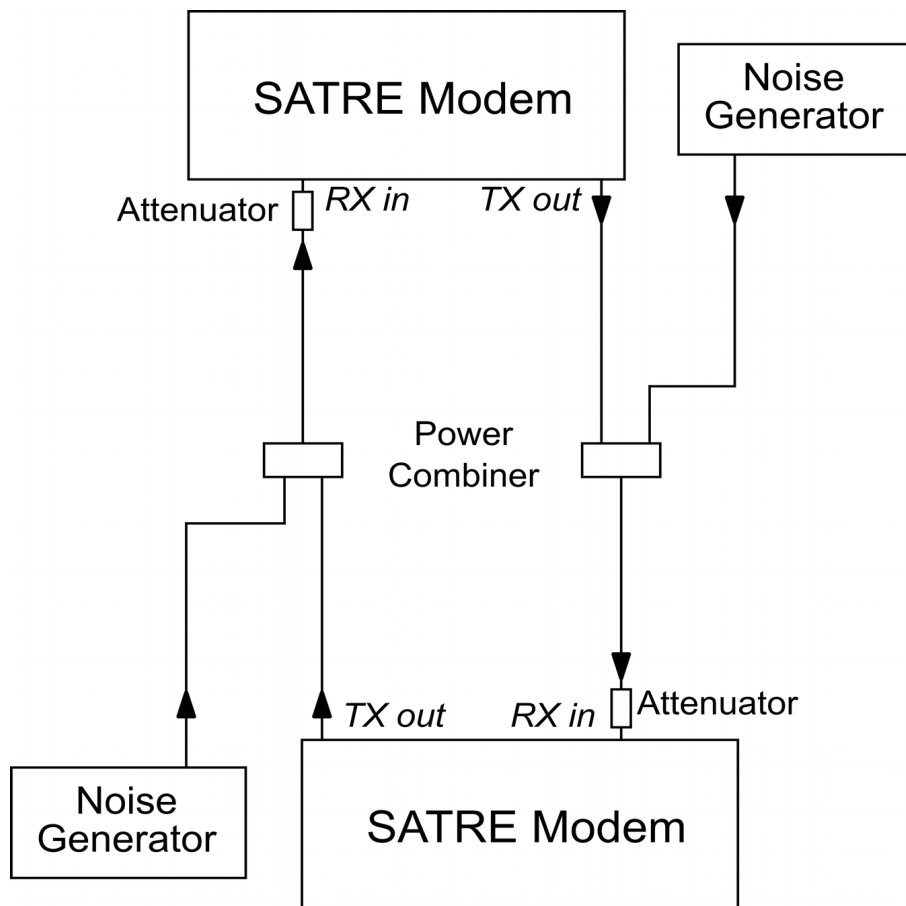
Modem performance: measurements at high bandwidth



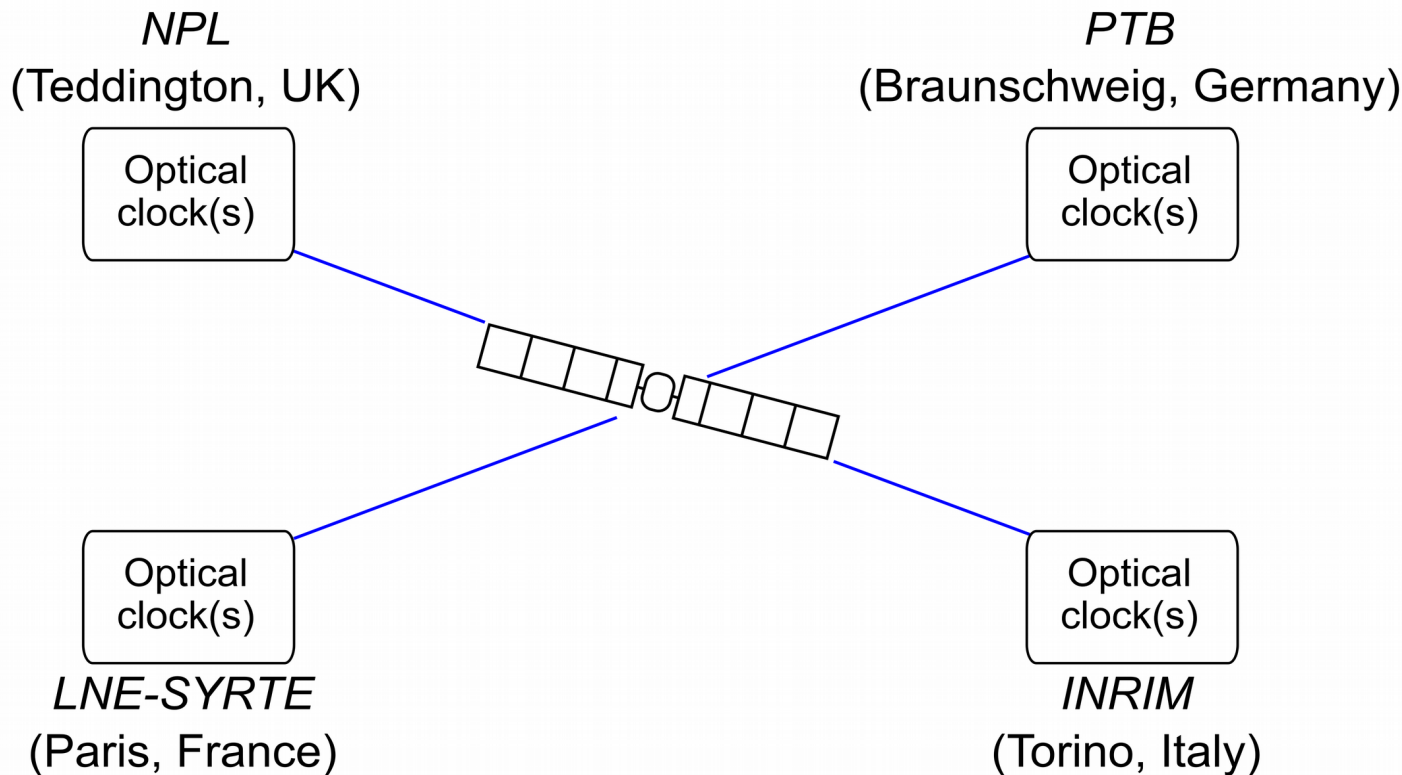
Each laboratory: modem from same manufacturer



SATRE Modem tests: long-term instability at $C/N_0 = 55$ dBHz



Goal: clock comparison with high-bandwidth TWSTFT



For the first time:

- TWSTFT with highest bandwidth
- International simultaneous clock comparison between > 2 countries and > 2 optical clocks
- Optical clock comparison for ~ 3 weeks

Introduction

Frequency comparison with satellites: Two-Way Satellite Time and Frequency Transfer (TWSTFT)

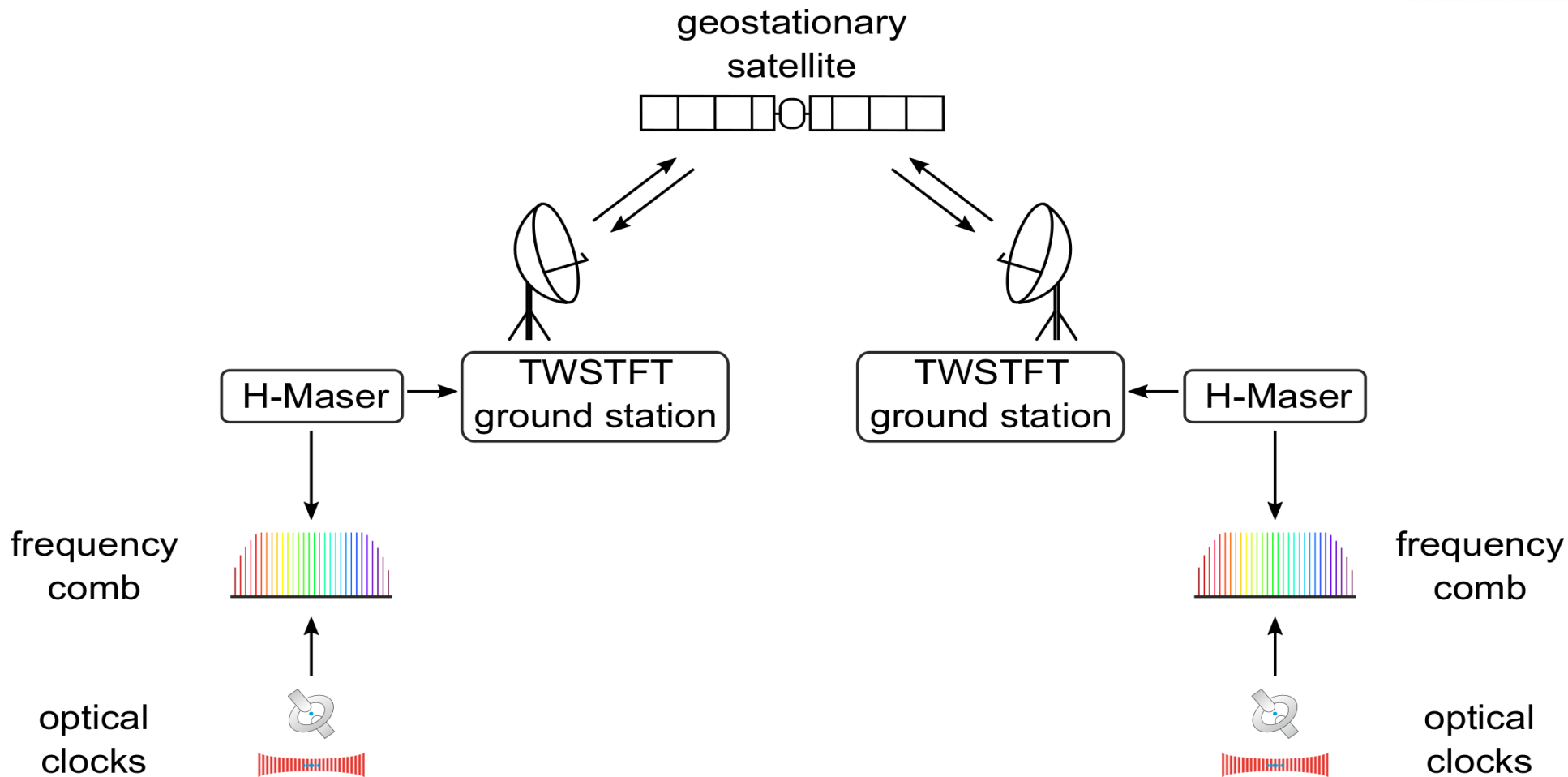
Planning and execution of the clock comparison campaign

Data analysis

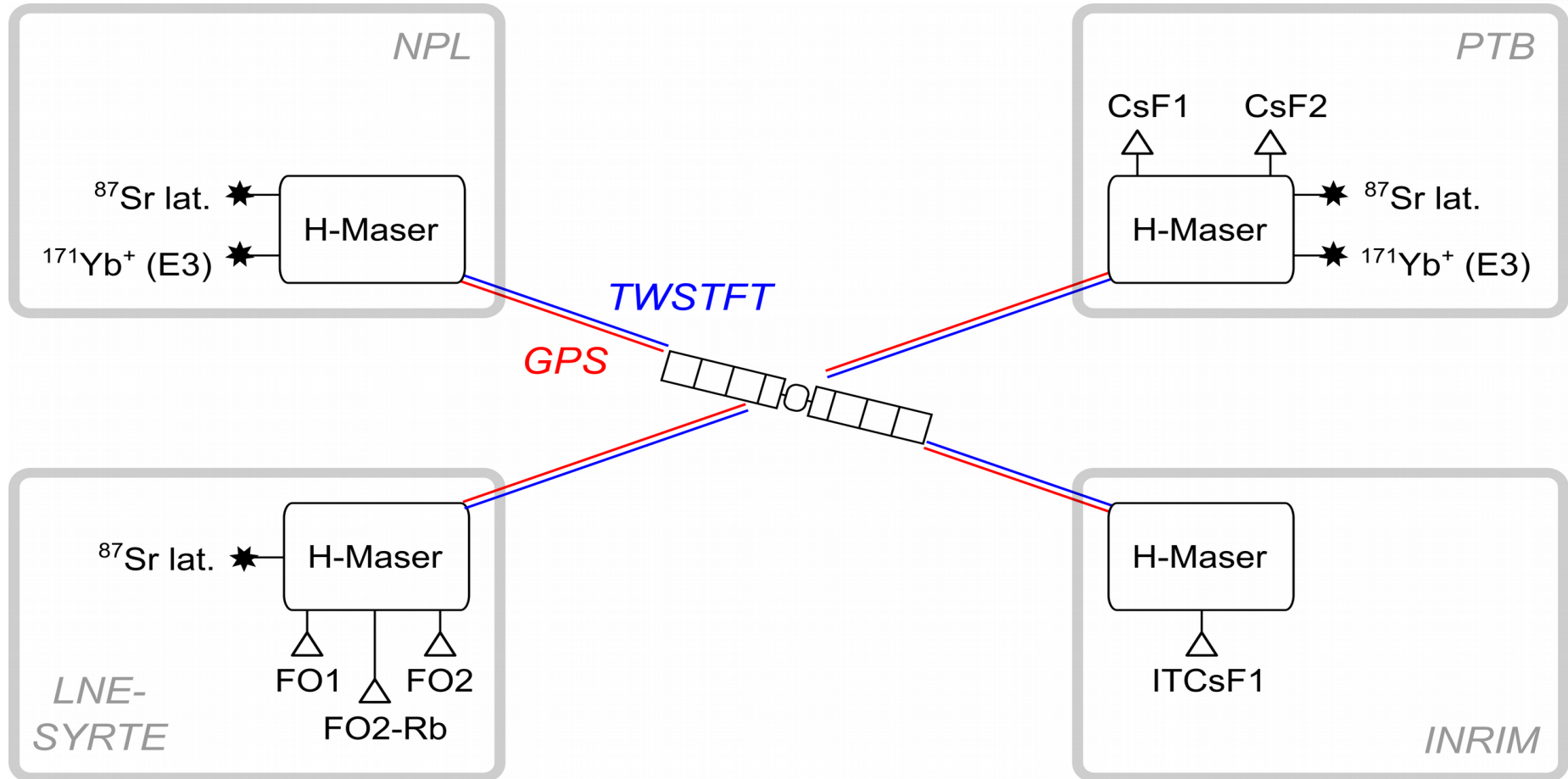
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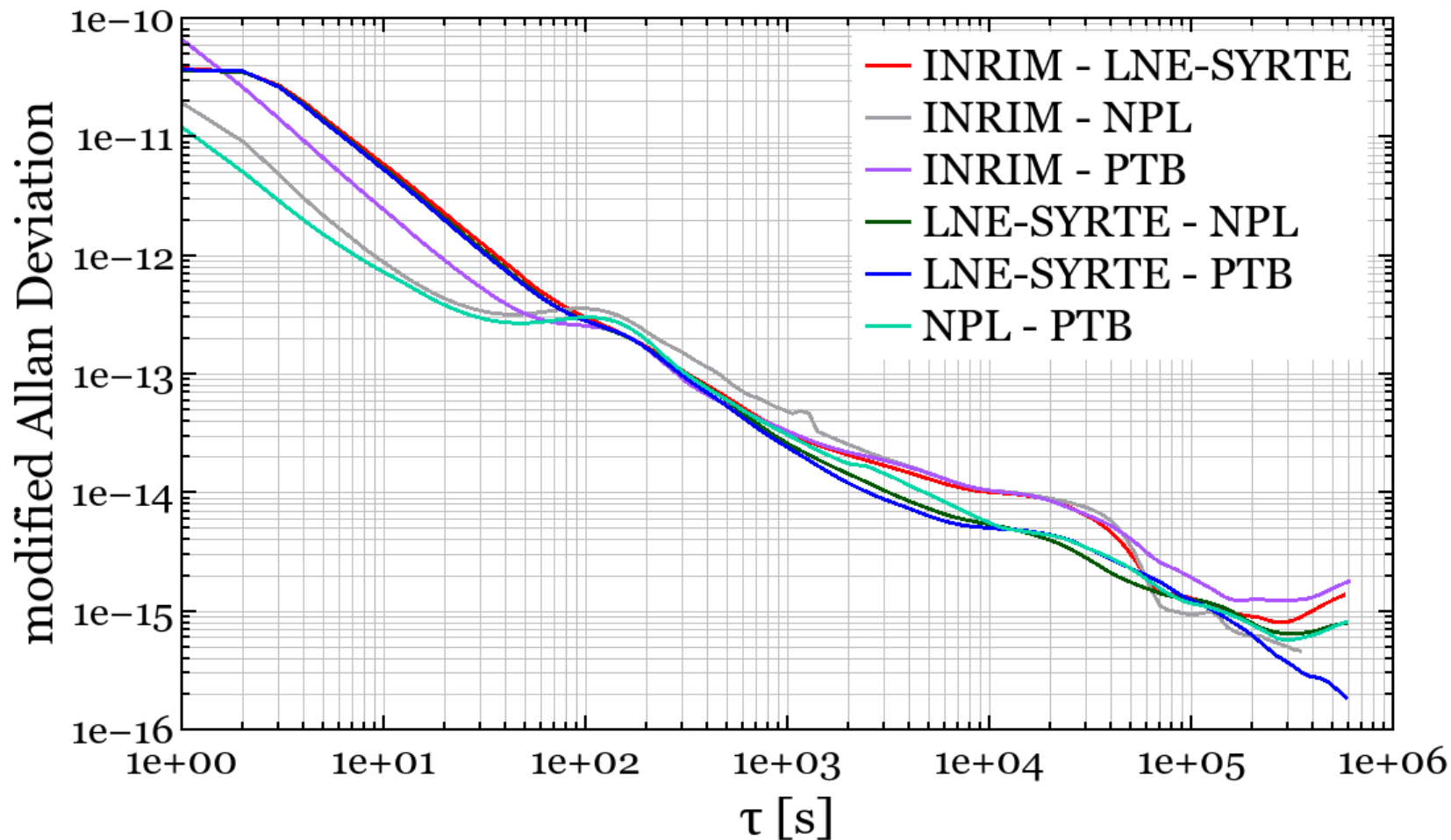
Comparison of optical clocks: Setup



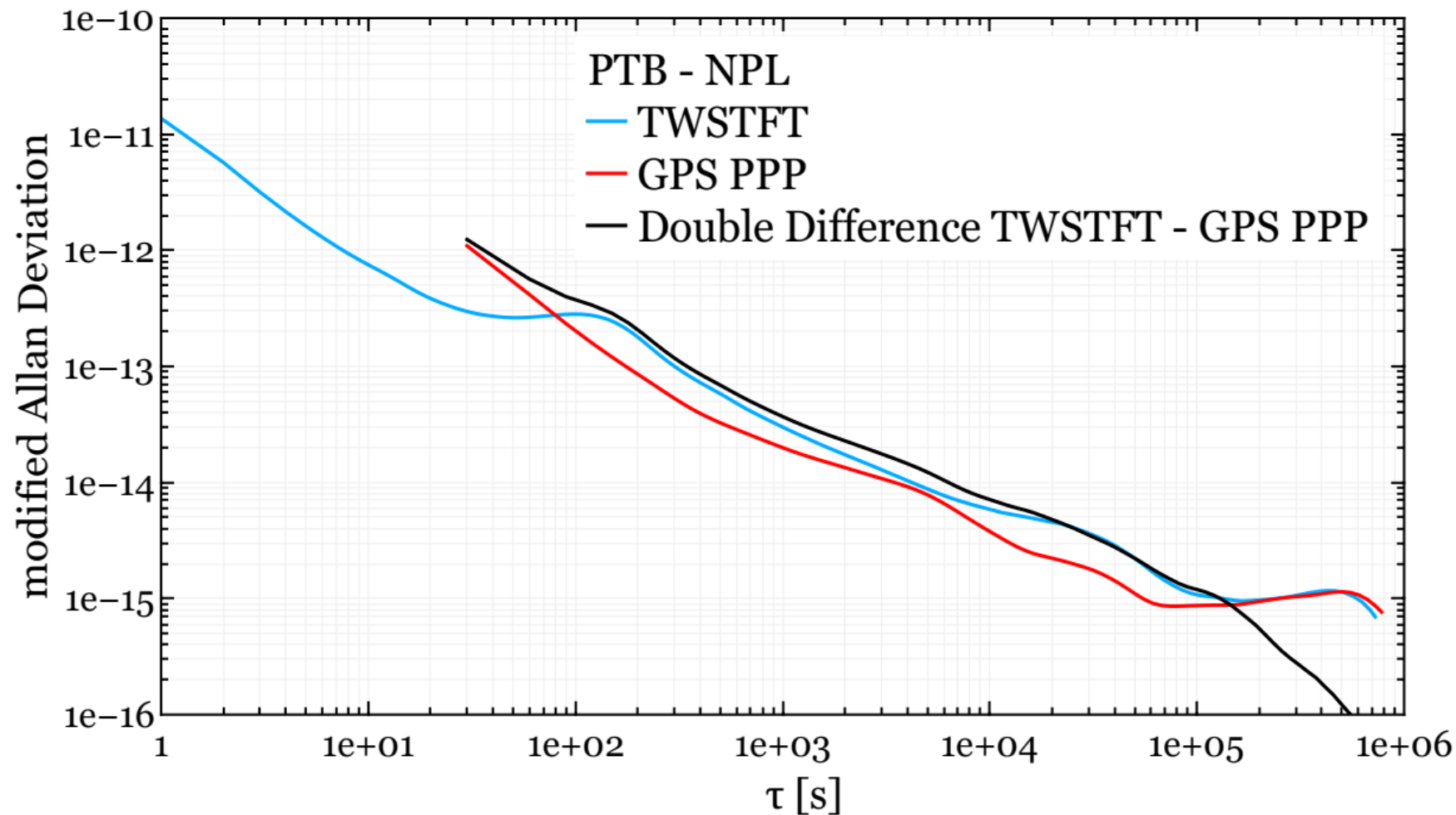
Clock comparison campaign: overview



Clock comparison campaign: TWSTFT instability



Clock comparison campaign: TWSTFT and GPS PPP



Introduction

Frequency comparison with satellites: Two-Way Satellite Time and Frequency Transfer (TWSTFT)

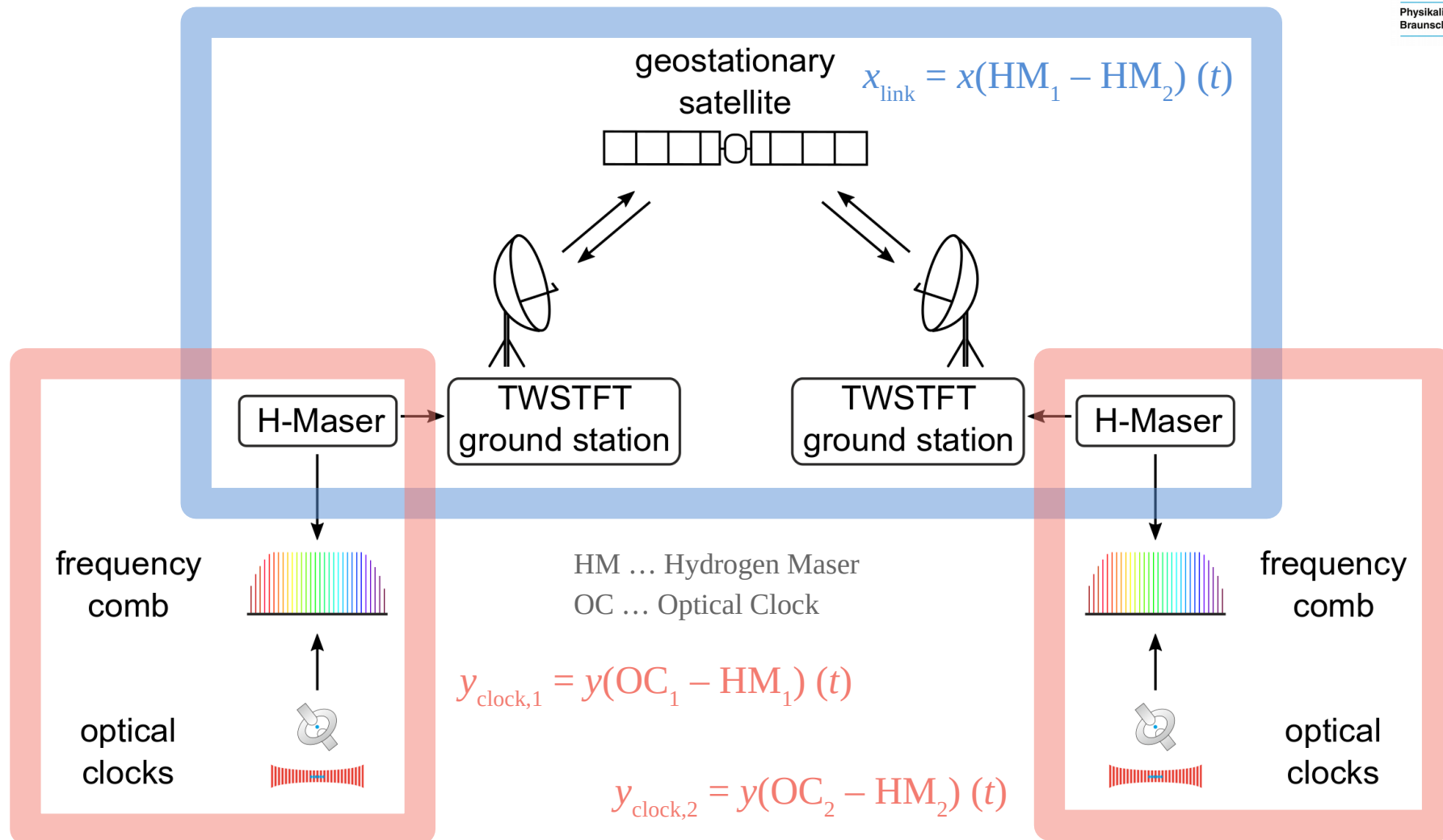
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Calculate relative frequency differences of optical clocks



Calculate relative frequency differences of optical clocks

$$x_{\text{link}}(t) = x(\text{HM}_1 - \text{HM}_2)(t)$$

$$y_{\text{clock},1}(t) = y(\text{OC}_1 - \text{HM}_1)(t)$$

$$y_{\text{link}}(t) = \frac{x_{\text{link}}(t + \frac{\Delta t}{2}) - x_{\text{link}}(t - \frac{\Delta t}{2})}{\Delta t}$$

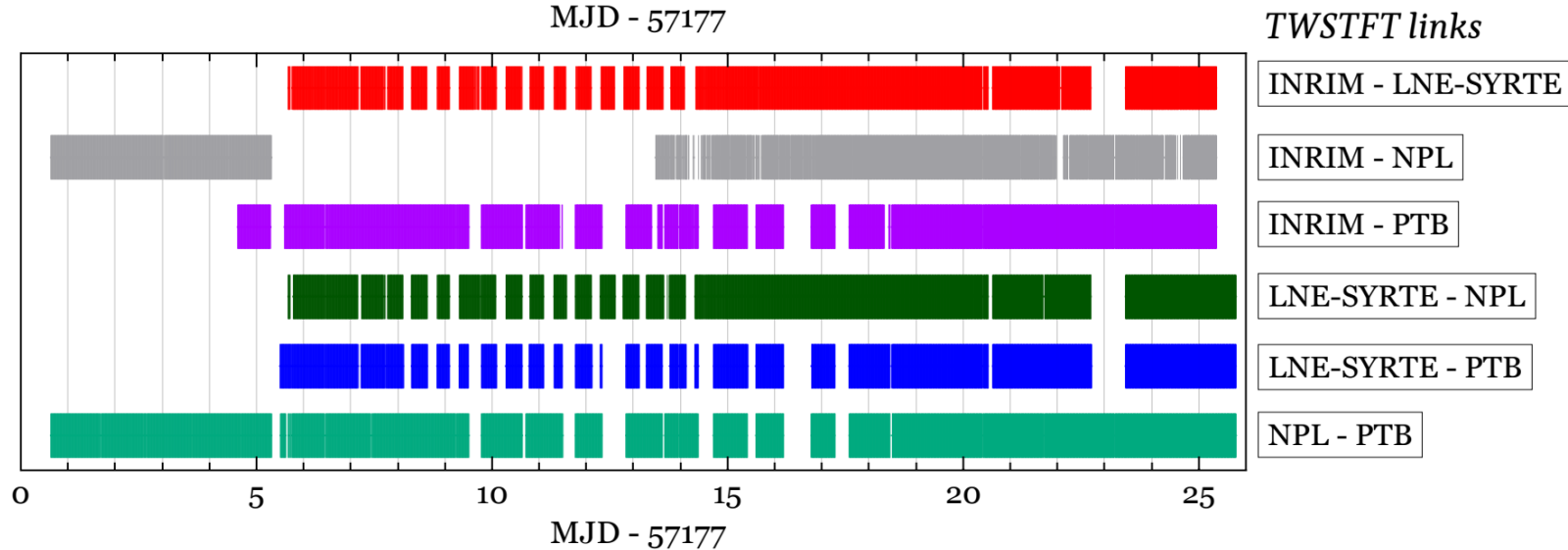
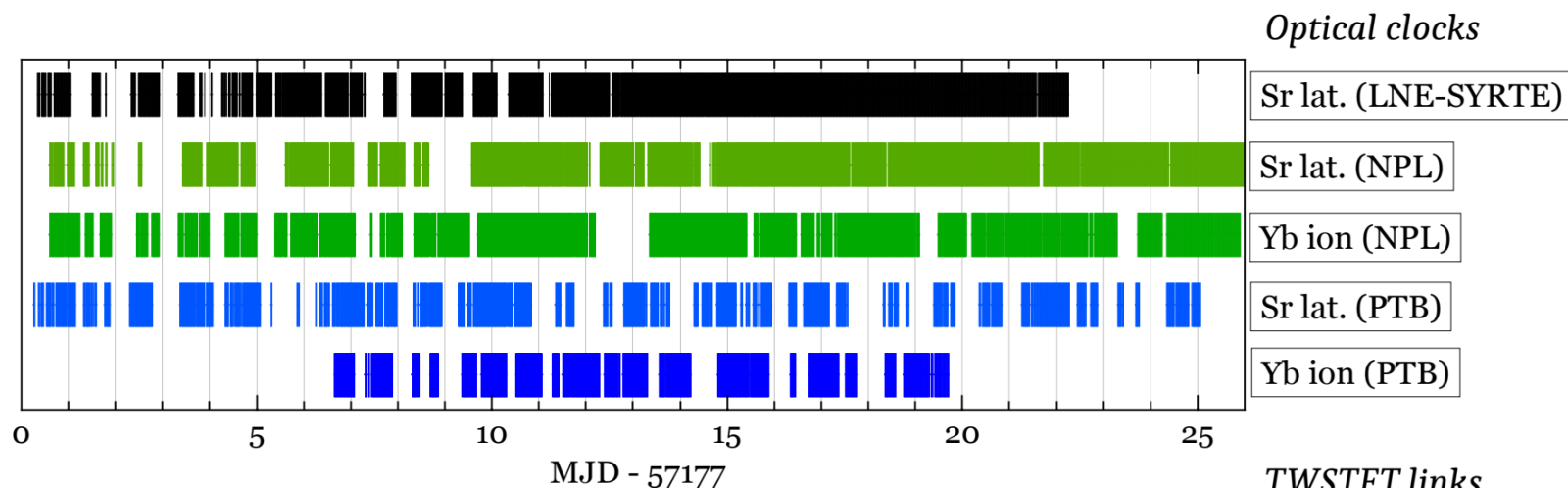
$$y_{\text{clock},2}(t) = y(\text{OC}_2 - \text{HM}_2)(t)$$

$$= y(\text{HM}_1 - \text{HM}_2)(t)$$

$$\rightarrow y(\text{OC}_1 - \text{OC}_2)(t) = y_{\text{clock},1}(t) - y_{\text{clock},2}(t) + y_{\text{link}}(t)$$

But: $y_{\text{clock},x}(t)$ dominated by noise of HM, $y_{\text{link}}(t)$ by phase noise of link

Calculate relative frequency differences of optical clocks



Calculate relative frequency differences of optical clocks

Find a compromise for:

- Minimize phase noise on link data with pre-averaging
- Use only overlapping data to have the HMs cancel out
- Discard as few data as possible

→ obtain $y(\text{OC}_1 - \text{OC}_2)(t) = y_{\text{clock},1}(t) - y_{\text{clock},2}(t) + y_{\text{link}}(t)$

→ calculate weighted mean

Calculate relative frequency differences of optical clocks

$$\bar{y} = \sum_{i=1}^n w_i \cdot y_i \quad \text{with} \quad \sum_{i=1}^n w_i = 1 \quad \text{gaps: } y(t_i) = [] \rightarrow w(t_i) = 0$$

Uncertainty? $u = \sqrt{u_{A, \text{clocks}}^2 + u_{A, \text{link}}^2 + u_{B, \text{clocks}}^2 + u_{B, \text{link}}^2}$

$u_{B, \text{link}}$: contributions estimated from the temperature measured during the campaign

u_A = square root of estimator for variance of the mean $s_{\bar{y}}^2$

$$s_{\bar{y}}^2 = \sum_{i=1}^n w_i^2 \sum_{i=1}^n w_i (y_i - \bar{y})^2$$

But:

non-white noise on data, gaps on data: biased estimator for standard deviation of the mean

Calculate relative frequency differences of optical clocks

reduced biased:

$$s_{\bar{y}}^2 = R_0 \sum_{i=1}^n w_i^2 + 2 \sum_{l=1}^{l_{\text{cut}}} R_l \sum_{j=1}^{l_{\text{cut}}} w_j w_{j+l}$$

with

$$R_l = \frac{\sum_{i=1}^{N-l} \sqrt{w_i w_{i+l}} (y_i - \bar{y})(y_{i+l} - \bar{y})}{\sum_{i=1}^{N-l} \sqrt{w_i w_{i+l}}}$$

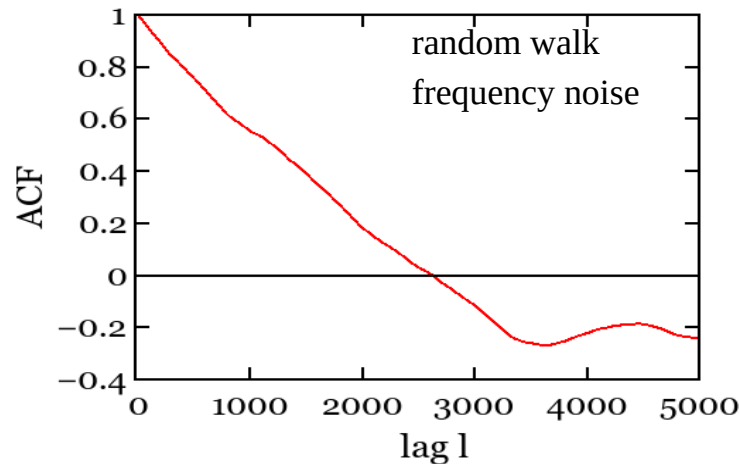
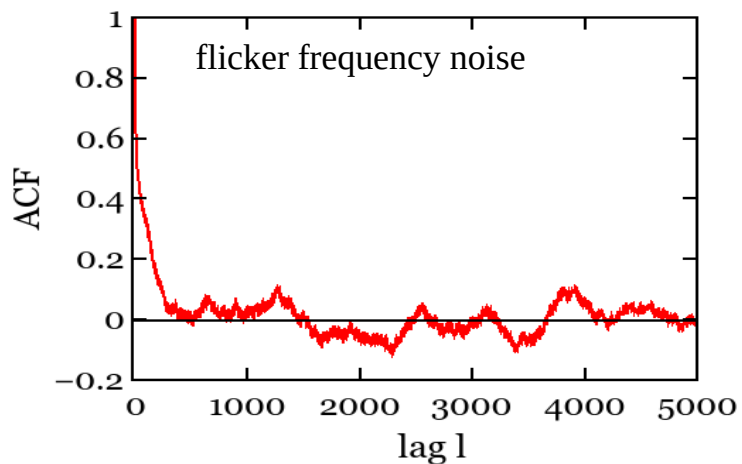
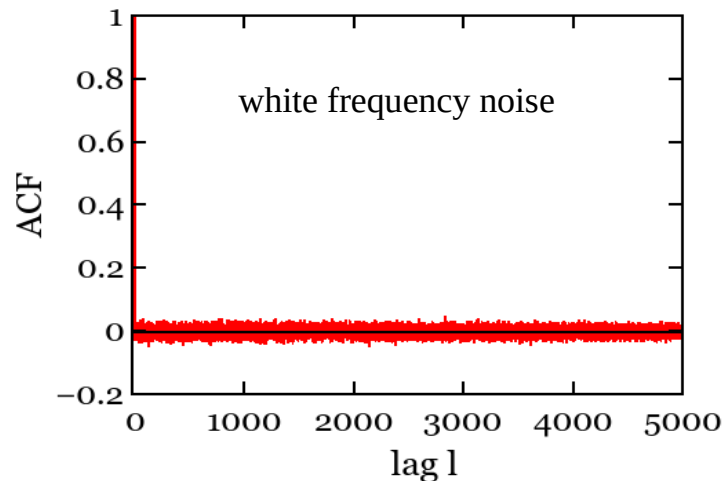
Autocorrelation function (ACF)

$$r_l = \frac{R_l}{R_0}$$

ACF, normalized

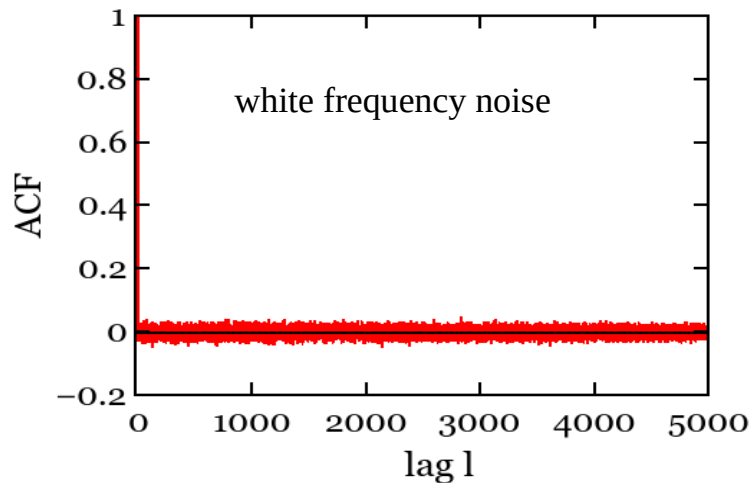
Calculate relative frequency differences of optical clocks

Autocorrelation function (of relative frequency data) at different noise types

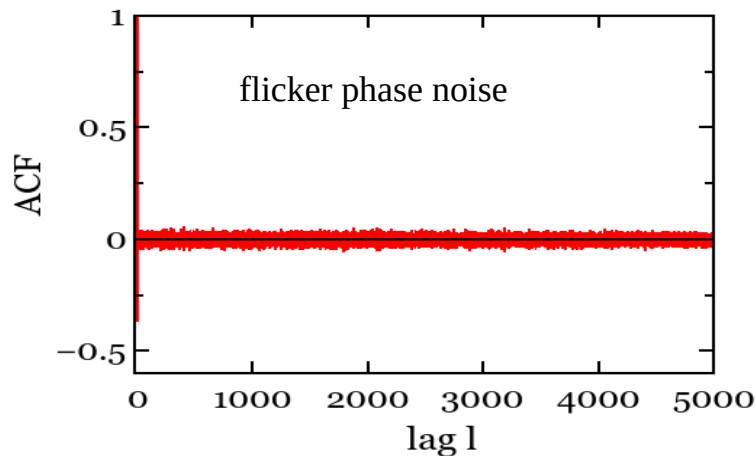
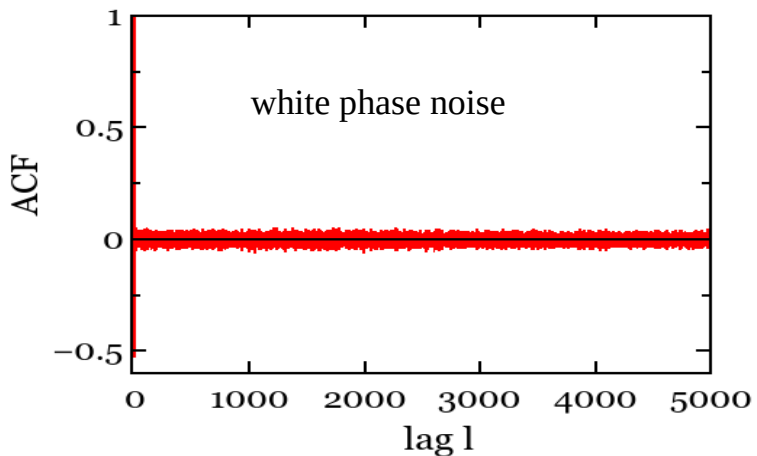


Calculate relative frequency differences of optical clocks

Autocorrelation function (of relative frequency data) at different noise types



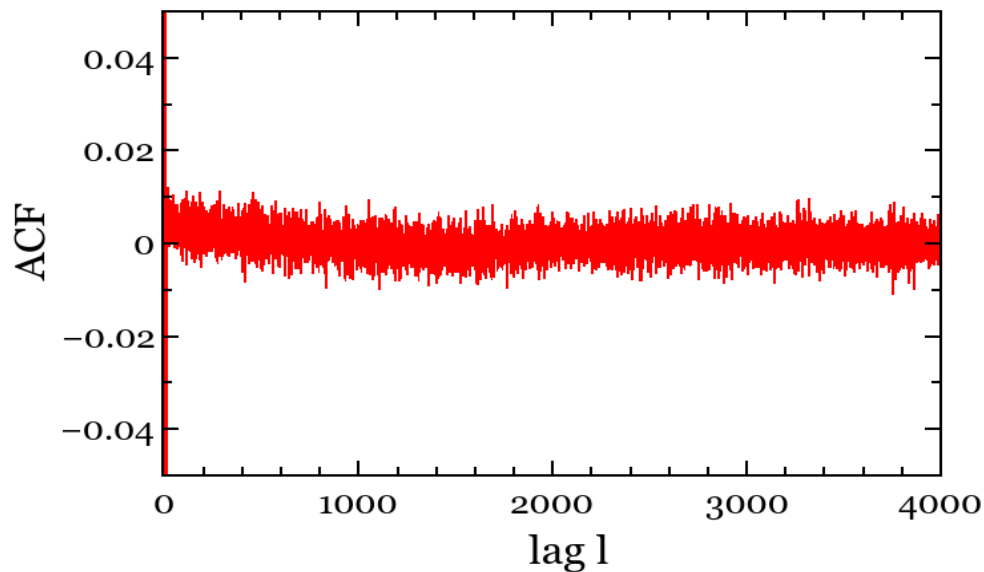
$$s_{\bar{y}}^2 = R_0 \sum_{i=1}^n w_i^2 + 2 \sum_{l=1}^{l_{\text{cut}}} R_l \sum_{j=1}^{l_{\text{cut}}} w_j w_{j+l}$$



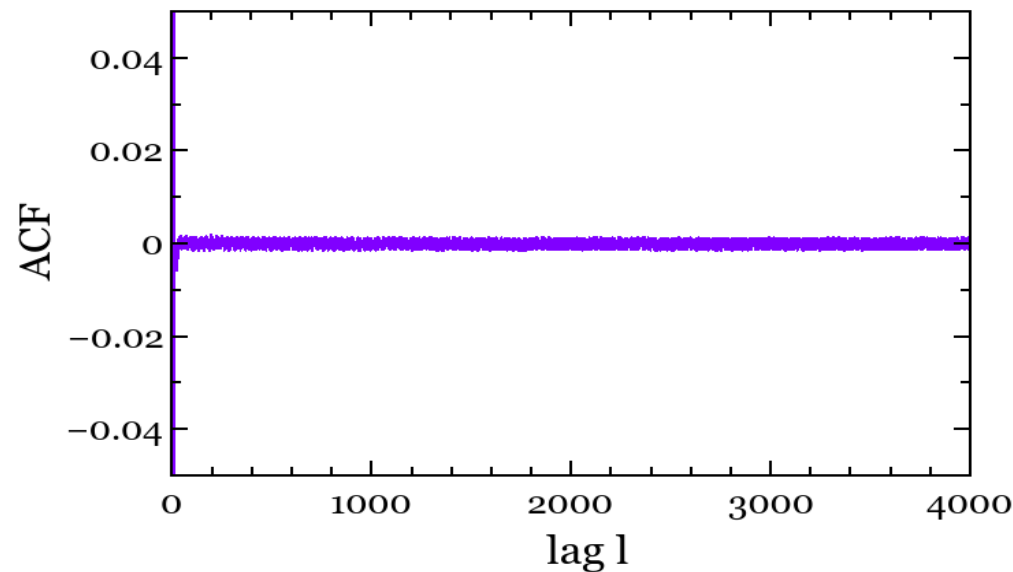
Calculate relative frequency differences of optical clocks

Autocorrelation function of relative frequency data from clock comparisons

$y(\text{OC} - \text{HM})$



$y(\text{OC}_1 - \text{OC}_2)$



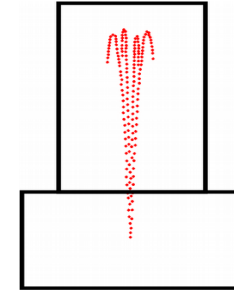
$$l_{\text{cut}} = 4000$$

Clock comparison campaign: Other comparisons

Fountain clocks:

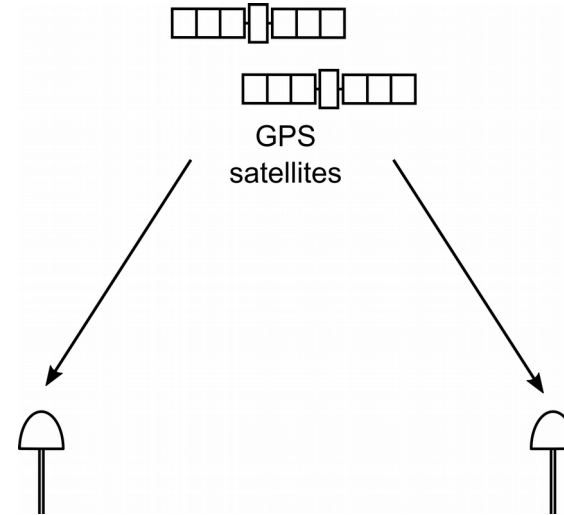
different requirements for calculation

→ pre-average each set of data over certain interval,
use $u_{A, \text{link}}$ of optical clock comparison



GPS:

same as TWSTFT, but on 30s interval



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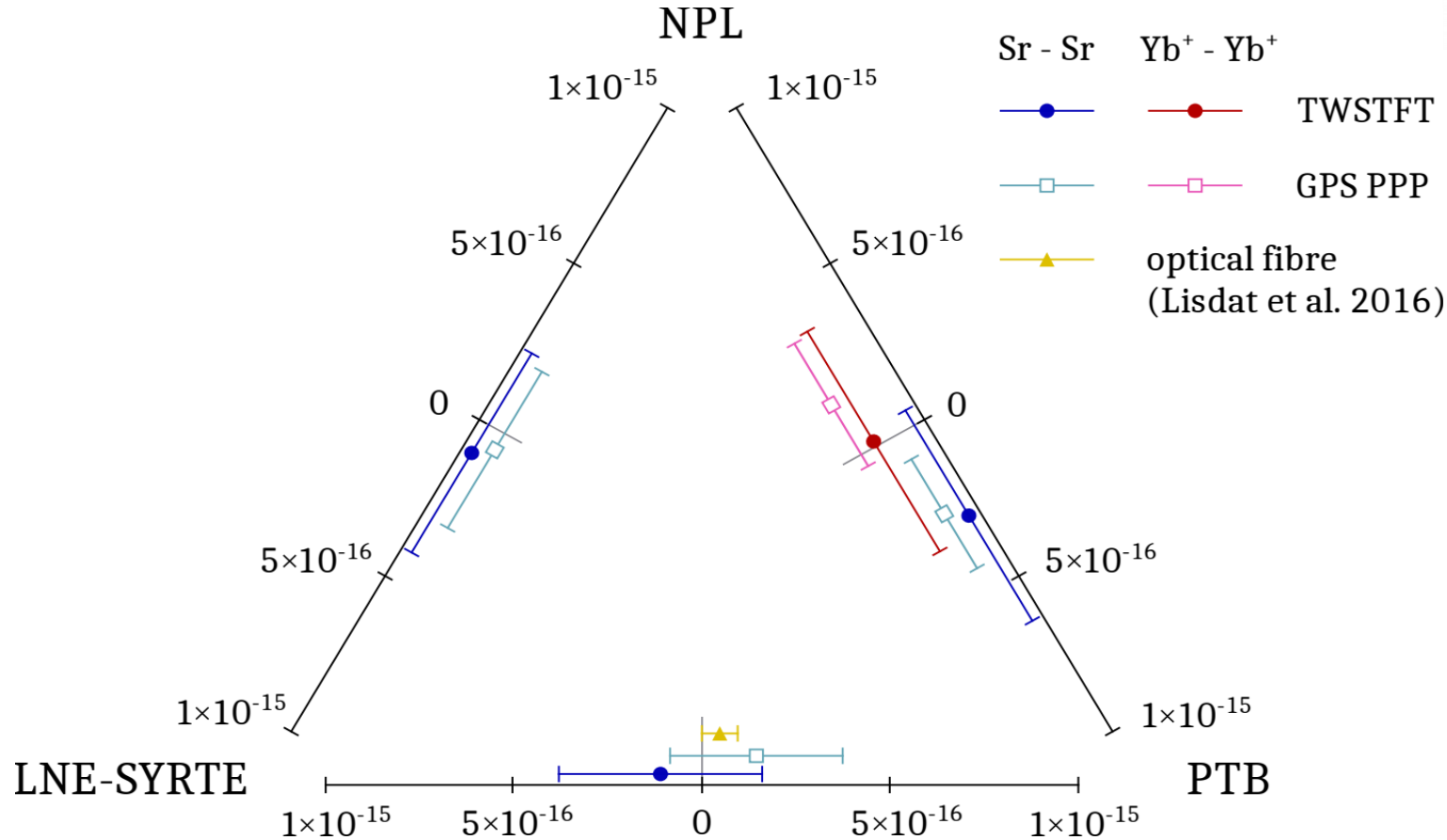
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Results for optical clocks (same type)

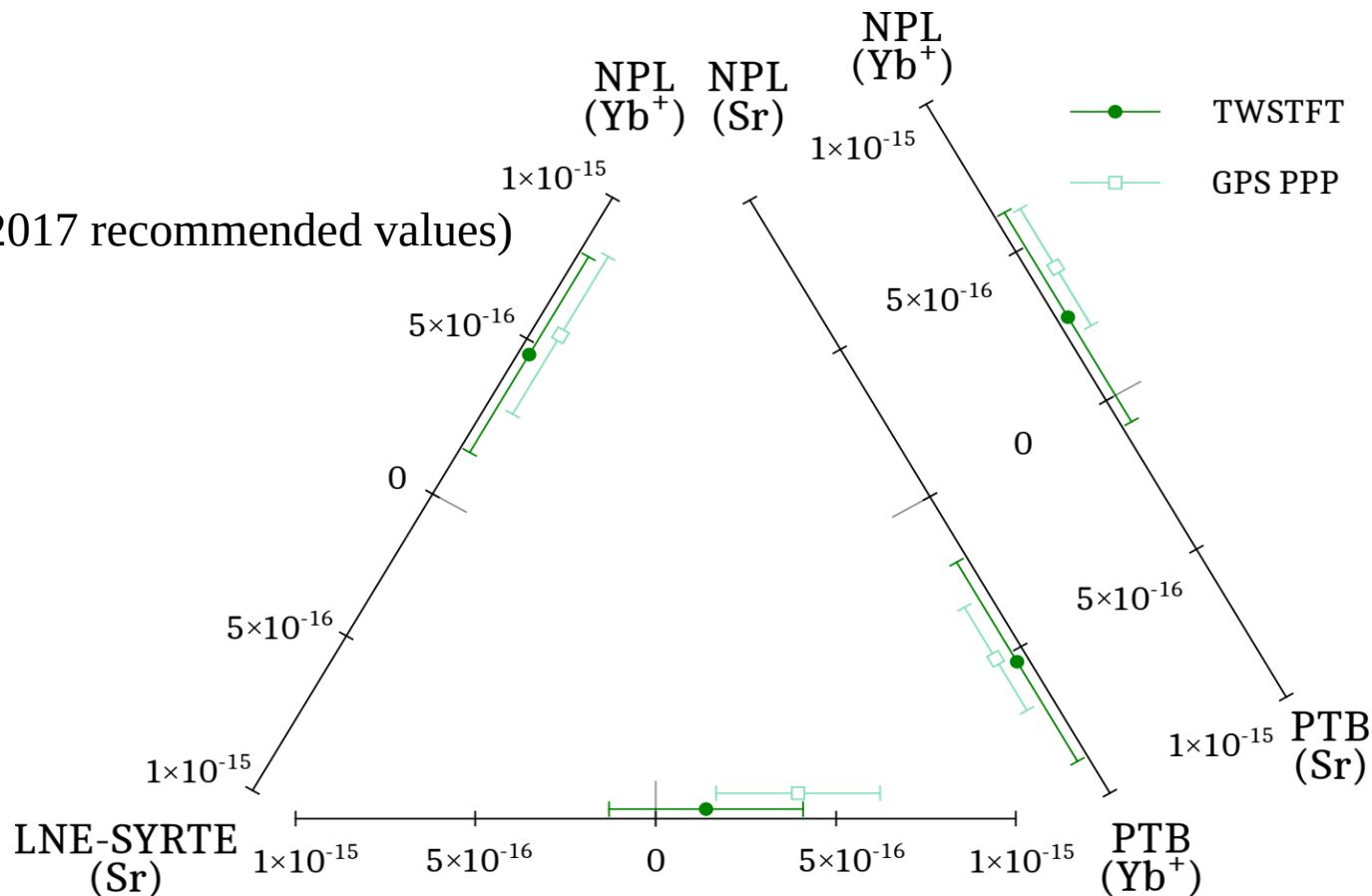


Different clocks: agreement, but slight offset in Sr (NPL) indicated

Different techniques: agreement

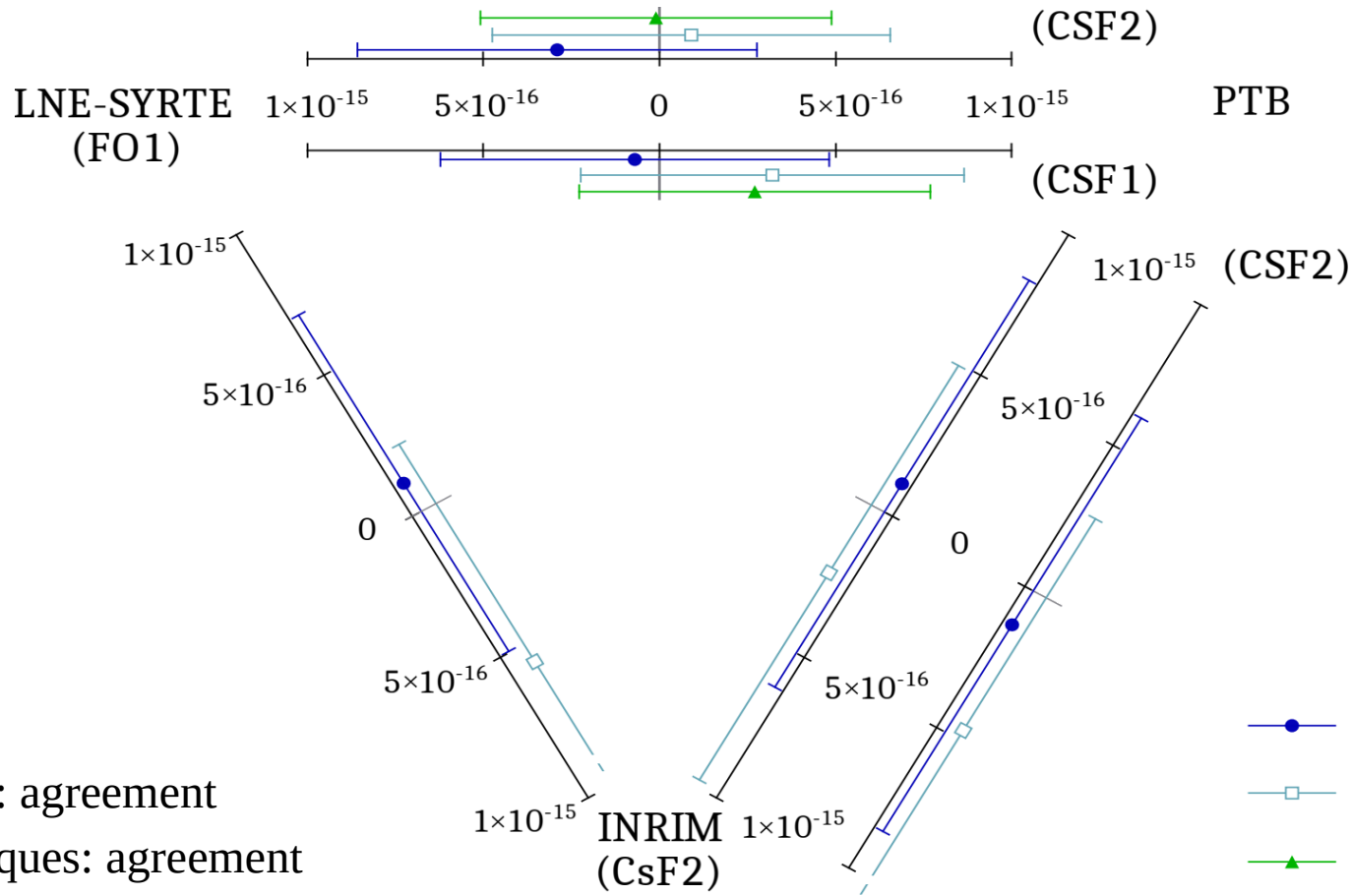
Results for optical clocks (different type)

("0" = CIPM 2017 recommended values)



Different clocks: agreement (combined uncertainty), but systematic offset
Different techniques: agreement

Results for fountain clocks (part 1)

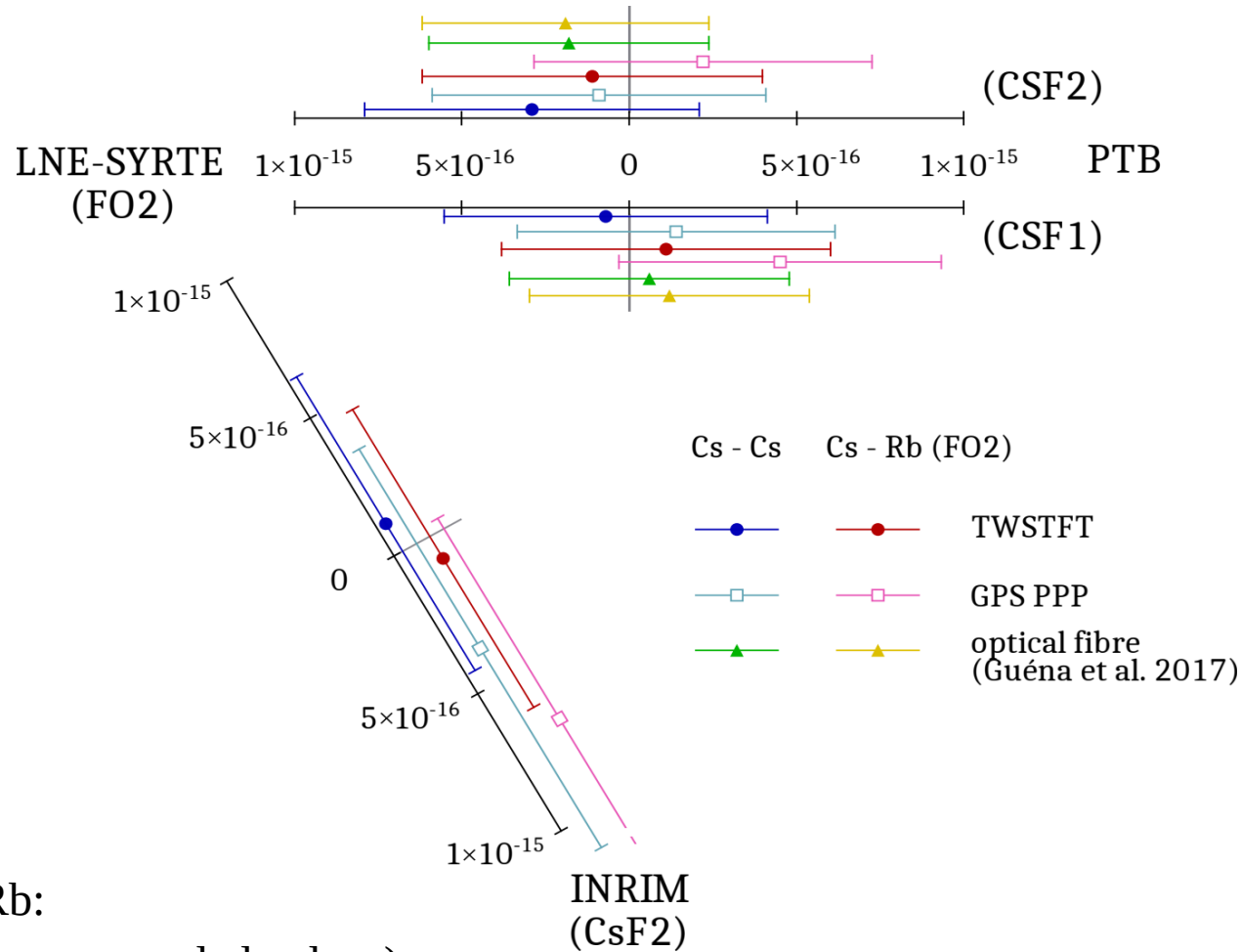


Fountain clocks: agreement

Different techniques: agreement

Larger uncertainties: larger contributions by the clocks to u_A and u_B

Results for fountain clocks (part 2)



Comparisons with Rb:
("0" = CIPM 2017 recommended values)

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Summary

- First simultaneous international optical clock comparison (> 2 clocks)
- Characterization of TWSTFT equipment, Analysis of effects impacting precise TWSTFT measurements
 - Corrections (relativistic/ionospheric) required for uncertainty of $1 \cdot 10^{-16}$ or lower
- TWSTFT improves respectively with higher chip rate
- Development of analysis procedure taking into account gaps and correlation
- Limitations:
 - disturbances increasing noise/gaps on data (clocks and links)
 - determination of u_B
- Results for clock types of same type agree with each other
- Results of clocks of different type agree with CIPM 2017 recommended values, but indicate an offset
 - advantage of simultaneity

Outlook: suitable techniques for clock comparisons

- Comparison via optical fibers:
Network set up in Europe, but still only very limited baselines available
- Transportable optical clocks:
limitation in uncertainty and operation due to technical compromises,
only subsequent measurements
- Satellite-based techniques:
 - GPS integer PPP (iPPP): lower instability than PPP for averaging times $>$ a few hours
 - TW Carrier Phase: superior to all other satellite-based techniques at short averaging times up to a few hours, but averages with $\tau^{-1/2}$
 - TWSTFT with SDR (software-defined radio): some systematic effects can be suppressed, lower instabilities observed at low chip rates

Acknowledgements

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Thank you for your attention!