Towards the development of an autonomous satellite orbit determination process via Inter-Satellite Links (ISLs)

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Les séminaires temps-fréquence

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SYRTE, Theory and Metrology Group

Motivation

Examine the observability and accuracy of the satellite orbit determination problem:

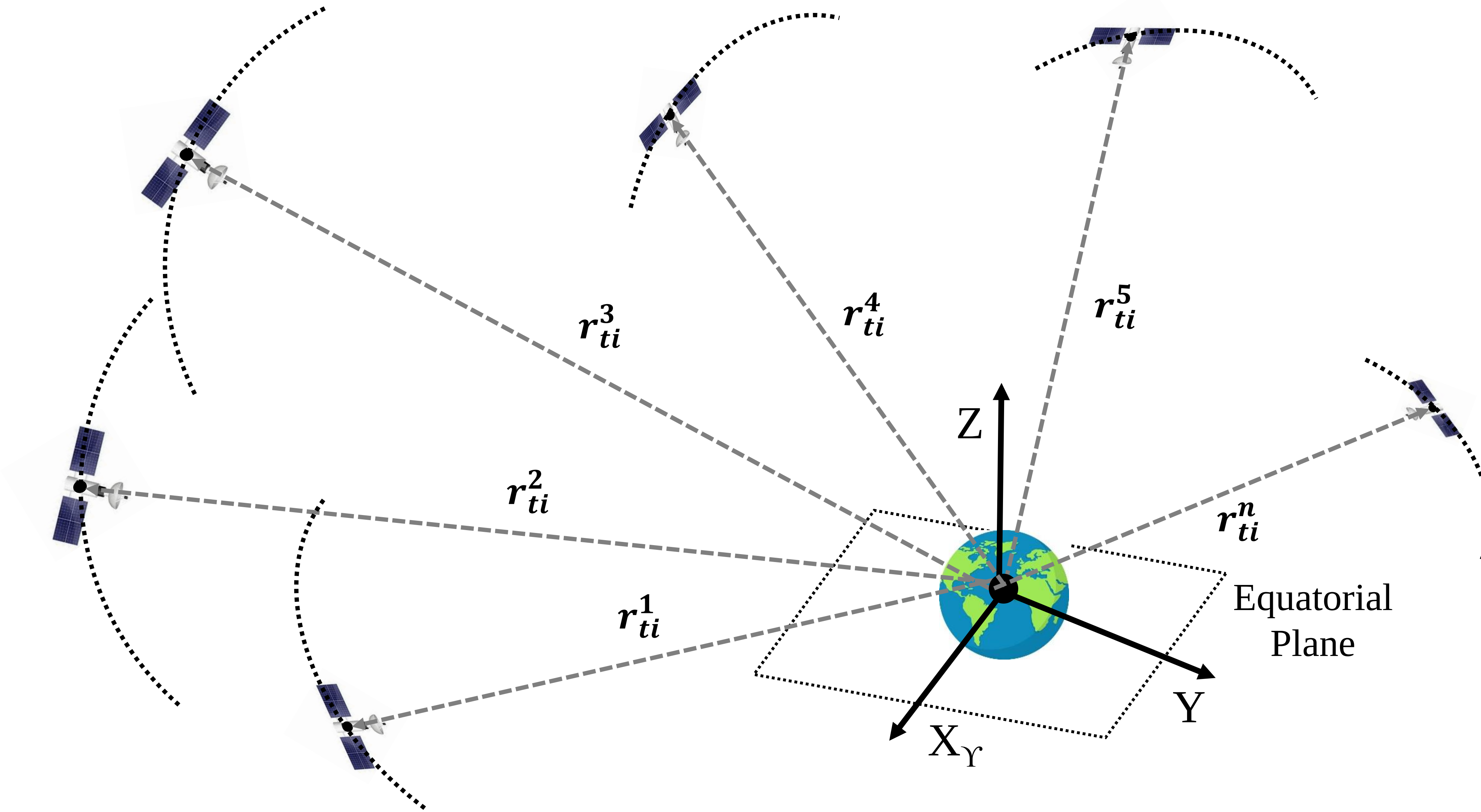
- using only inter-satellite range measurements and
- in constellations consisting of different number of satellites.

considering the impact of:

- the satellite constellation geometry,
- different reference frame definition strategies and
- different parametrization models (Absolute and relative orbital elements).

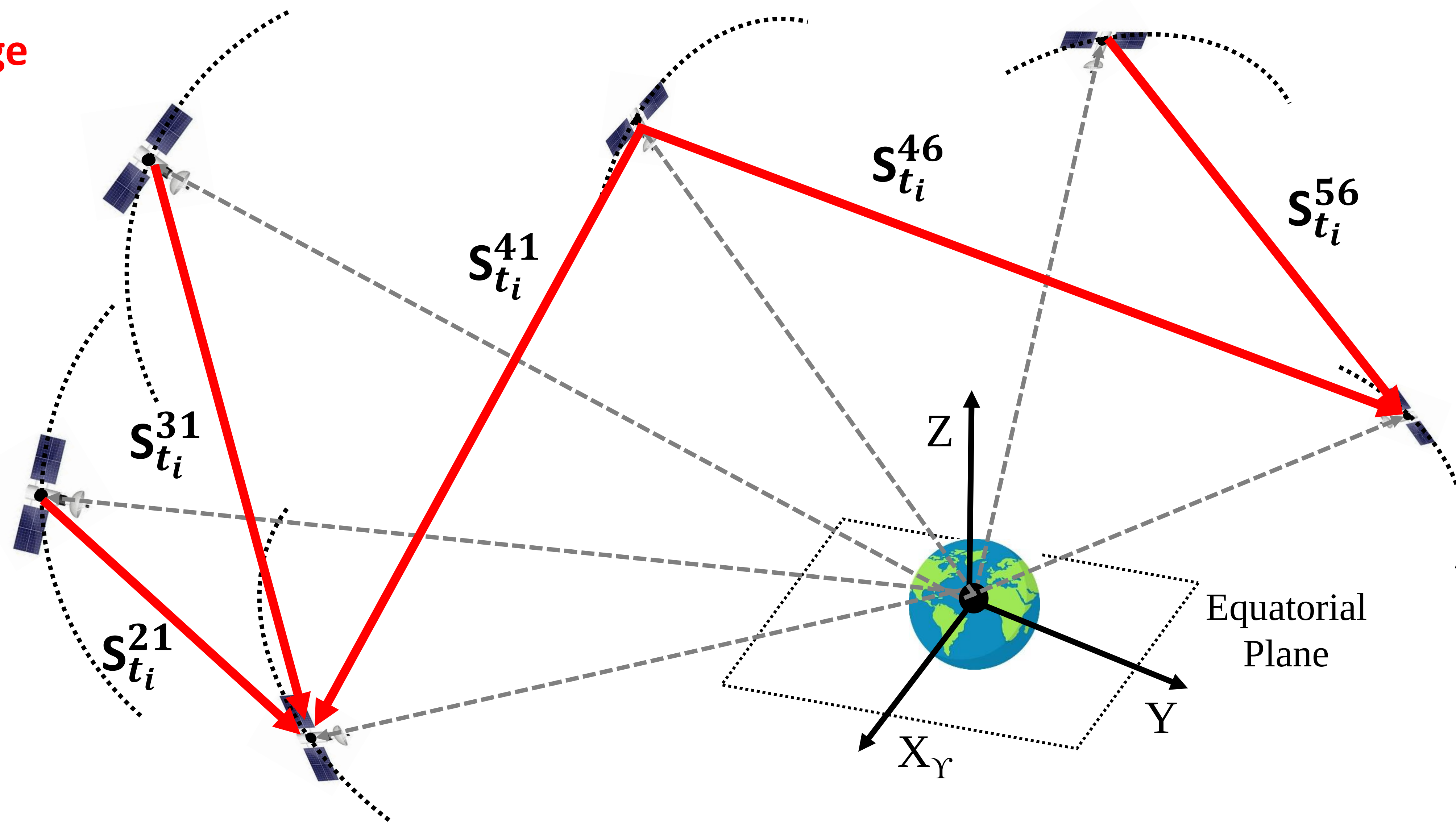
Description of the problem

Inertial Orbits



Description of the problem

Inter-satellite range measurements



Absolute and relative orbit determination via ISLs

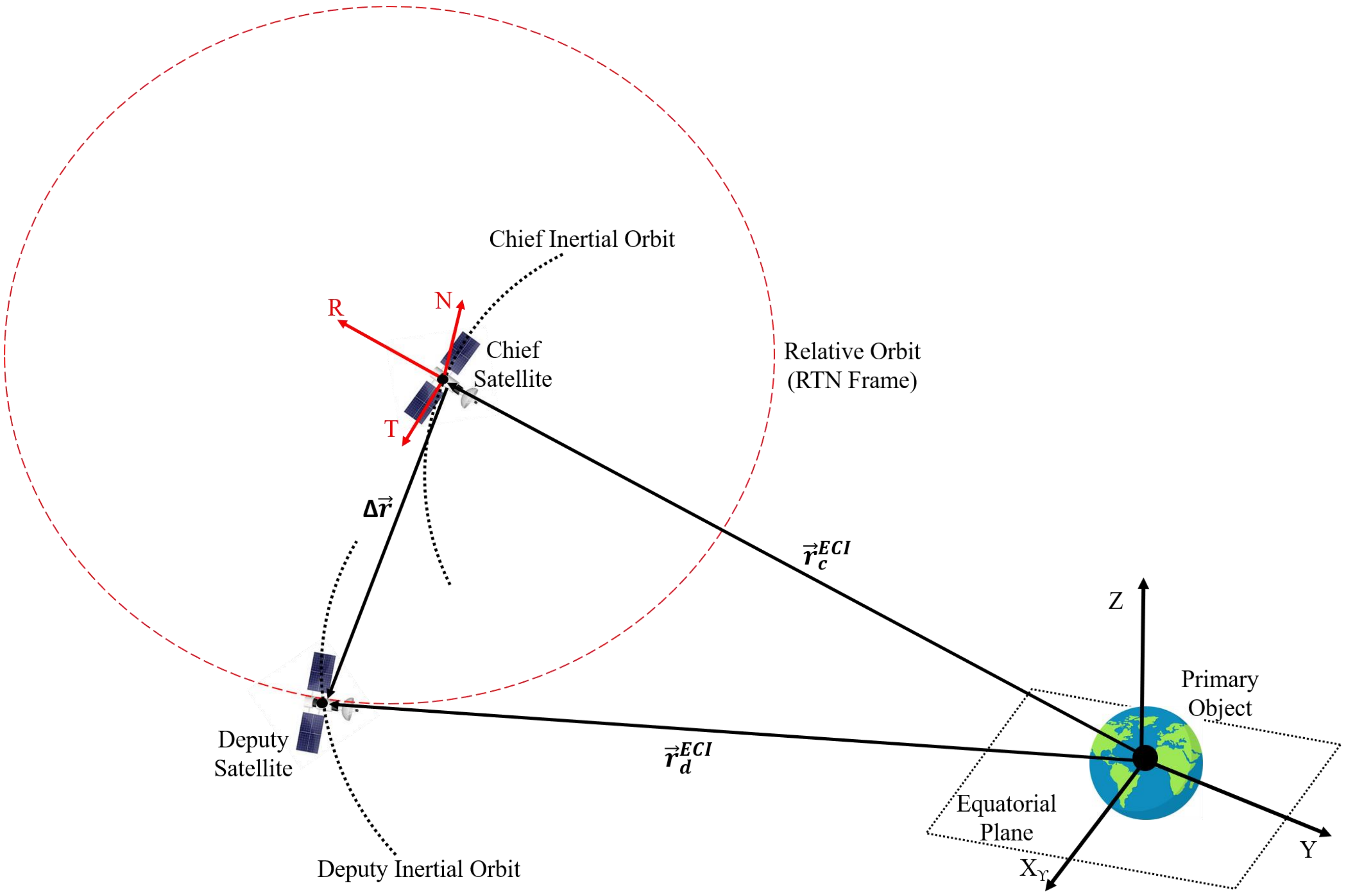
Satellites motion in the framework of Newtonian mechanics

$$\ddot{\boldsymbol{r}}_c = \boldsymbol{N}(\boldsymbol{r}_c; \mu) + \boldsymbol{F}(\boldsymbol{r}_c, \boldsymbol{v}_c, t; \boldsymbol{q})$$

$$\ddot{\boldsymbol{r}}_d = \boldsymbol{N}(\boldsymbol{r}_d; \mu) + \boldsymbol{F}(\boldsymbol{r}_d, \boldsymbol{v}_d, t; \boldsymbol{q})$$

Relative motion in the ECI frame

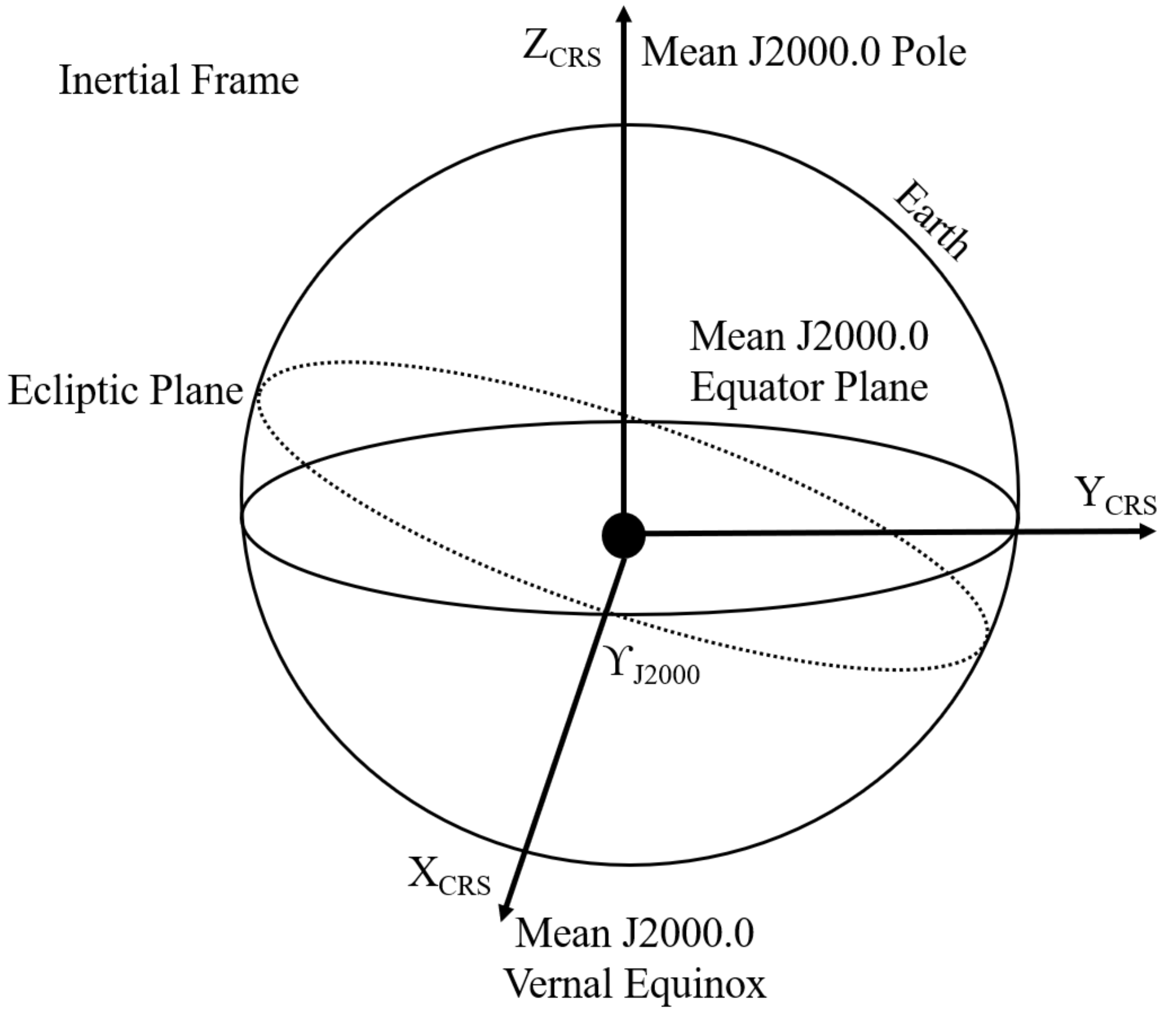
$$\delta\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_d^{ECI} - \ddot{\boldsymbol{r}}_c^{ECI}$$

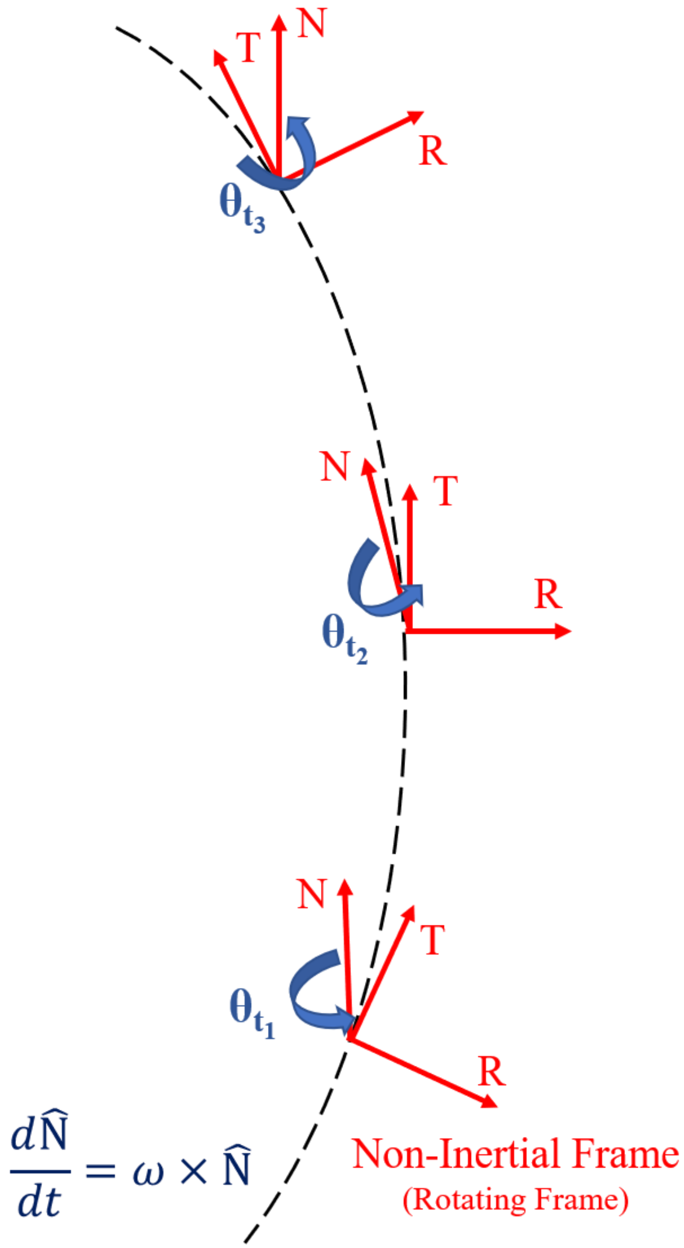


Inertial vs Satellite based reference frame

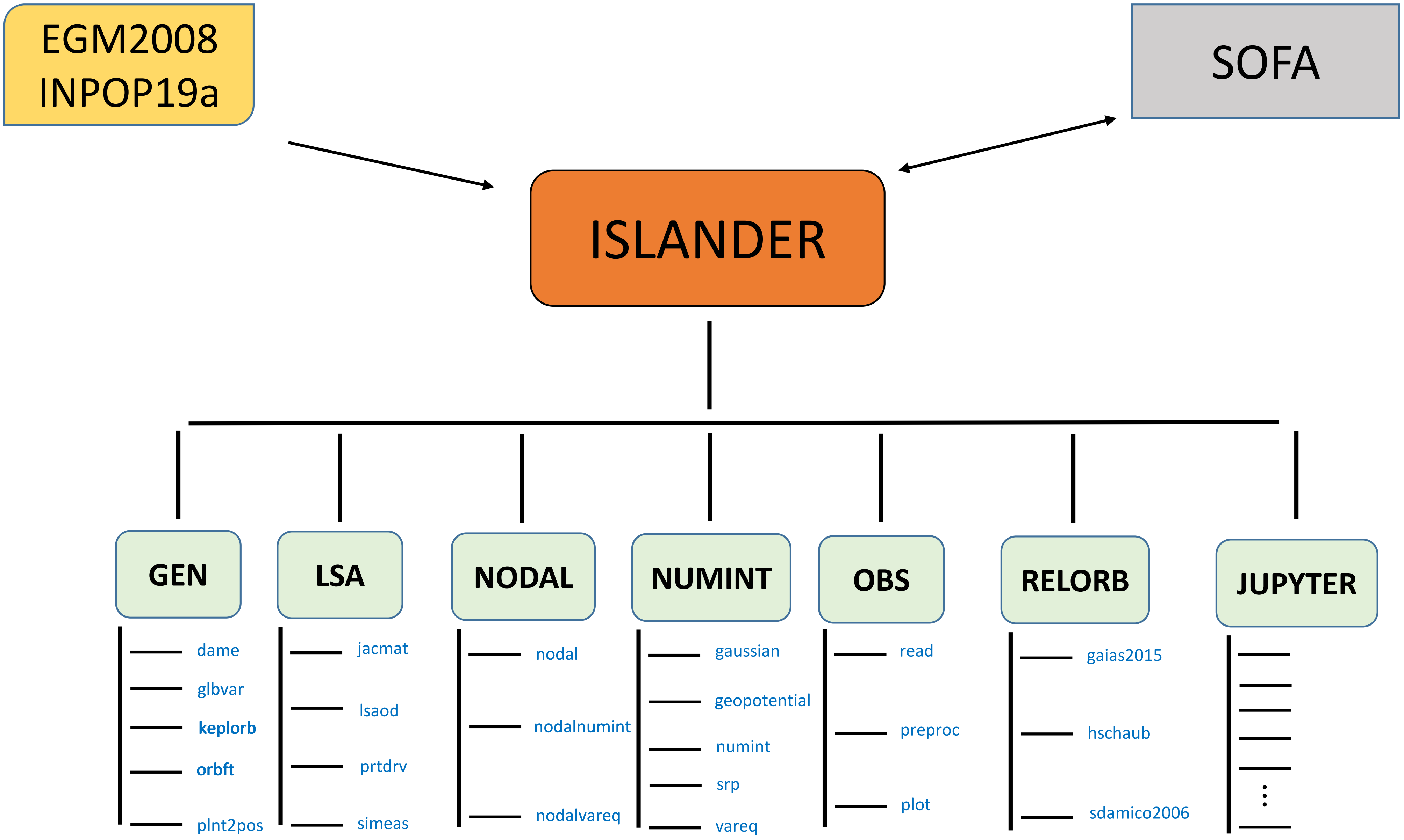
$$R_{ECI}^{RTN} = R_z(u) R_x(i) R_z(\Omega)$$

Euler Rotations
from ECI -> RTN





Simulator


EGM2008
INPOP19a
SOFA
ISLANDER
GEN
dame
glbvar
keplorb
orbft
plnt2pos
LSA
jacmat
lsaod
prtdrv
simeas
NODAL
nodal
nodalnumint
nodalvareq
NUMINT
gaussian
geopotential
numint
srp
vareq
OBS
read
preproc
plot
RELORB
gaias2015
hschaub
sdamico2006
JUPYTER

Iterative least-squares adjustment procedure

$$(A^T PA)(x - x_0) = A^T Pb$$

$$(H^T WH)(x - x_0) = H^T W(x^{ext} - x_0)$$

Normal Equations and Datum Constraints

$$\delta x = (N + H^T WH)^{-1}(U + H^T WH(x^{ext} - x_0))$$

$$\hat{\sigma}^2_{apost} = \frac{V^T PV}{n - m + k}$$

$$x = x_0 + \delta x$$

$$C_x = \hat{\sigma}^2_{apost}(N + H^T WH)^{-1}$$

Constrained solution

$$x^{ext} = x^{ext}$$

$$x_0 = x$$

$$P = \hat{\sigma}^2_{apost} P$$

$$W = \hat{\sigma}^2_{apost} W$$

Rank defect analysis of the NEQ matrix

1) **Spectral Analysis** of the NEQ matrix via the SVD strategy: $\boldsymbol{N} = \boldsymbol{UDU}^T$
U: eigenvectors, D: eigenvalues

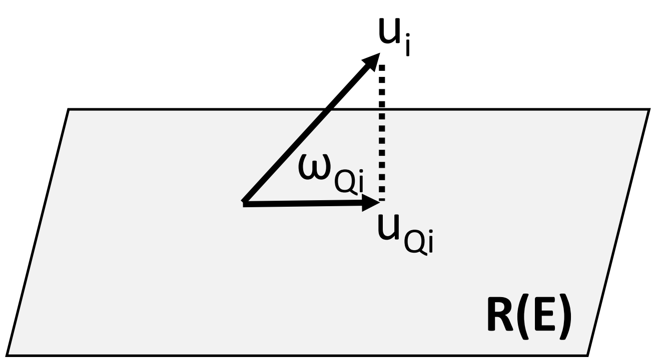
2a) **Geometrical analysis** of the Normal equation matrix: $\boldsymbol{C}_\theta = (\mathbf{E}\ \boldsymbol{N}\ \mathbf{E}^{\mathbf{T}})^{-1}$ (*Sillard and Boucher 2001*),
θ: Transformation Parameters, $\mathbf{E}$: Inner Constraint matrix

2b) **Geometrical analysis** of the Normal equation matrix

$$\tan(\omega_{\mathrm{Qi}}) = \frac{\|u_{\mathrm{i}} - u_{\mathrm{Qi}}\|}{\|u_{\mathrm{Qi}}\|}$$

Angle between zero eigevectors and the theoretical null space

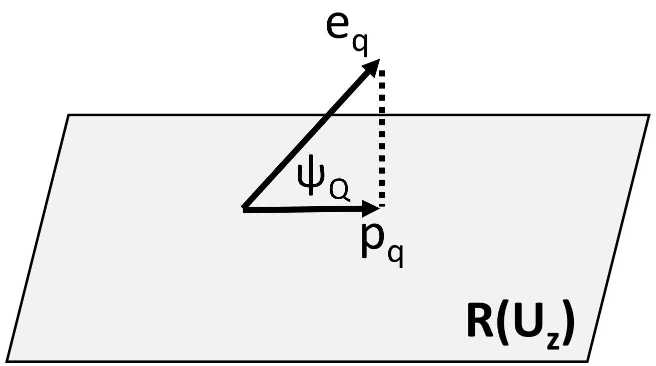
$$u_{\mathrm{Qi}} = E_Q(E_Q^T E_Q)^{-1} E_Q u_{\mathrm{i}}$$



$$\tan(\psi_{\mathrm{Q}}) = \frac{\|e_{\mathrm{q}} - p_{\mathrm{q}}\|}{\|p_{\mathrm{q}}\|}$$

Angle between numerical null space and the theoretical null vectors

$$p_{\mathrm{q}} = U(U^T U)^{-1} U e_{\mathrm{q}}$$



Simulator's Features

Relative orbit determination/propagation:

- Parametrization model:
  - Classical orbital element differences
  - Eccentricity/Inclination vector seperation
  - Nodal Elements
  - Relative orbital elements
- Solutions via:
  - Analytical formula
  - State Transition Matrix
  - Numerical integration
- Dynamical model:
  - Central gravity field
  - J_2
  - Atmospheric Drag
- Reference Frame: RTN (rotating frame)

Relative orbit determination via classical orbital elements

Deputy's orbit position in its RTN frame

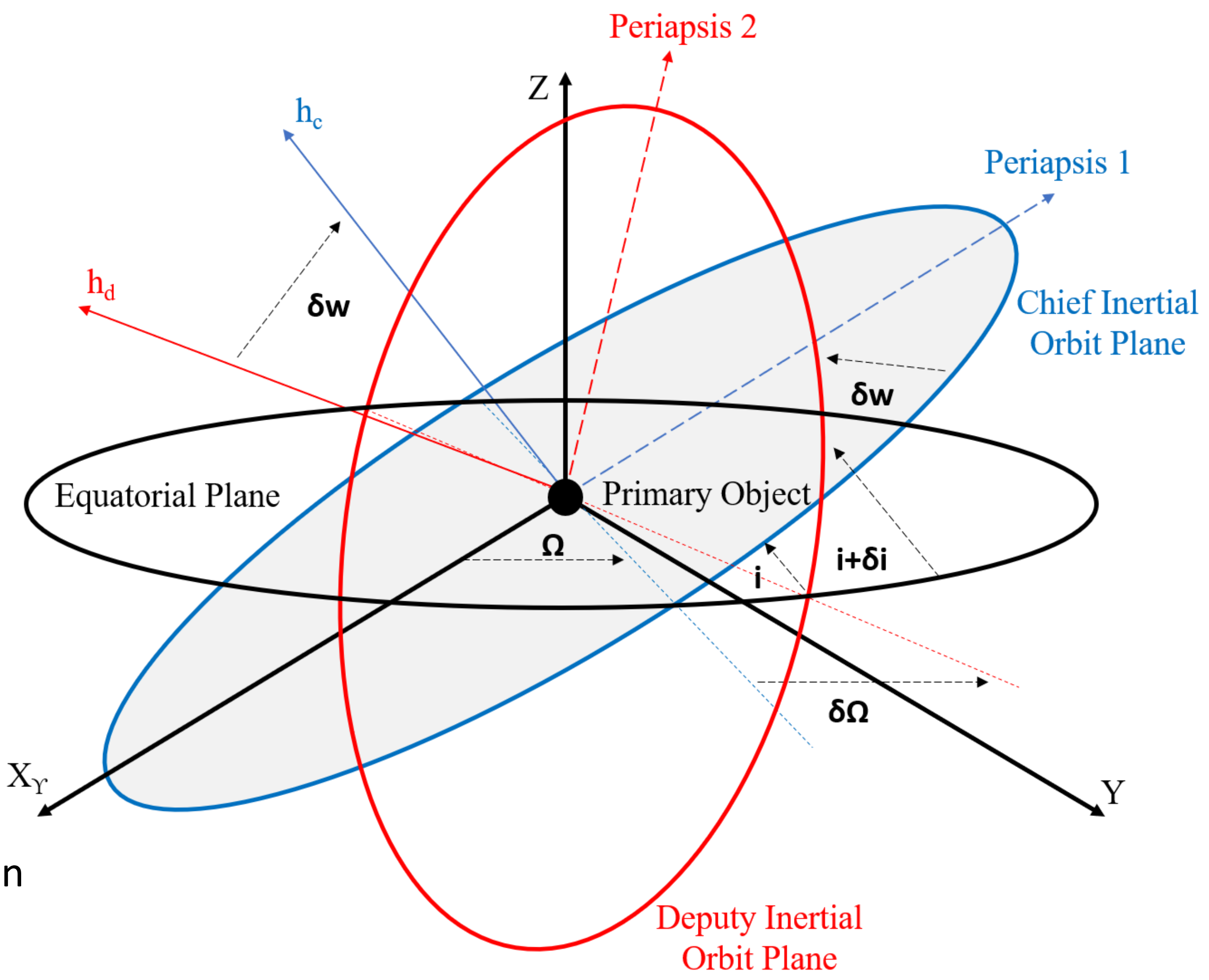
$$\boldsymbol{r}_d^{RTN_d} = [R_d \quad 0 \quad 0]^T$$

Deputy's orbit position in the chief's RTN frame

$$\boldsymbol{r}_d^{RTN_c} = [R_c + x \quad y \quad z]^T$$

$$\begin{bmatrix} R_c + x \\ y \\ z \end{bmatrix} = \boldsymbol{R}_c^{RTN_c/ECI} \boldsymbol{R}_d^{ECI/RTN_d} \begin{bmatrix} R_d \\ 0 \\ 0 \end{bmatrix}$$

Fundamental equation for the relative motion in the RTN frame through orbital element



Relative orbit determination via Nodal Elements

According to Leomanni et al. (2020):

$$\boldsymbol{R}_{RTN_d}^{RTN_c} = \boldsymbol{R}_z(\theta_1)\boldsymbol{R}_x(-\gamma)\boldsymbol{R}_z(-\theta_2)$$

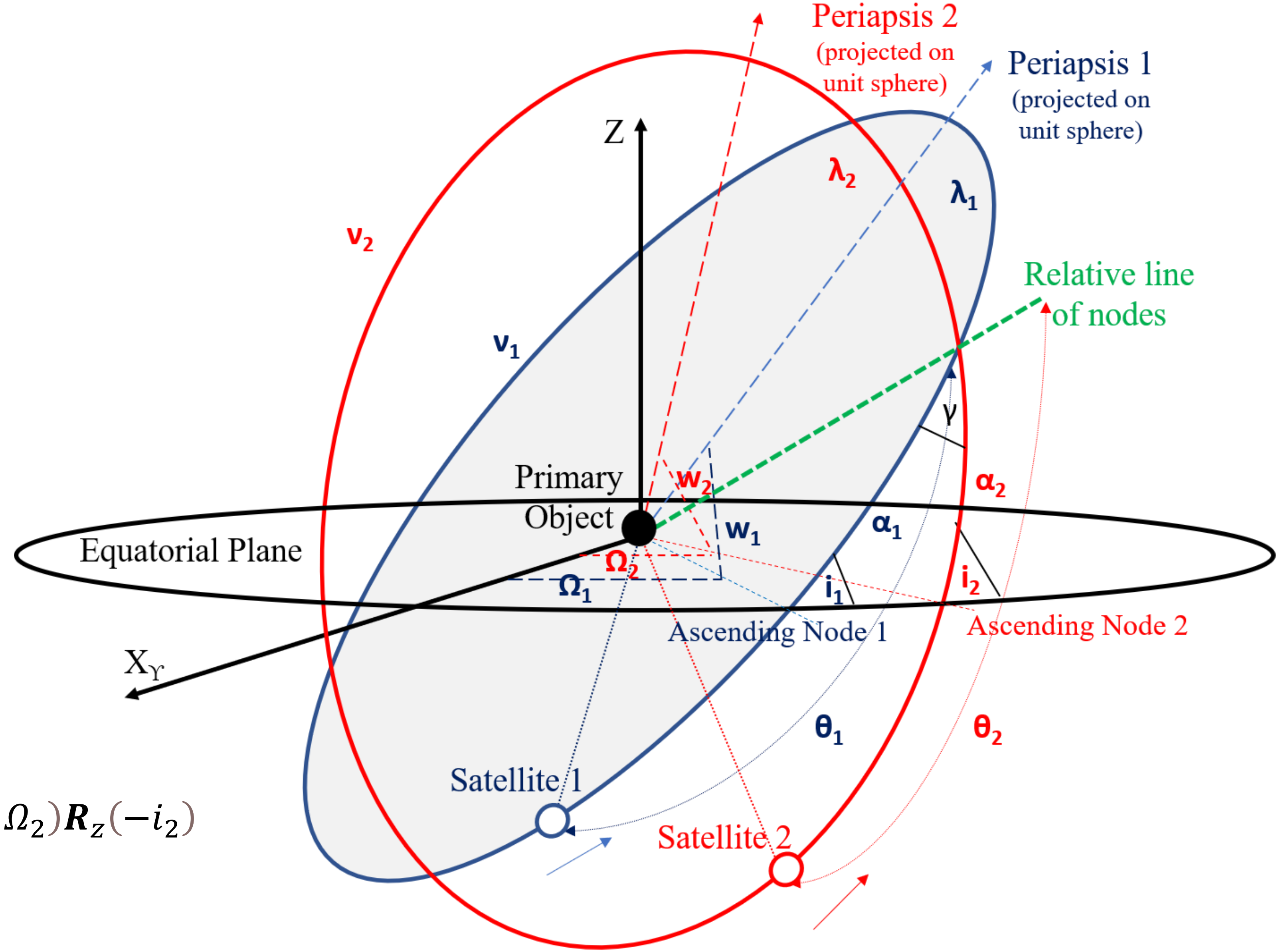
Nodal Elements

$$\lambda_j = \omega_j - a_j$$

$$\theta_j = \nu_j + \lambda_j, (i = 1, 2)$$

Relative Inclination γ

$$\boldsymbol{R}_z(-\alpha_1)\boldsymbol{R}_x(\gamma)\boldsymbol{R}_z(\alpha_2) = \boldsymbol{R}_z(i_1)\boldsymbol{R}_x(\Omega_1 - \Omega_2)\boldsymbol{R}_z(-i_2)$$



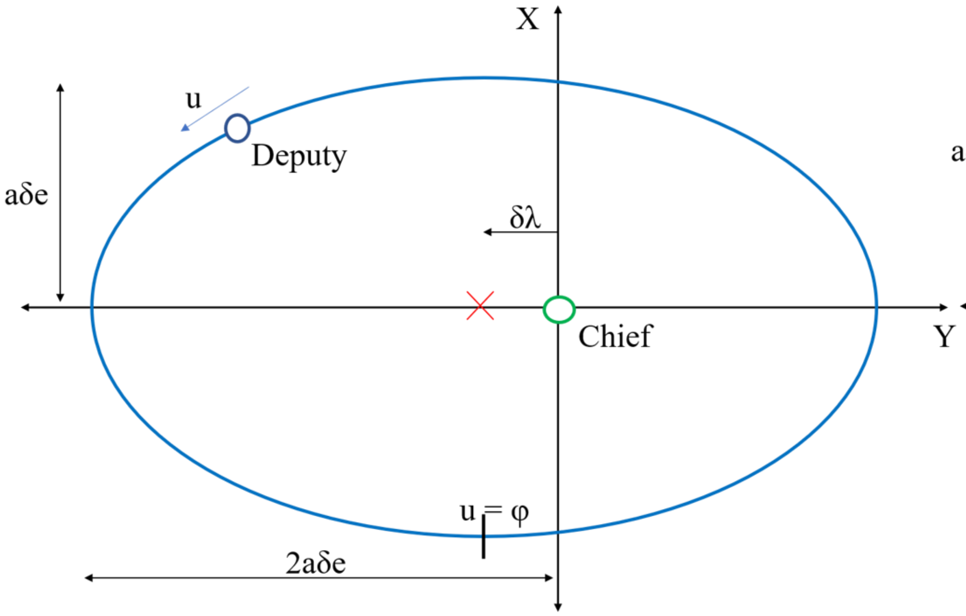
Relative orbit determination via Relative Elements

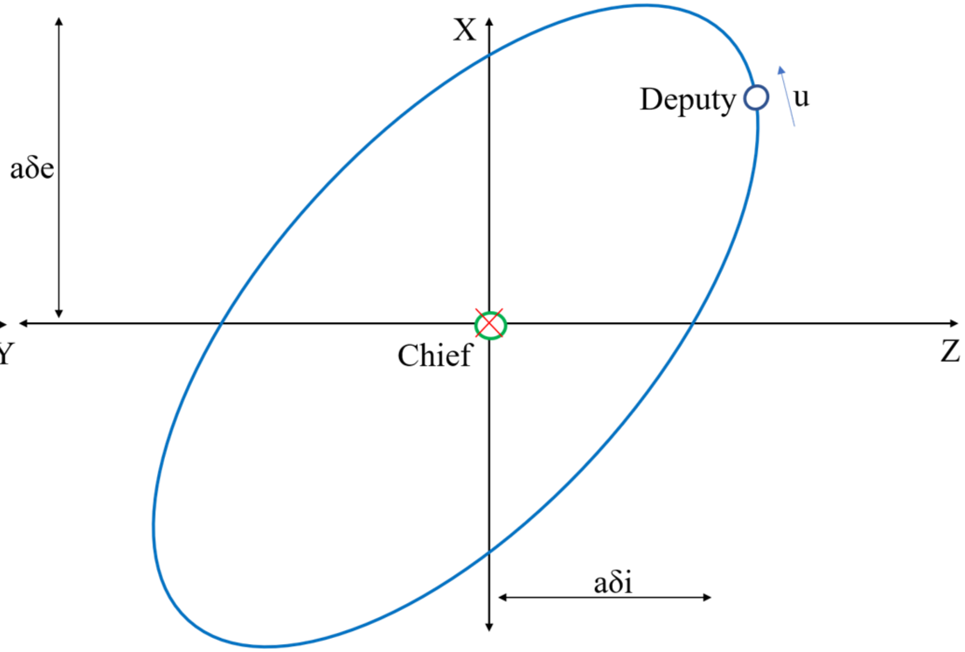
According to D'Amico 2005:

$$\delta\boldsymbol{\sigma} = \begin{bmatrix} \delta a \\ \delta\lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{bmatrix} = \begin{bmatrix} \frac{a_d - a_c}{a_c} \\ (u_d - u_c) + (\Omega_d - \Omega_c)\cos(i_c) \\ e_d\cos(\omega_d) - e_c\cos(\omega_c) \\ e_d\sin(\omega_d) - e_c\sin(\omega_c) \\ i_d - i_c \\ (\Omega_d - \Omega_c)\sin(i_c) \end{bmatrix}$$

$\delta a = 0$, $\delta\lambda = 0$ →

$$x = a_c\delta e\cos(u_c - \varphi)$$
$$y = 2a_c\delta e\sin(u_c - \varphi)$$
$$z = a_c\delta i\cos(u_c - \theta)$$





Assessment of relative orbital models

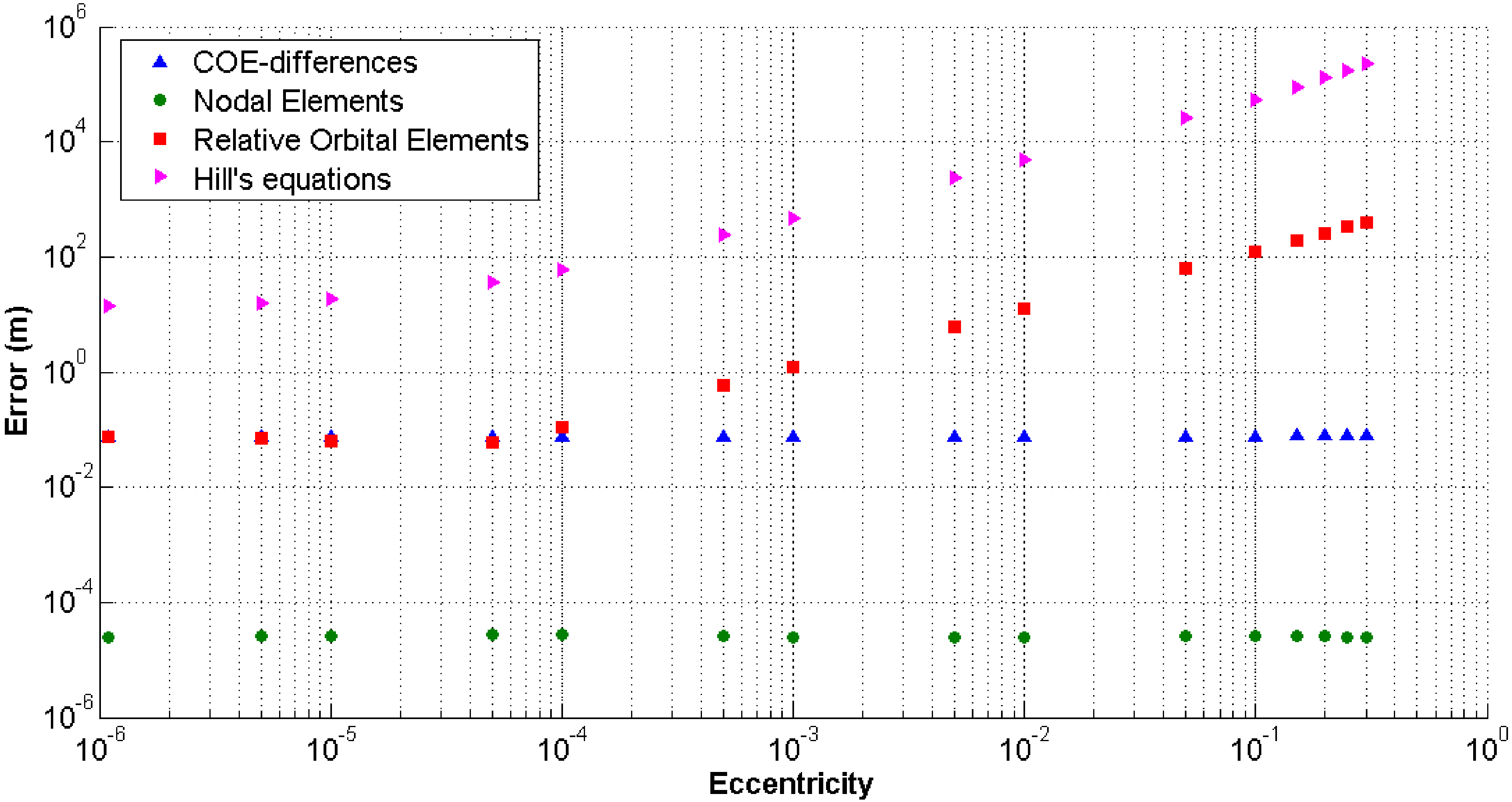
Relative orbit determination

Method	Orbit type	Derived from	Solution	Perturbation
Hill’s Equations	Circular	Kinematic Equation	Linearization	None
COE Differences	Any e_c	Kinematic Equation	Linearization	None
Relative Orb. Elem.	Near-Circular	Hill's Equations	Linearization	None
Nodal Elements	Any e_c	Kinematic Equation	Exact	None

Assessment of relative orbital models

Maximum relative state error between the modeled and the true relative orbit

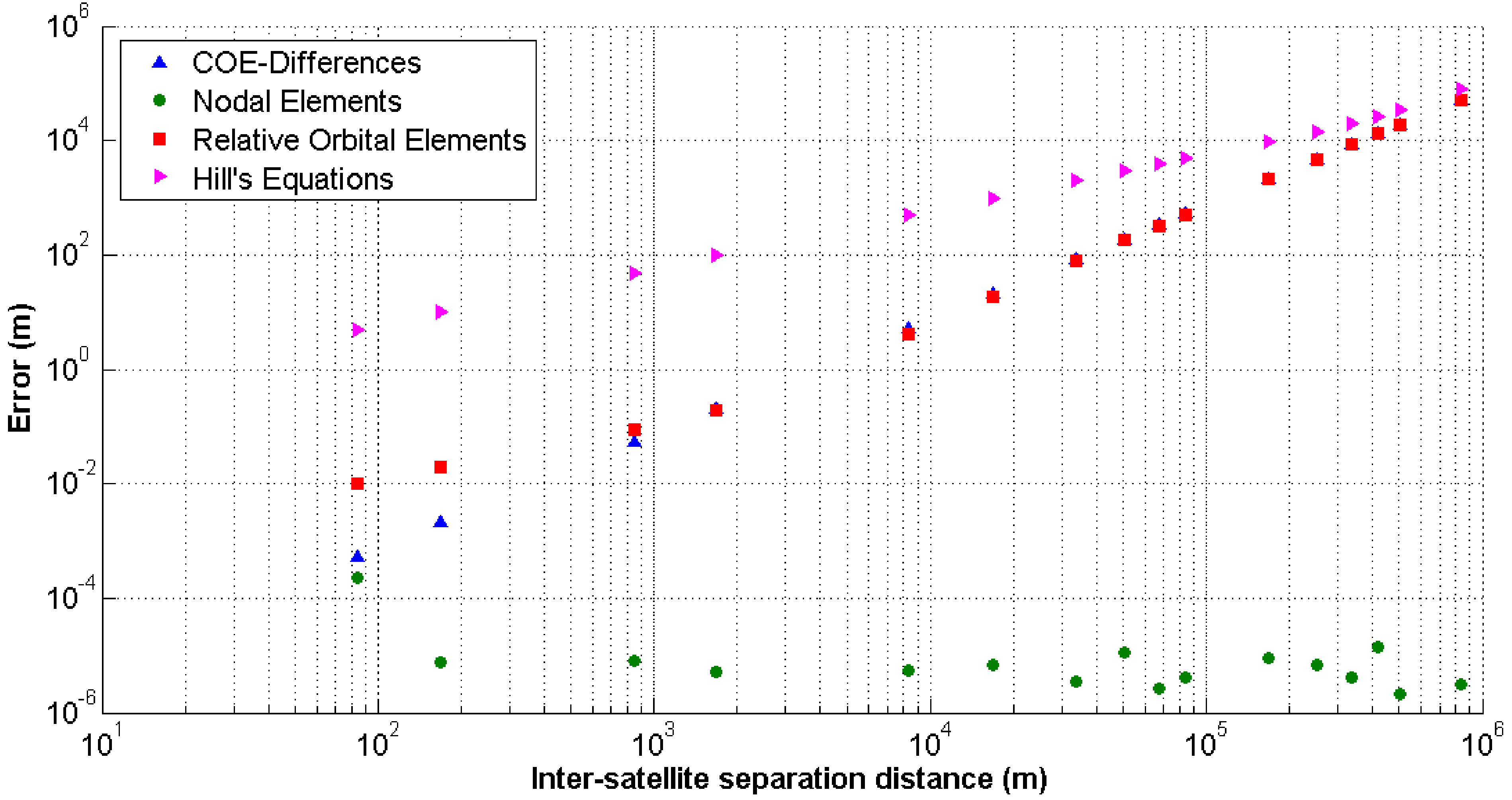
Intersatellite distance fixed = 600 meters



Assessment of relative orbital models

Maximum relative state error between the modeled and the true relative orbit

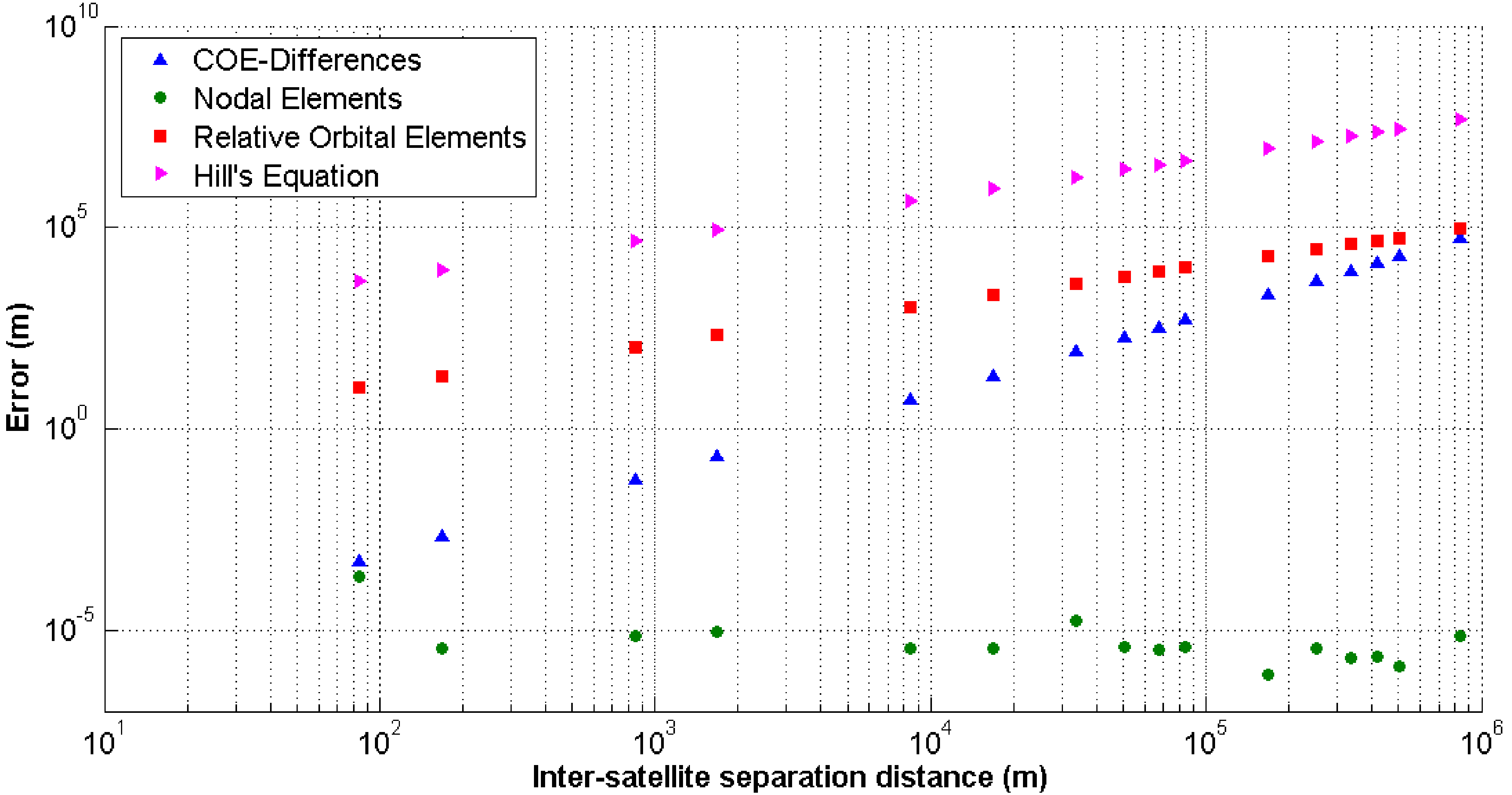
Eccentricity of chief orbit fixed = 10^{-4}



Assessment of relative orbital models

Maximum relative state error between the modeled and the true relative orbit

Eccentricity of chief orbit fixed = 0,1



Relative orbit determination via Nodal Elements

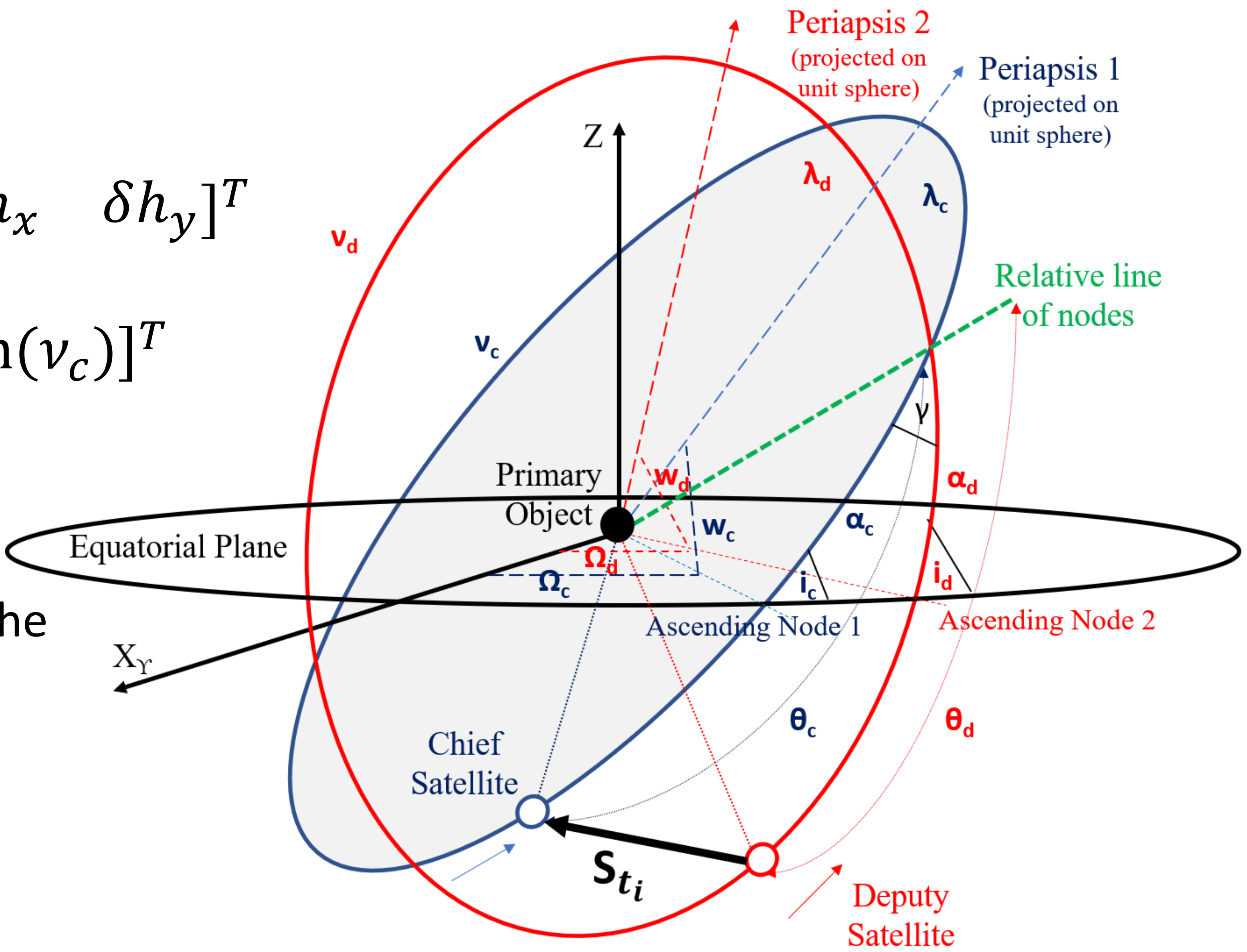
9 parameters model

$$\boldsymbol{\sigma} = [\delta\theta \quad \delta p \quad \delta\xi_x \quad \delta\xi_y \quad \delta h_x \quad \delta h_y]^T$$

$$\boldsymbol{\eta} = [p_c \quad e_c\cos(\nu_c) \quad e_c\sin(\nu_c)]^T$$

Intersatellite range with respect to the 9 parameter model

$$\rho = r_c\sqrt{1 + q^2 - 2\text{b1}q}$$



Relative orbit determination via Nodal Elements

J2 Perturbation Case

J2 perturbation force

$$\boldsymbol{u}^j_{J_2} = -\frac{3J_2R_\oplus^2G_e}{2r_j^4}\begin{bmatrix}1-3\sin^2(i_j)\sin^2(\alpha_j+\theta_j)\\ \sin^2(i_j)\sin(2\alpha_j+2\theta_j)\\ \sin(2i_j)\sin(\alpha_j+\theta_j)\end{bmatrix}$$

New vector **υ**

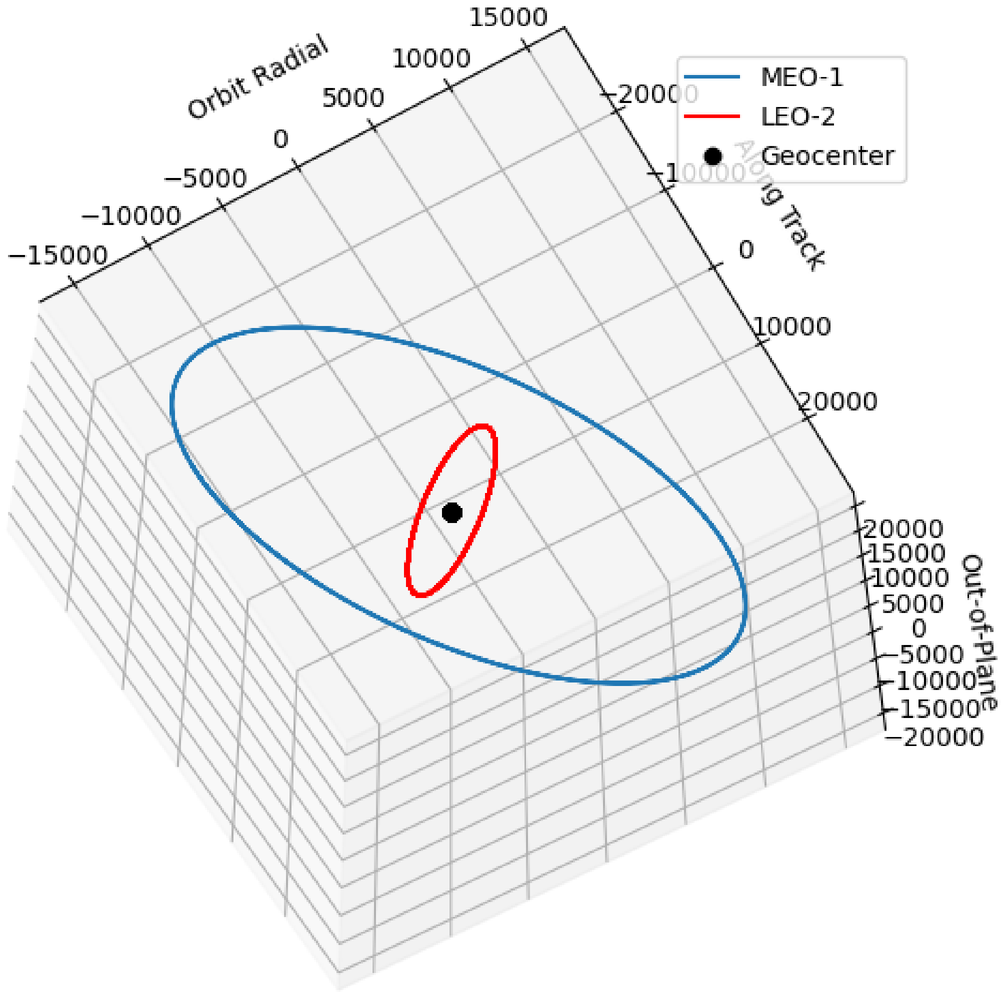
$$\boldsymbol{\upsilon} = \begin{bmatrix}\upsilon_1\\ \upsilon_2\end{bmatrix} = \begin{bmatrix}\tan(\frac{i_c}{2})\cos(\alpha_c+\theta_c)\\ \tan(\frac{i_c}{2})\sin(\alpha_c+\theta_c)\end{bmatrix}$$

11 parameters model

$$\boldsymbol{x}' = \begin{bmatrix}\boldsymbol{o}\\ \boldsymbol{\eta}\\ \boldsymbol{\upsilon}\end{bmatrix}$$

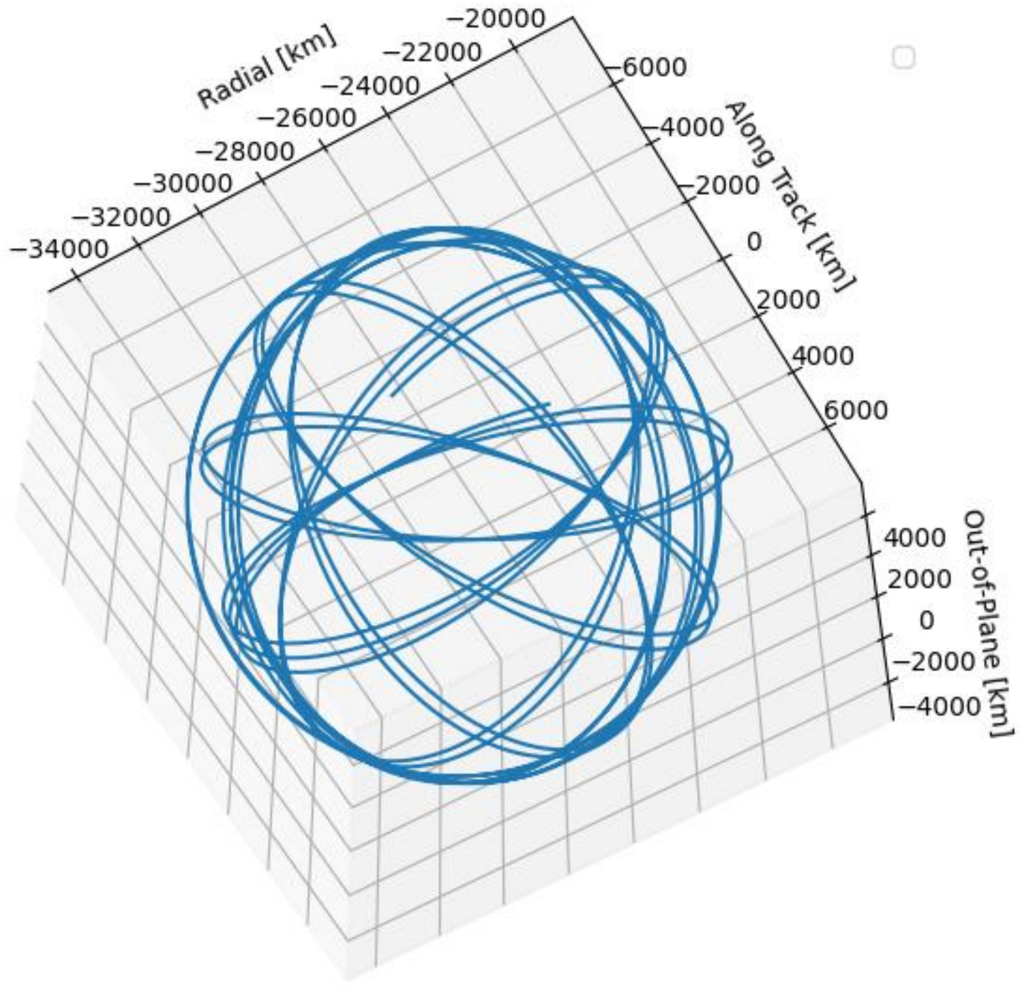
Observability analysis of the relative orbit determination problem

Inter-satellite distance btw 19292 and 33857 Km



Application of

Nodal Elements procedure

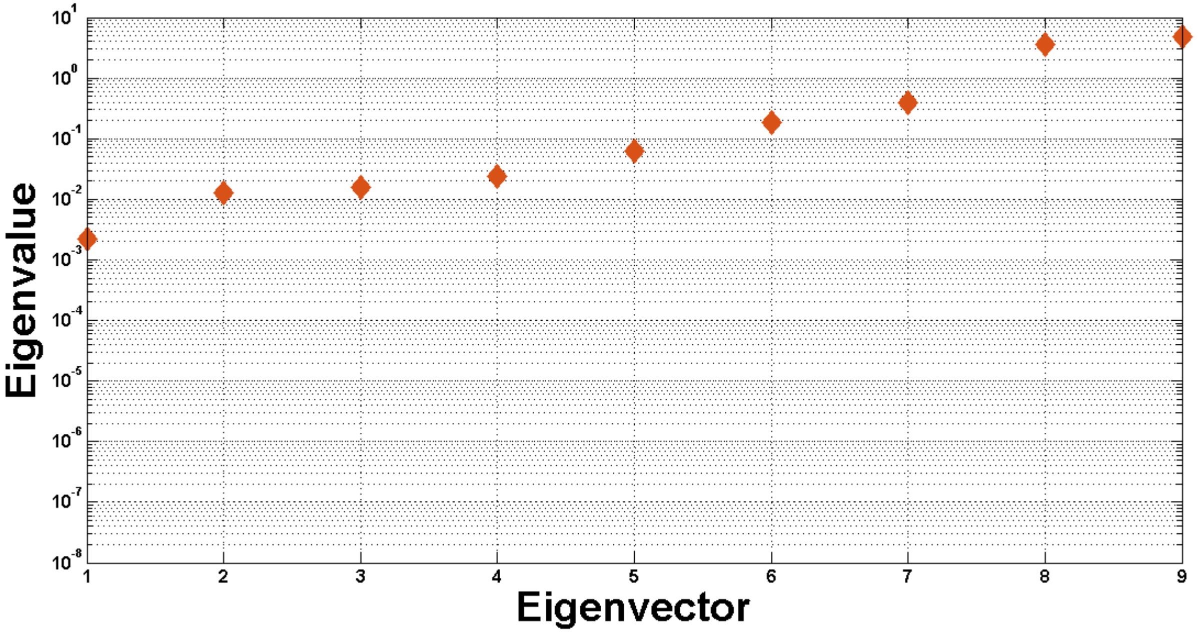


Absolute Orbits

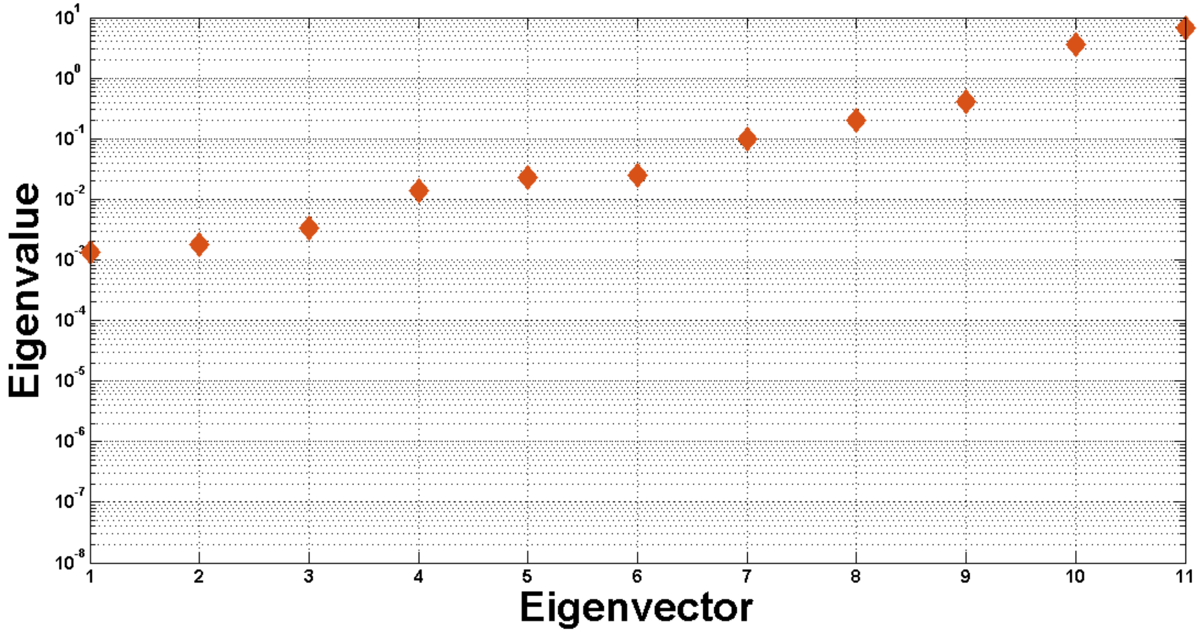
Relative Orbit

Observability analysis of the relative orbit determination problem

Spectral Analysis of Normal Equation (NEQ) matrix



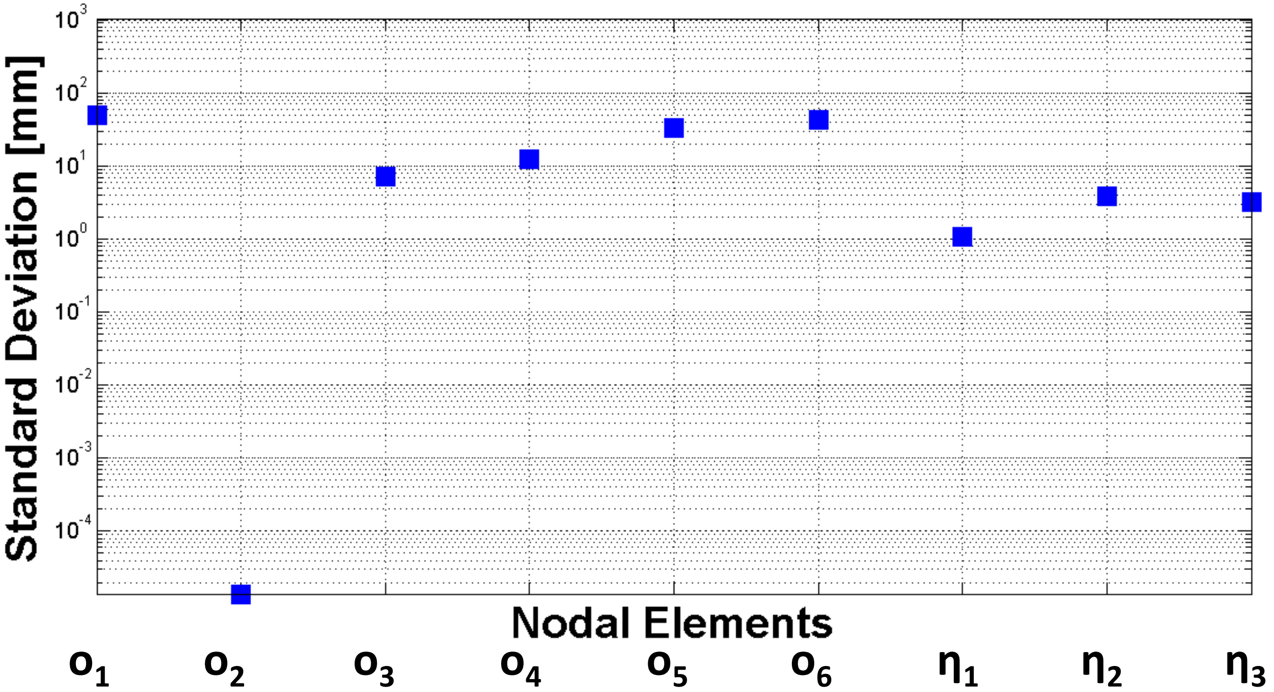
Unperturbed relative orbit



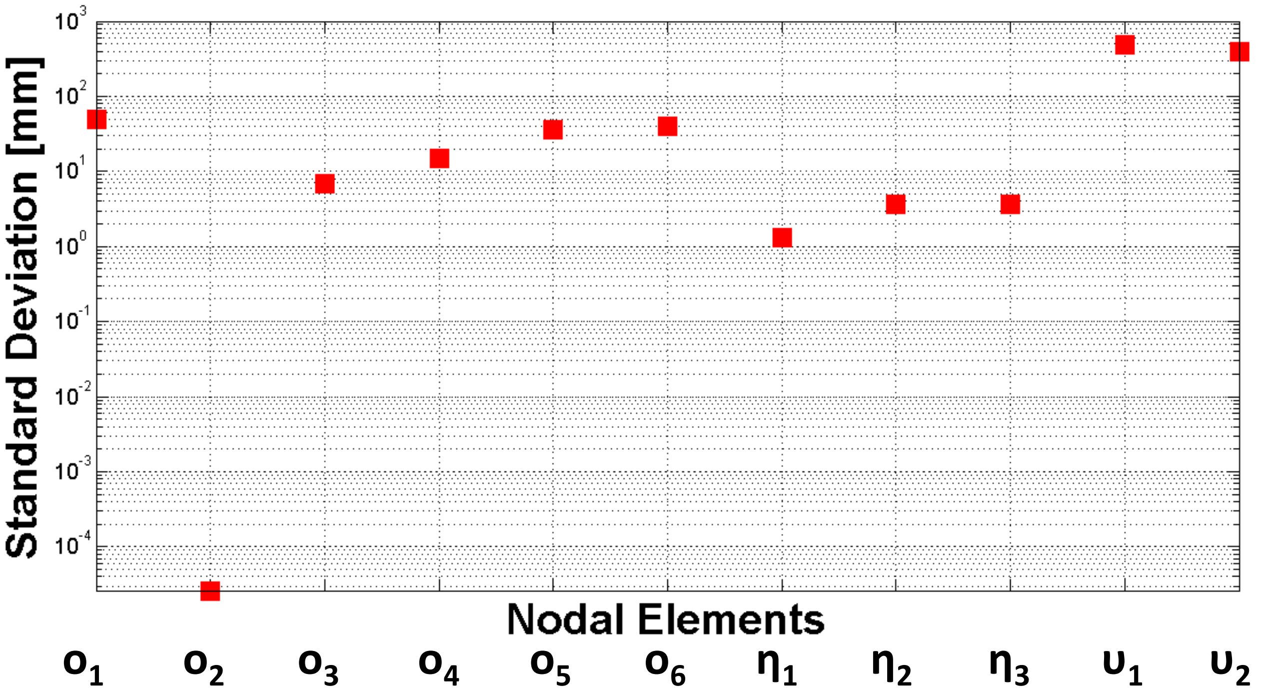
J2-perturbed relative orbit

Observability analysis of the relative orbit determination problem

Geometrical Analysis of Normal Equation (NEQ) matrix



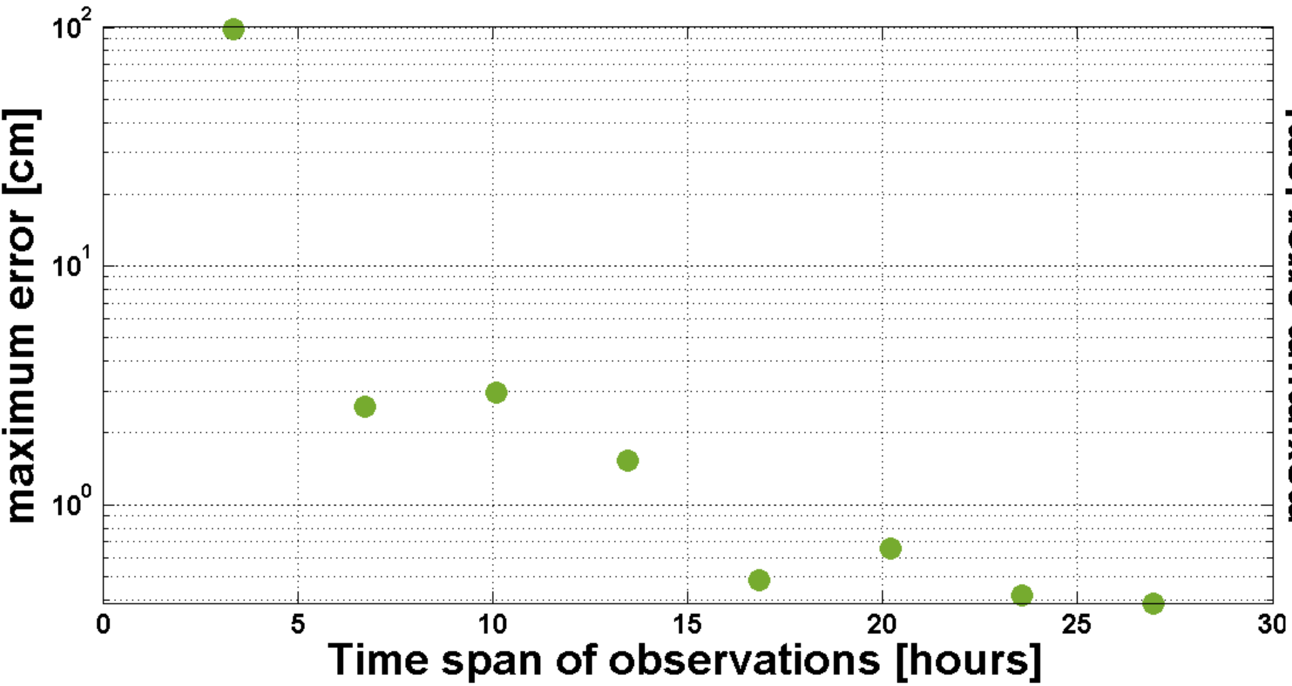
Unperturbed relative orbit



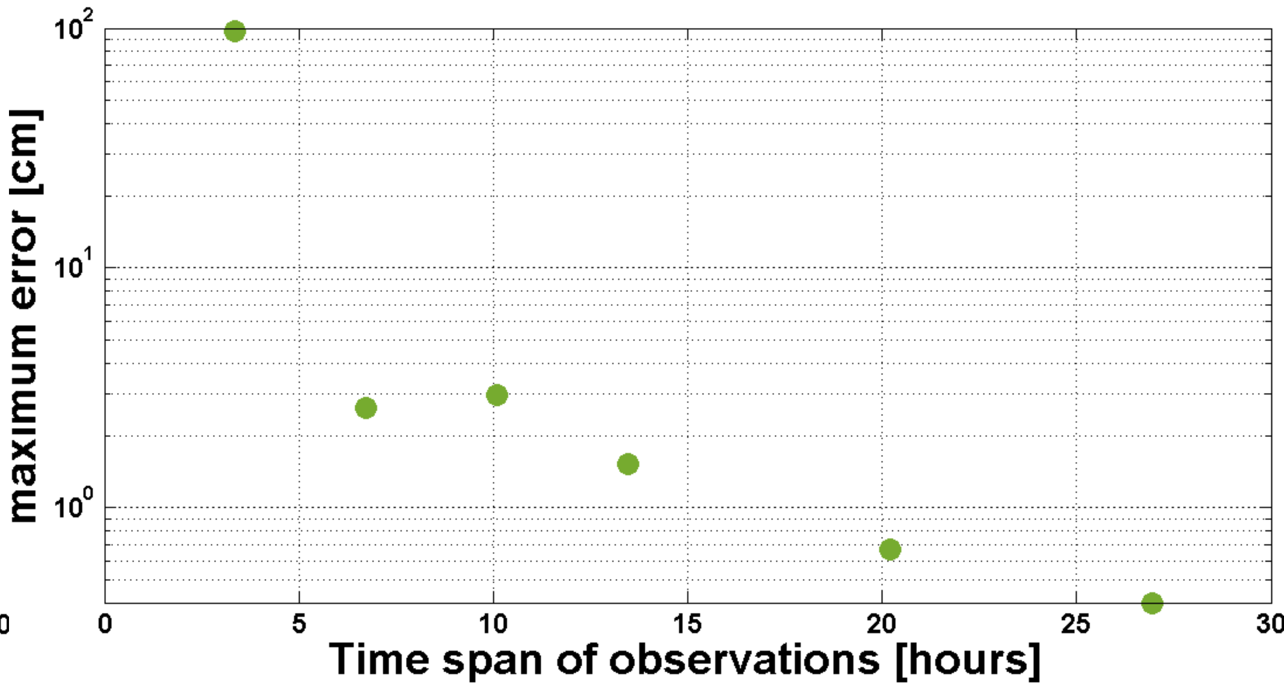
J2-perturbed relative orbit

Accuracy assessment of the relative orbit determination problem

Unperturbed relative orbit



J2-perturbed relative orbit

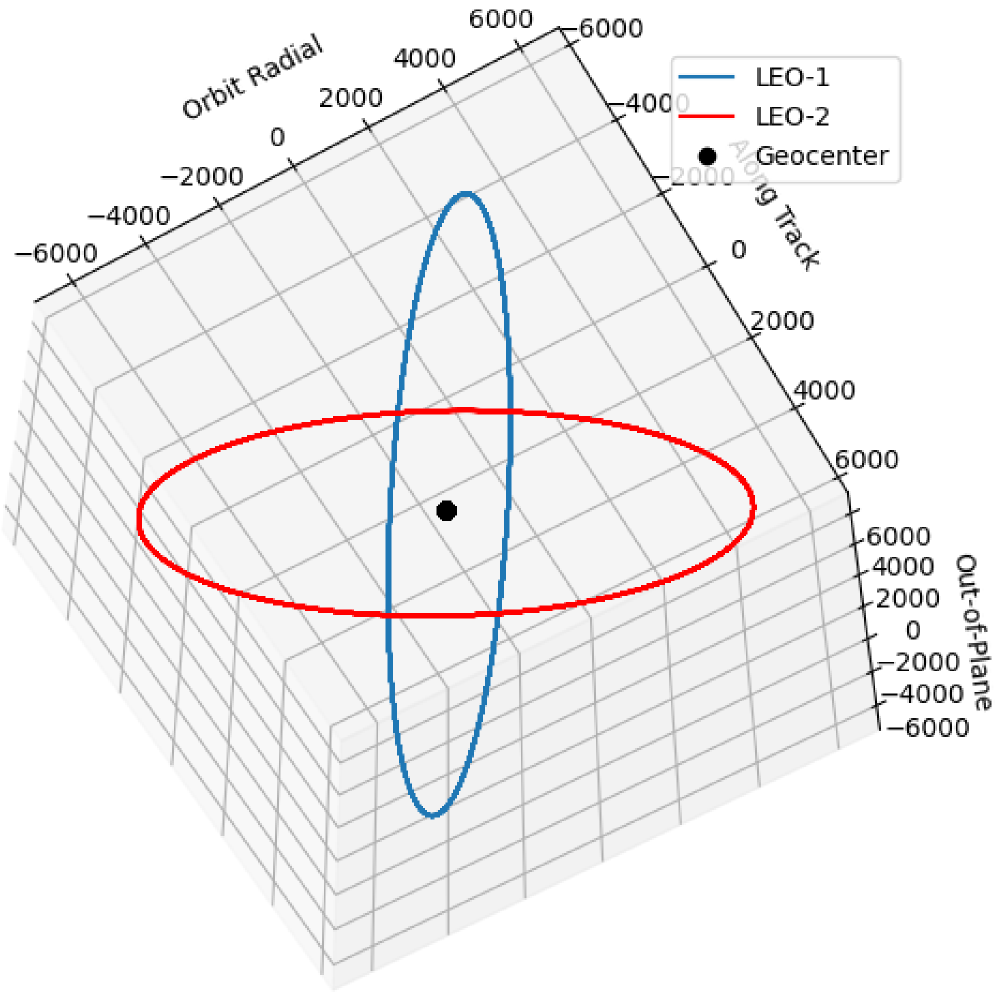


Unconstrained Solution

Constrained Solution
The **υ** vector was fixed

Observability analysis of the relative orbit determination problem

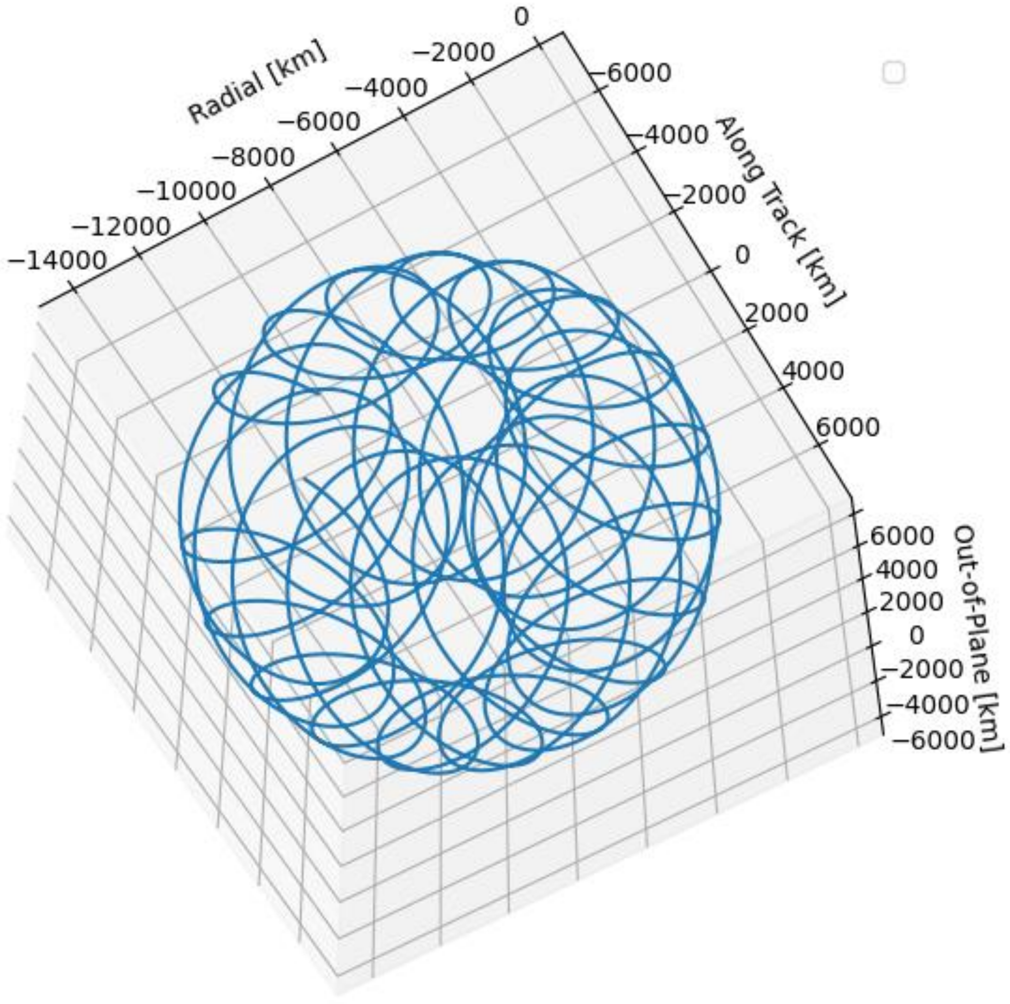
Inter-satellite distance btw 650 and 14003 Km



Absolute Orbits

Application of

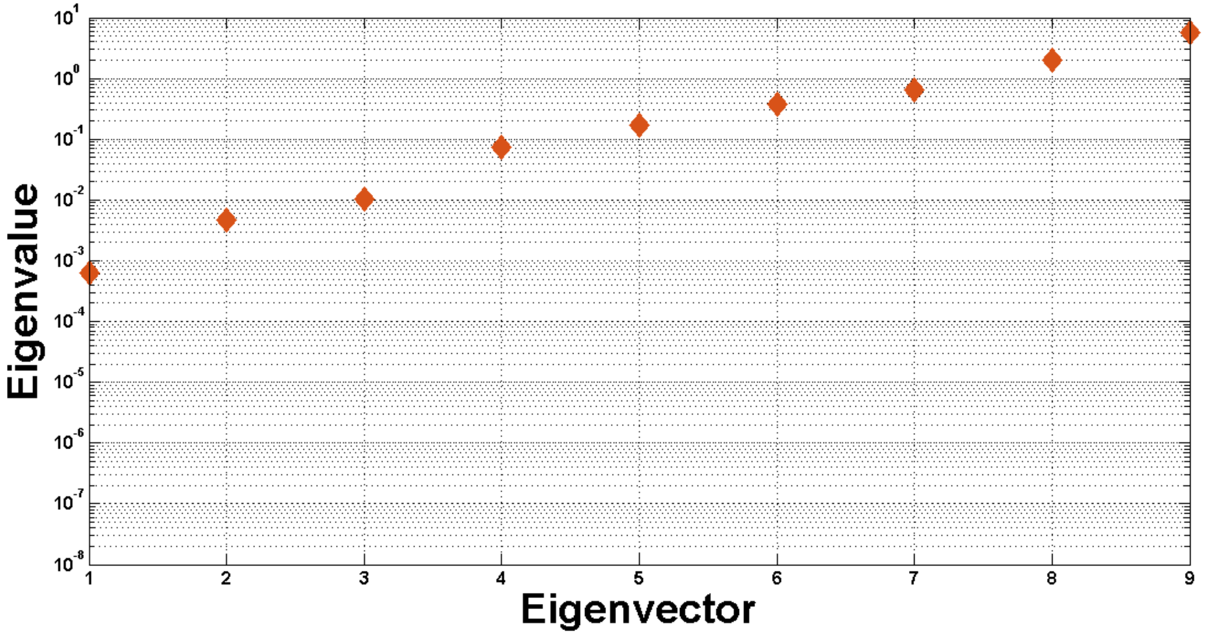
Nodal Elements procedure



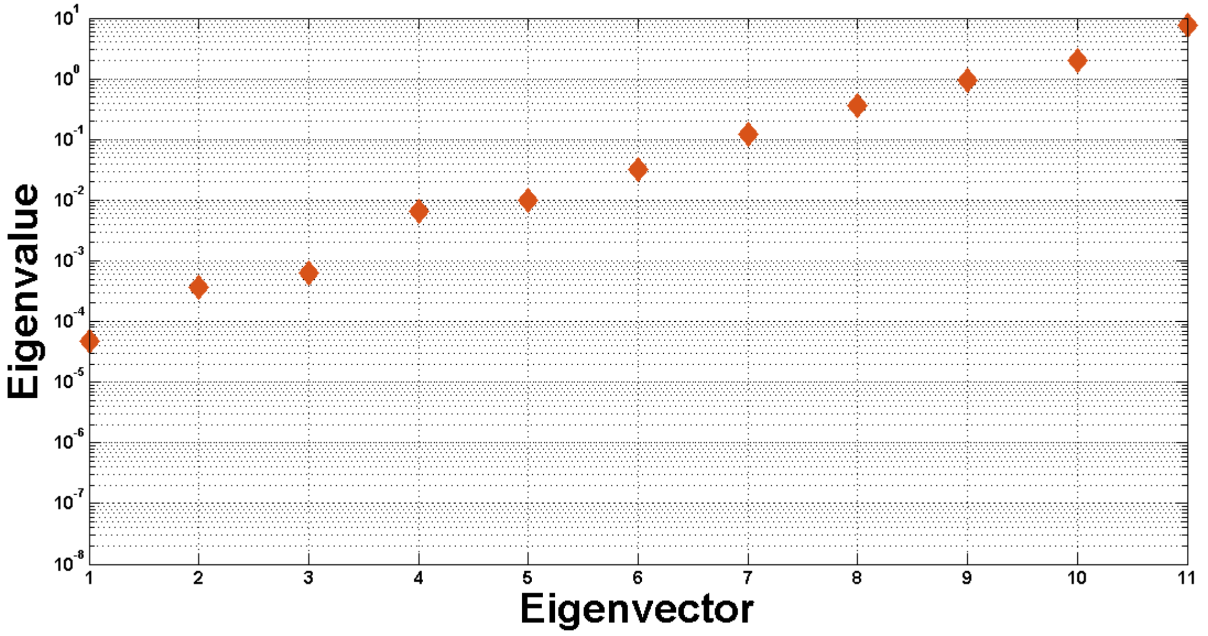
Relative Orbit

Observability analysis of the relative orbit determination problem

Spectral Analysis of NEQ matrix



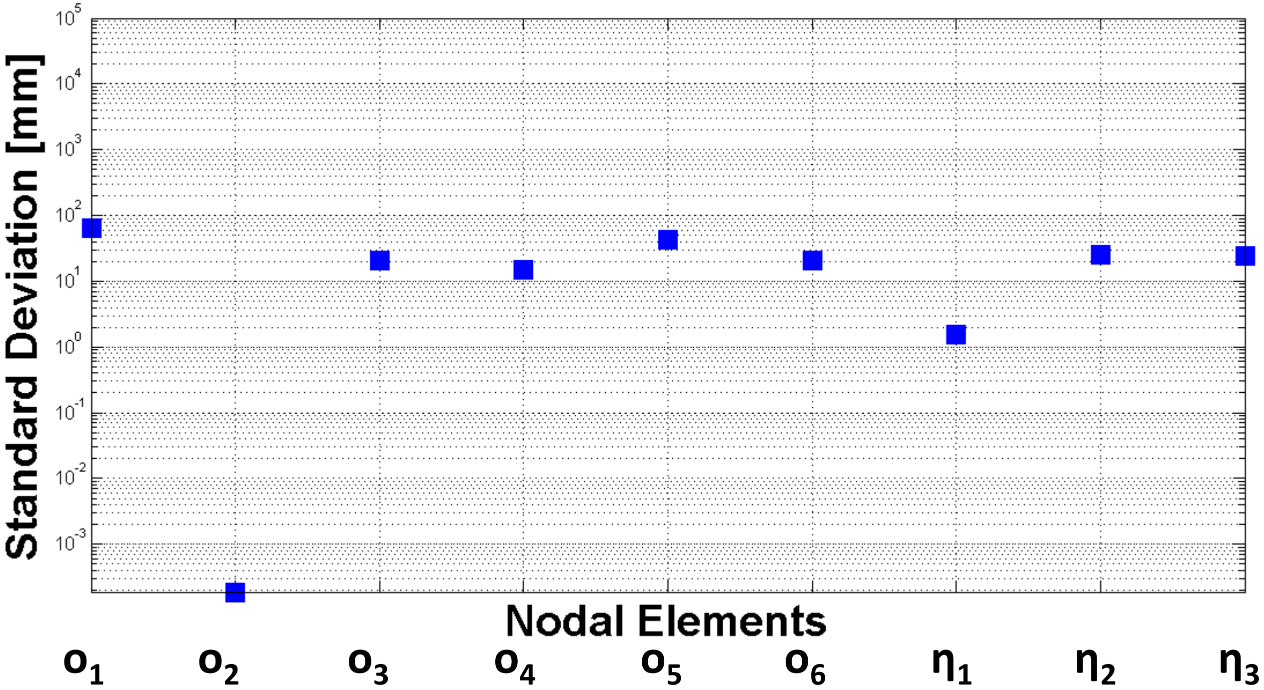
Unperturbed relative orbit



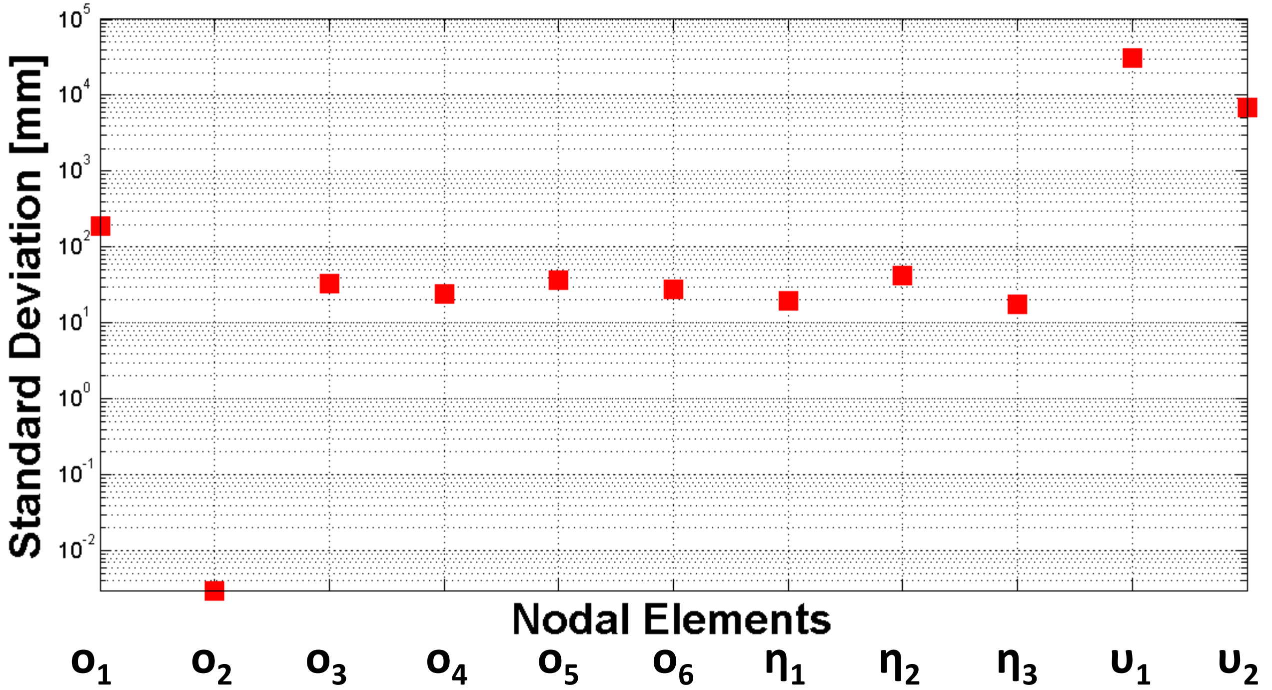
J2-perturbed relative orbit

Observability analysis of the relative orbit determination problem

Geometrical Analysis of NEQ matrix



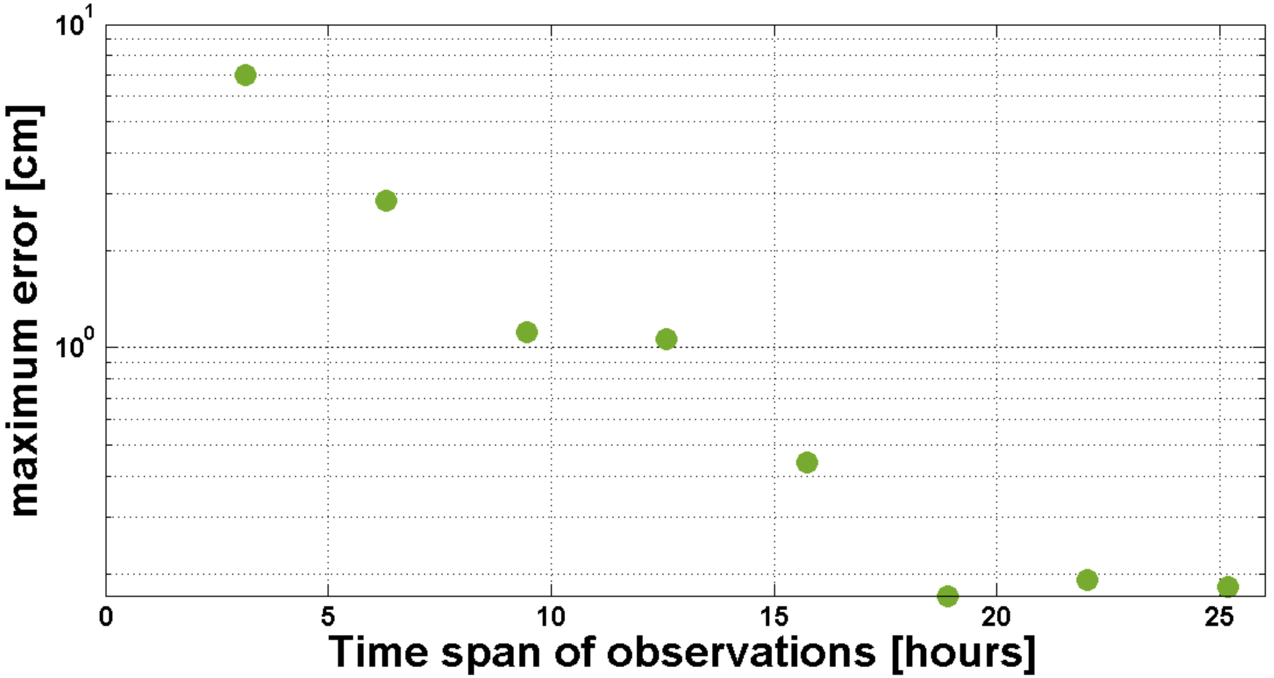
Unperturbed relative orbit



J2-perturbed relative orbit

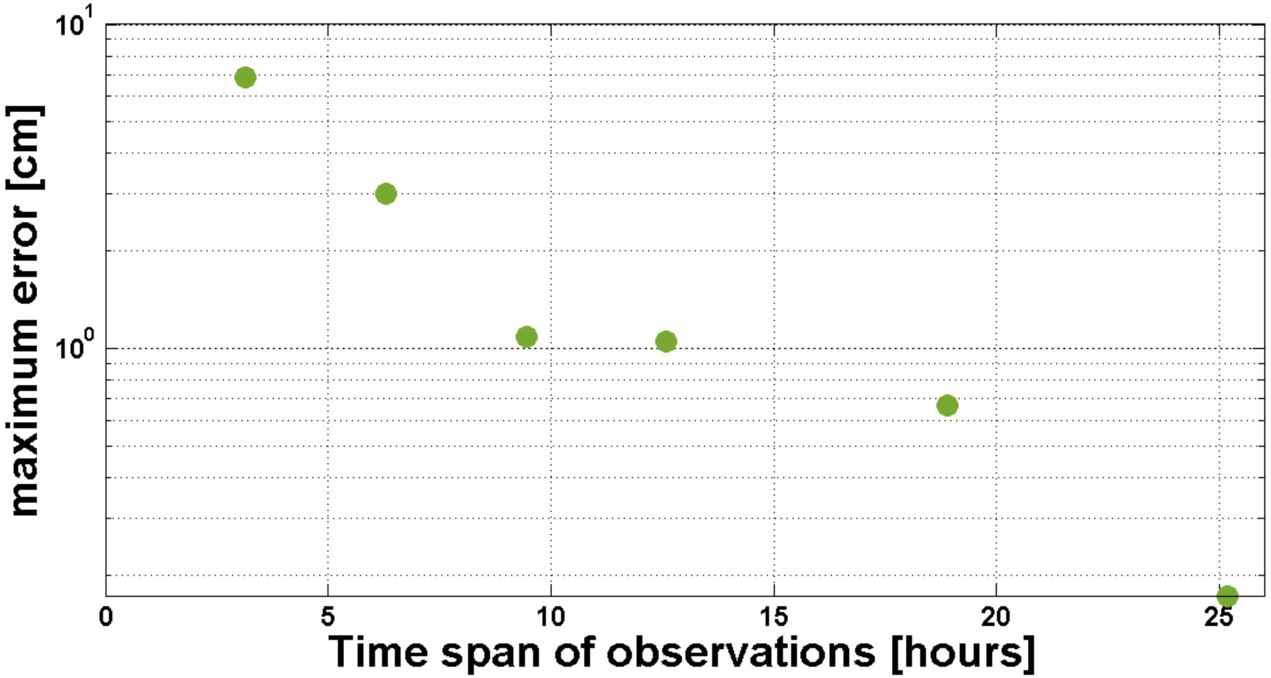
Accuracy assessment of the relative orbit determination problem

Unperturbed relative orbit



Unconstrained Solution

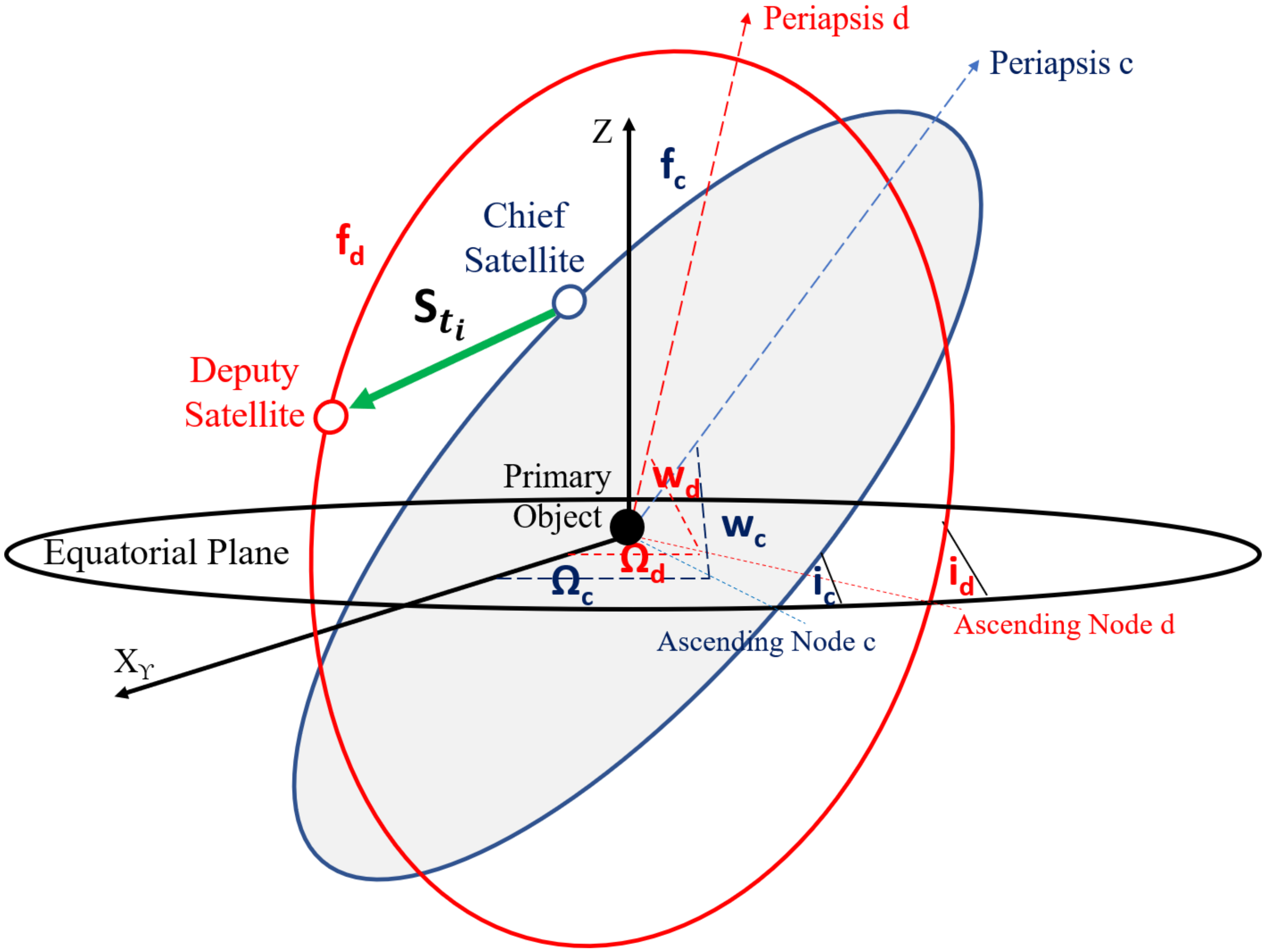
J2-perturbed relative orbit



Constrained Solution

The **υ** vector was fixed

Absolute orbit estimation of satellite constellations via ISLs



Simulator's Features

Absolute orbit determination/propagation:

- Parametrization model:
  - Cartesian System (CRD/VEL)
  - Keplerian Elements

Simulator's Features

Cartesian model

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ \vdots \\ \vdots \\ \vdots \\ x_{6\times n} \end{bmatrix} = \begin{bmatrix} x_{t_0}^1 \\ y_{t_0}^1 \\ z_{t_0}^1 \\ vx_{t_0}^1 \\ vy_{t_0}^1 \\ vz_{t_0}^1 \\ \vdots \\ \vdots \\ x_{t_0}^n \\ y_{t_0}^n \\ z_{t_0}^n \\ vx_{t_0}^n \\ vy_{t_0}^n \\ vz_{t_0}^n \end{bmatrix}$$

Initial position and velocity per satellite

Keplerian model

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ \vdots \\ \vdots \\ \vdots \\ x_{6\times n} \end{bmatrix} = \begin{bmatrix} a_{t_0}^1 \\ e_{t_0}^1 \\ i_{t_0}^1 \\ w_{t_0}^1 \\ \Omega_{t_0}^1 \\ f_{t_0}^1 \\ \vdots \\ \vdots \\ a_{t_0}^n \\ e_{t_0}^n \\ i_{t_0}^n \\ w_{t_0}^n \\ \Omega_{t_0}^n \\ f_{t_0}^n \end{bmatrix}$$

6 Keplerian elements per satellite

Simulator's Features

Absolute orbit determination/propagation:

- Parametrization model:
  - Cartesian System (CRD/VEL)
  - Keplerian Elements

- Numerical integration:
  - n^{th} order Runge-Kutta (single-step)
  - Adams-Bashforth-Moulton (multi-step)

- Dynamical model:
  - Full Gravity potential (Geopotential model: e.g EGM2008)
  - Moon, Sun + 8 planets (Planets' Ephemerides: INPOP19a)
  - Solar Radiation Pressure (Box-Wind model)
  - Atmospheric Drag
  - Earth Tides

- Earth Rotation Parameters: EOP (IERS) 14 C04 TIME SERIES

- Reference Frame: Inertial (ICRF/GCRS)

Datum constraints

$$\delta x = (N + H^T W H)^{-1}[U + H^T W c]$$

or

$$\delta x = (N + Q^T W Q)^{-1}[U + Q^T W c]$$

Addition of constraints to the NEQ System

The ISLANDER module has the option to apply **different types** of datum constraints:

- $H\delta x = c = H(x^{ext} - x_0)$

  Constraining of individual CRD/VEL or Keplerian Elements to known values in a number of satellites.

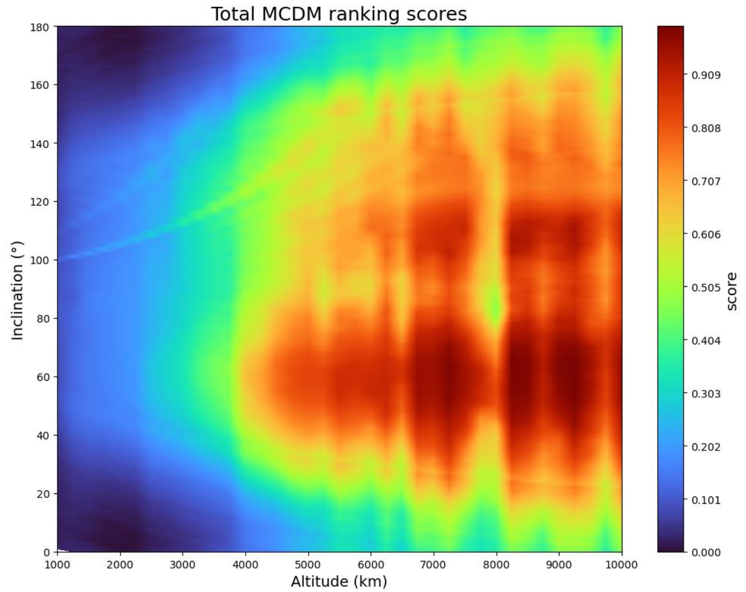
- $Q\delta x = c \qquad Q = (EE^T)^{-1}E \qquad c = Q\left(x^{ext} - x_0\right)$

  Nullification of Helmert transformation parameters wrt. prior info of initial conditions for satellite orbits (minimal constraints).

Simulated Application

Our aim is to estimate circular orbits of a number of GENESIS-like satellites derived from Aghouraf and Chang (2023) optimal search:

Altitude: 9000 km, Inclination: 64 deg
8250 km, 57 deg
8250 km, 67 deg
9250 km, 52 deg
9250 km, 67 deg
6000 km, 95.5 deg



Tested Scenarios

Parametrization model: Cartesian system

Used satellite constellation: 31 GPS satellites or 24 GALILEO

Applied datum constraints:

- Tight constraints on individual initial coordinates & velocities of all GPS or GALILEO satellites
- Minimal constraints (only for three rotation parameters)

Error added to the known position of the GPS or GALILEO satellites: 3 cm

Data processing settings

Date & duration of simulated data: 30/08/2018 - 21:00:00, ~12 hours (~1 GPS satellite arc)
03/01/2021 – 00:00:00, ~14 hours (~1 GALILEO satellite arc)

Observation Sampling: 120 sec

Numerical Integration Method: 8th order Runge-Kutta (10 sec step)

Noise type: Gaussian random noise (σ = 1 cm)

Reference Frame: GCRS

Dynamical Model: Earth's gravity field (20 × 20)
Moon + planets' forces
Solar Radiation Pressure
Earth tides

GNSS visibility Model for GENESIS

According to Montenbruck et al. (2023):

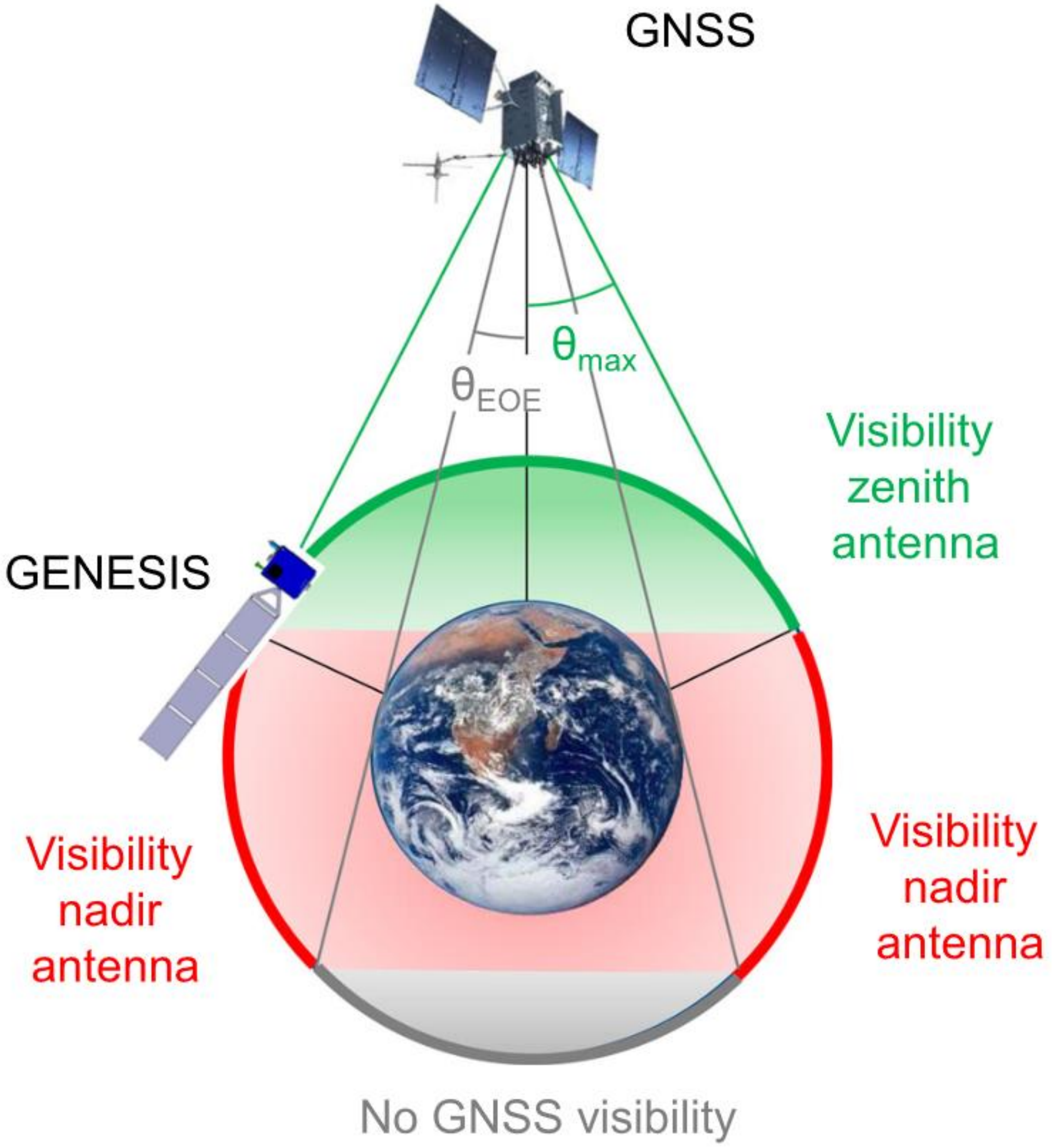
- $\Theta_{EOE} = \arcsin(\mathrm{REarth} + \mathrm{rGNSS})$
- $\Theta_{max} = \arcsin(\,(R_{Earth} + \mathrm{hGENESIS}) + \mathrm{rGNSS})$

Visibility of Zenith antenna:

$\Theta < \Theta_{max}$ and $r_{GNSS\text{-}GENESIS} < \sin(\Theta_{max})\, r_{GNSS}$

Visibility of Nadir antenna:

$\Theta_{EOE} < \Theta < \Theta_{max}$

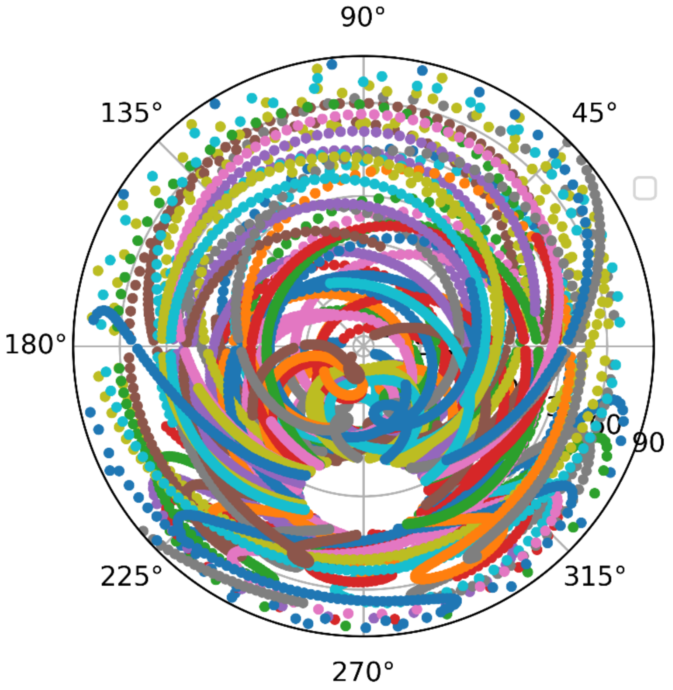


GNSS visibility Model for GENESIS

In order to monitor the measurements received by GENESIS, relative orbits were defined between GENESIS and each GNSS satellite.

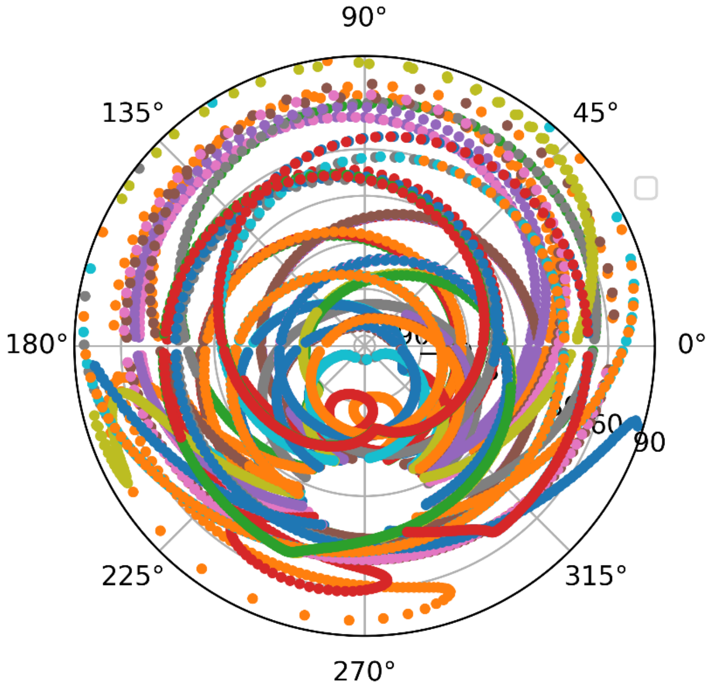
GENESIS Orbit: altitude **9000** km, inclination **64** deg, eccentricity **0**
Reference frame: RTN (rotating)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec

GPS constellation



Skyplot of the GNSS satellites tracked by GENESIS

GALILEO constellation



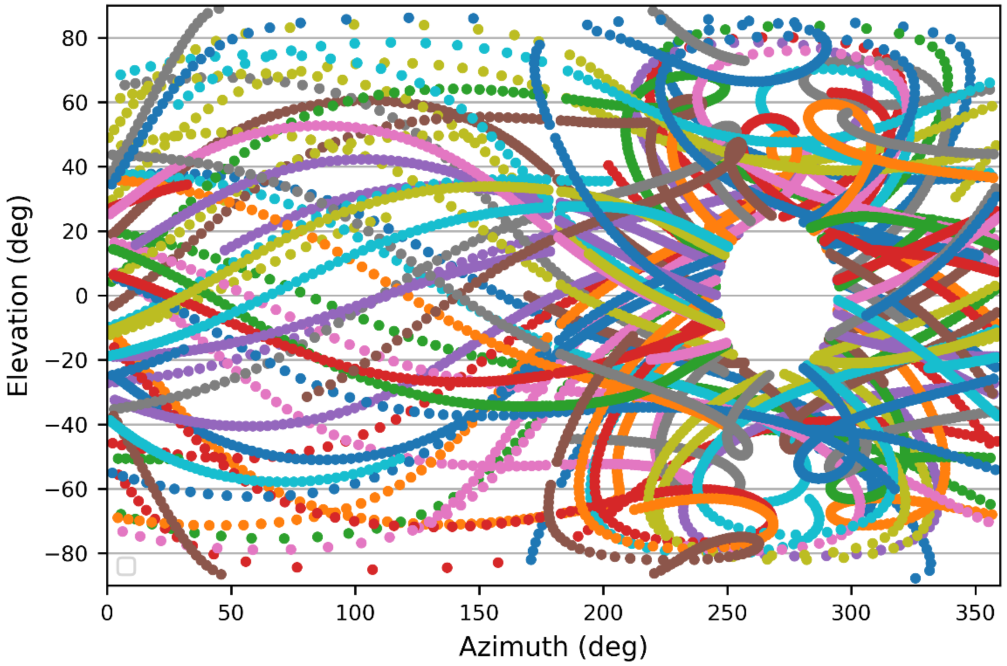
GNSS visibility Model for GENESIS

In order to monitor the measurements received by GENESIS, relative orbits were defined between GENESIS and each GNSS satellite.

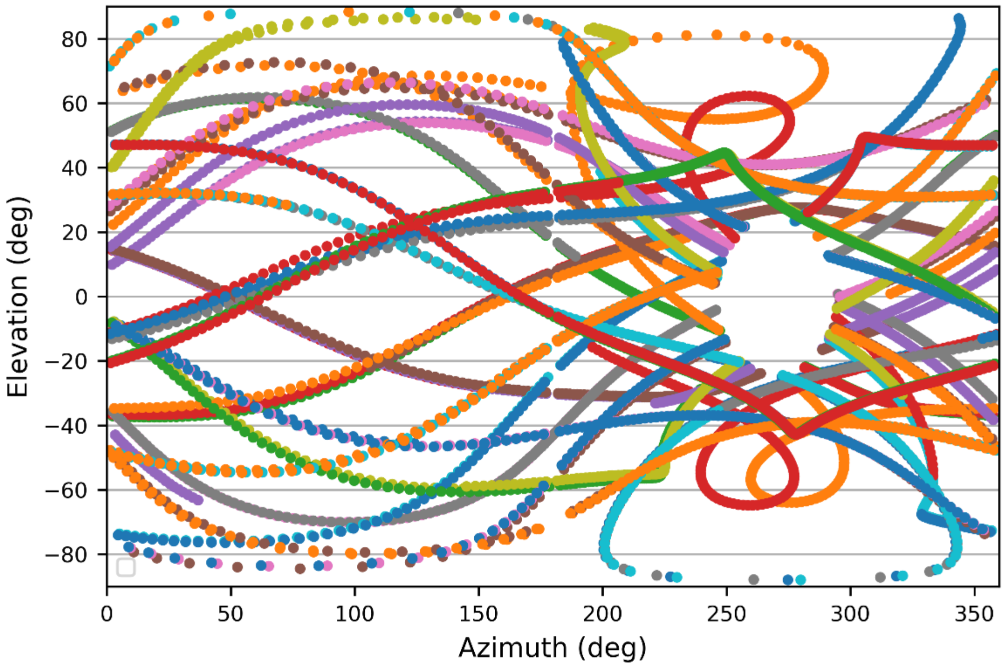
GENESIS Orbit: altitude **9000** km, inclination **64** deg, eccentricity **0**
Reference frame: RTN (rotating)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec

GNSS satellites tracked by GENESIS
Elevation and azimuth wrt the GENESIS orbit

GPS constellation



GALILEO constellation

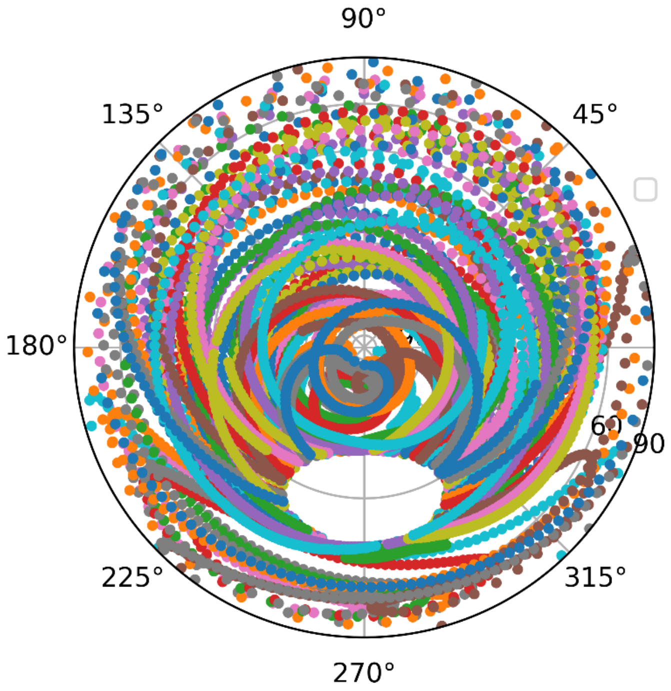


GNSS visibility Model for GENESIS

In order to monitor the measurements received by GENESIS, relative orbits were defined between GENESIS and each GNSS satellite.

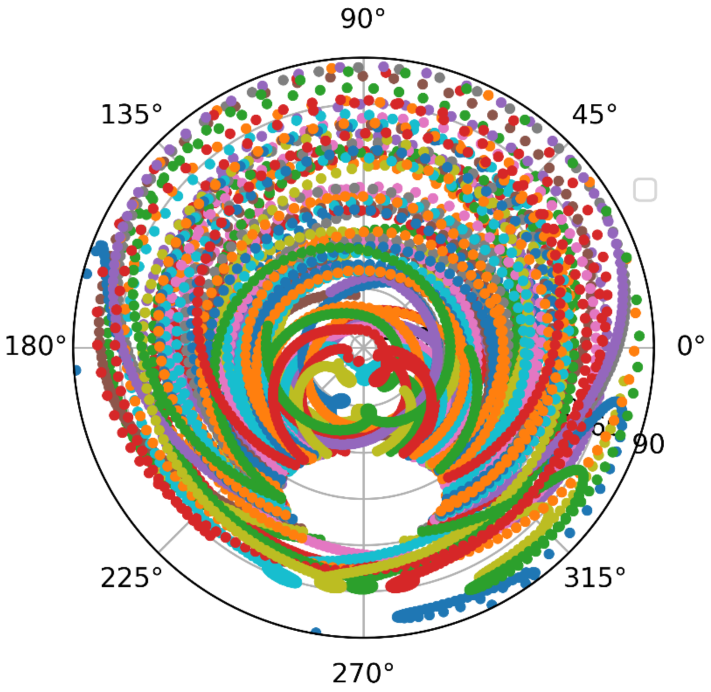
GENESIS Orbit: altitude **6000** km, inclination **95.5** deg, eccentricity **0**
Reference frame: RTN (rotating)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec

GPS constellation



Skyplot of the GNSS satellites tracked by GENESIS

GALILEO constellation



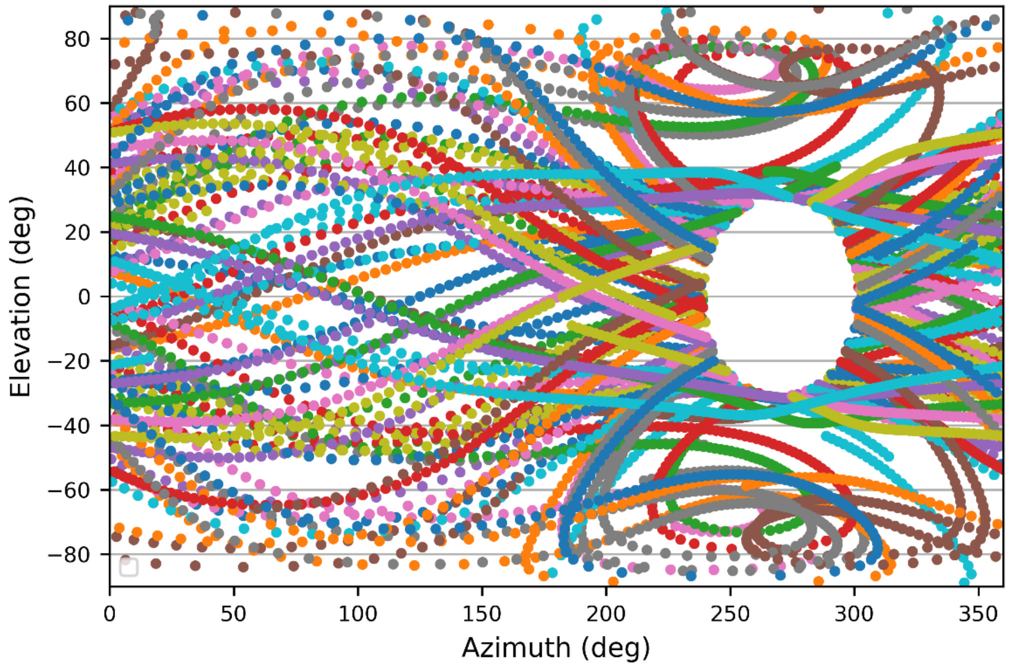
GNSS visibility Model for GENESIS

In order to monitor the measurements received by GENESIS, relative orbits were defined between GENESIS and each GNSS satellite.

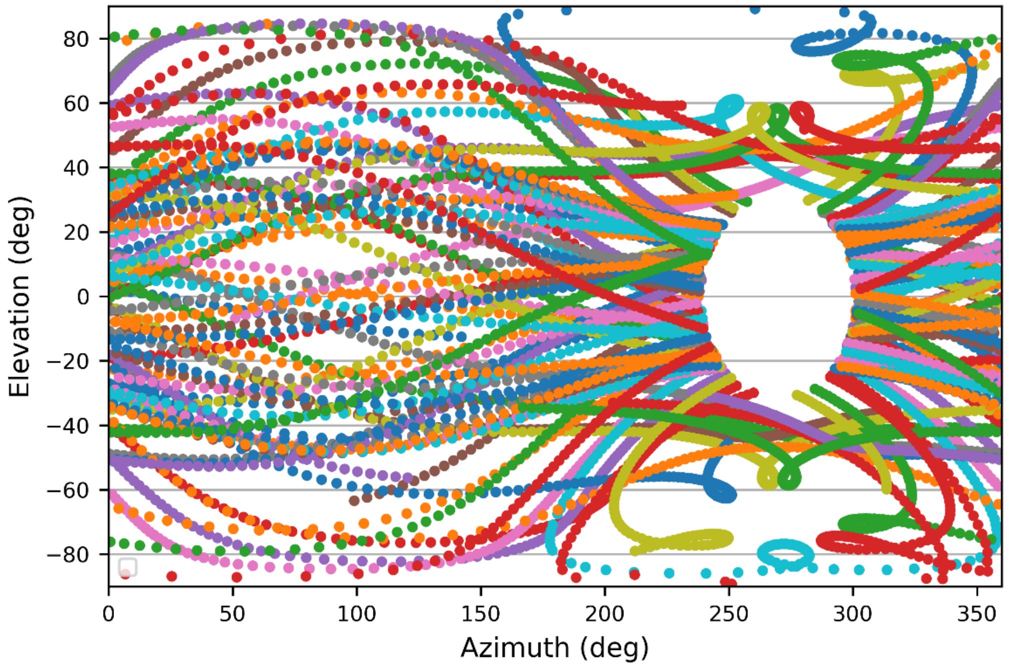
GENESIS Orbit: altitude **6000** km, inclination **95.5** deg, eccentricity **0**
Reference frame: RTN (rotating)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec

GNSS satellites tracked by GENESIS
Elevation and azimuth wrt the GENESIS orbit

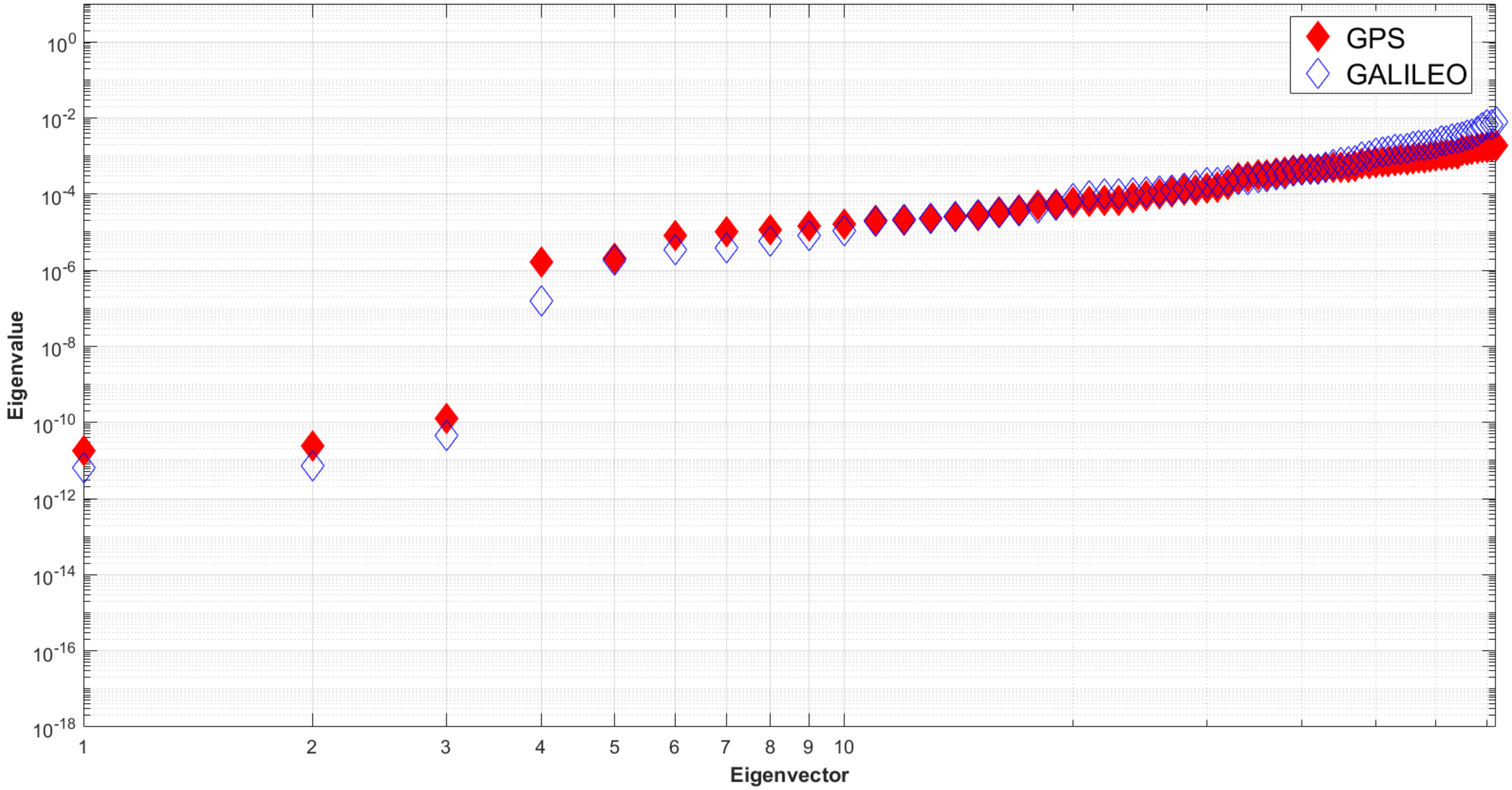
GPS constellation



GALILEO constellation



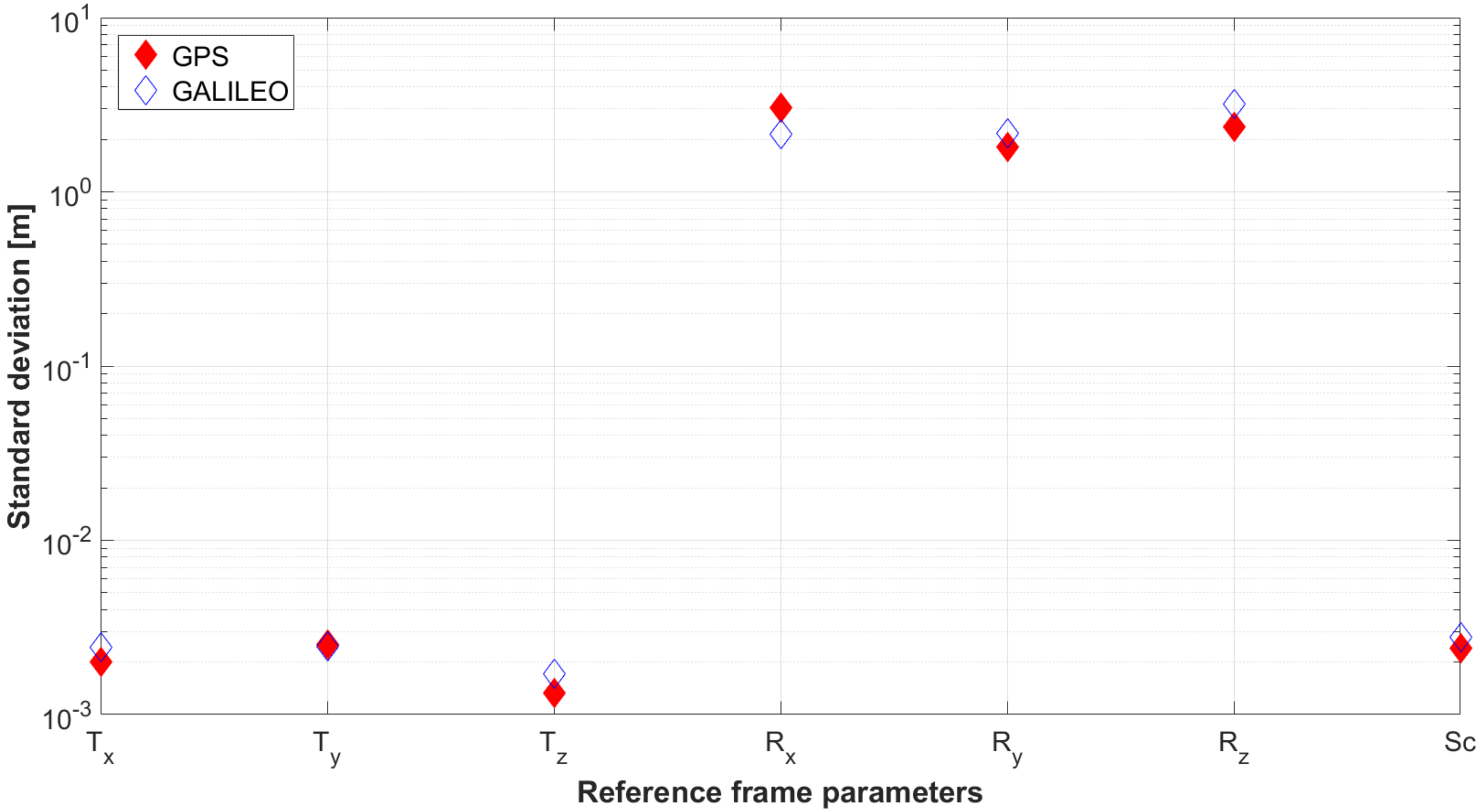
Spectral Analysis of the unconstrained NEQ matrix



Reference system effect

$C_\theta = (E\ N\ E^T)^{-1}$ (*Sillard and Boucher 2001*)

θ: frame parameters, E: Jacobian of 7p Helmert transf, N: **unconstrained** NEQ matrix



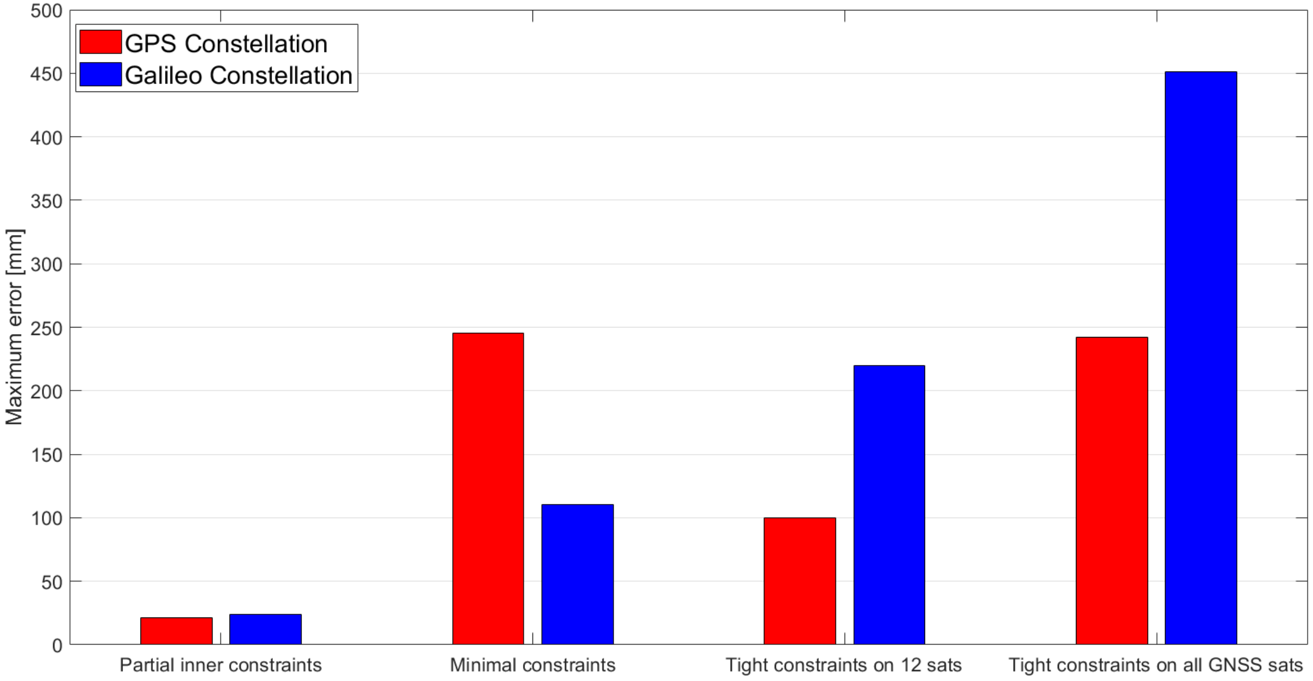
Influence of prior errors in GNSS initial positions and datum definition strategy

Maximum 2-norm of relative state error (estimated minus true orbit) over a fixed simulation period of 24 hours for the GENESIS satellite.

GENESIS Orbit: altitude **9000** km, inclination **64** deg, eccentricity **0**
Reference frame: GCRS (inertial)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec

Impact of datum definition strategy on the absolute orbit determination of GENESIS satellite

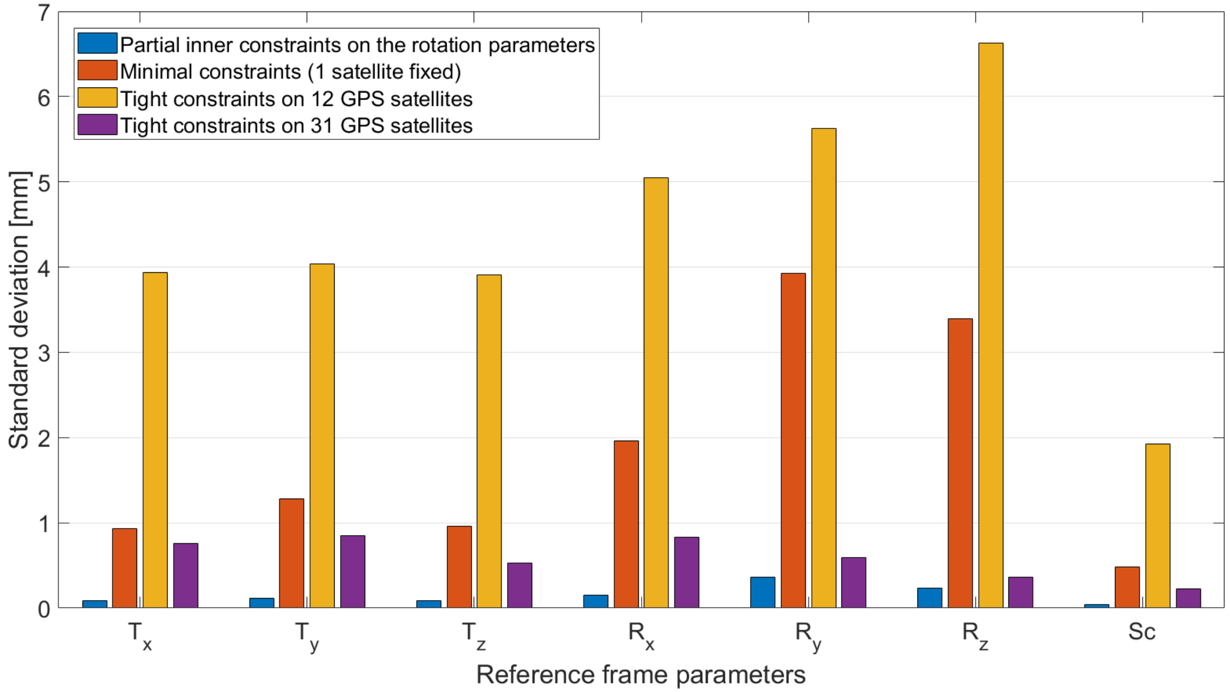
Scenario **3 cm**



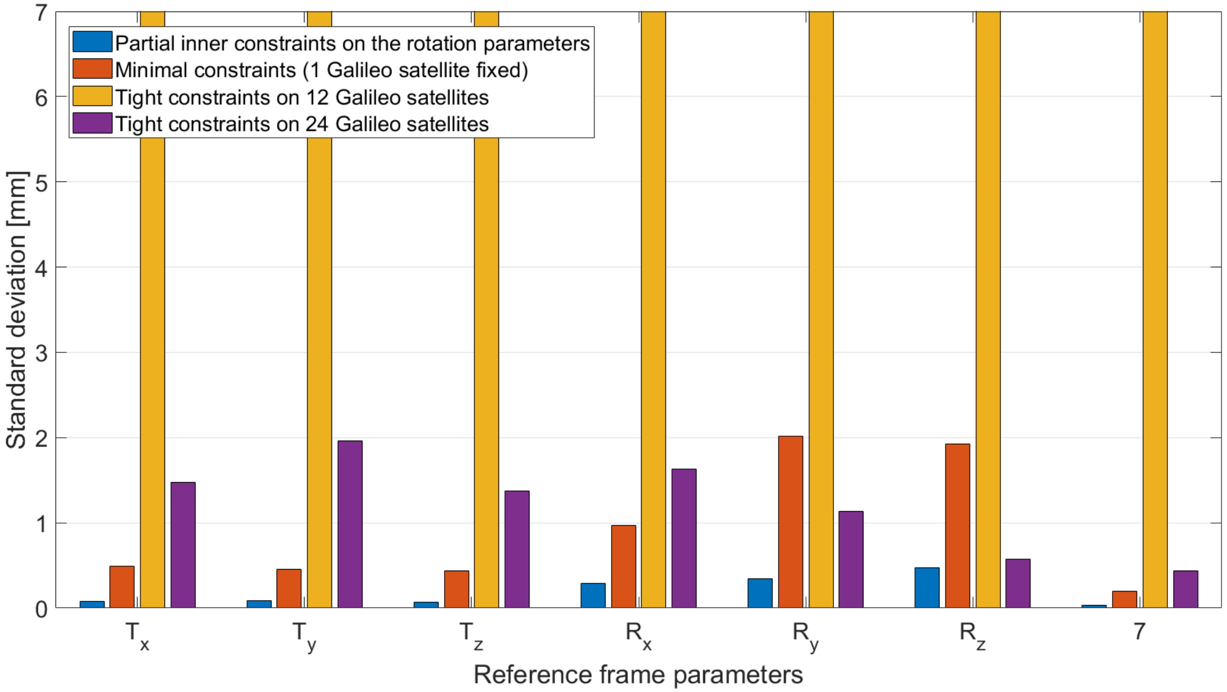
Assessment of estimated orbit at RF realization level

$$C_x = \hat{\sigma}^2_{apost}(N + H^T W H)^{-1} \Rightarrow C_\theta = (EE^T)^{-1} E C_x E^T (EE^T)^{-1}$$

GPS Constellation



Galileo Constellation



Impact of datum definition strategy, on the absolute orbit determination of GENESIS satellite, at the reference frame level.

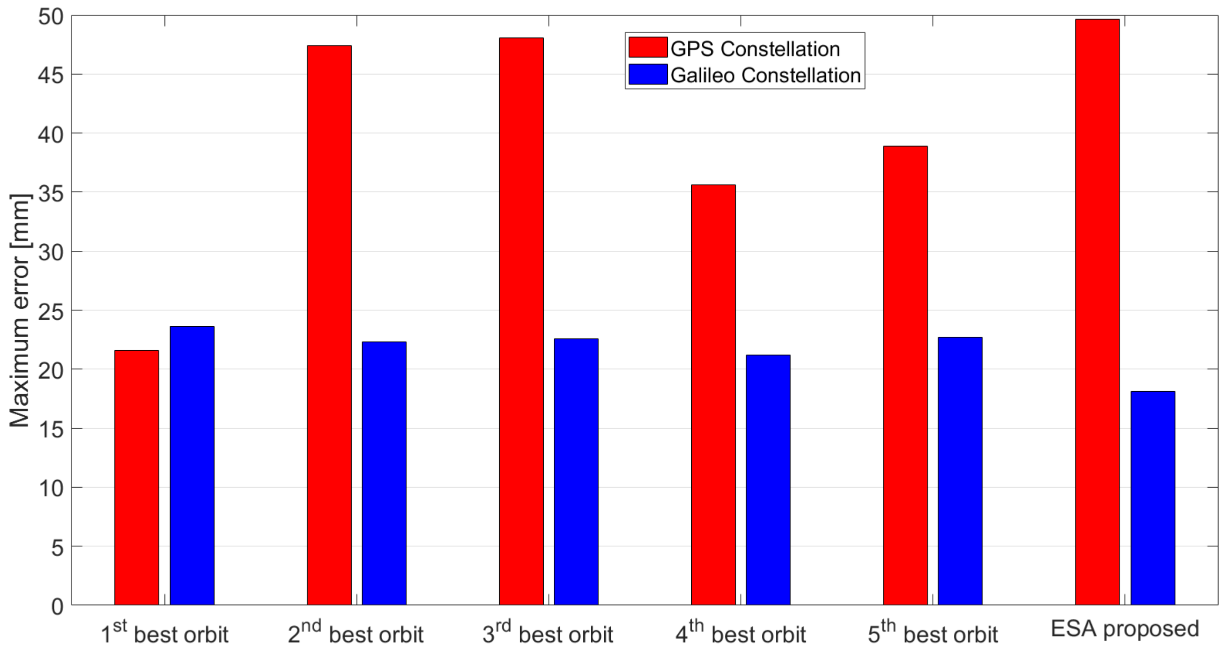
Influence of prior errors in GNSS initial positions and satellite constellation geometry

Maximum 2-norm of relative state error (estimated minus true orbit) over a fixed simulation period of 24 hours for the GENESIS satellite.

GENESIS Orbit: Altitude: 9000 km, Inclination: 64 deg
8250 km, 57 deg
8250 km, 67 deg
9250 km, 52 deg
9250 km, 67 deg
6000 km, 95.5 deg

Reference frame: GCRS (inertial)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec
Datum definition strategy: Partial inner constraints on the rotation parameters

Scenario **3 cm**

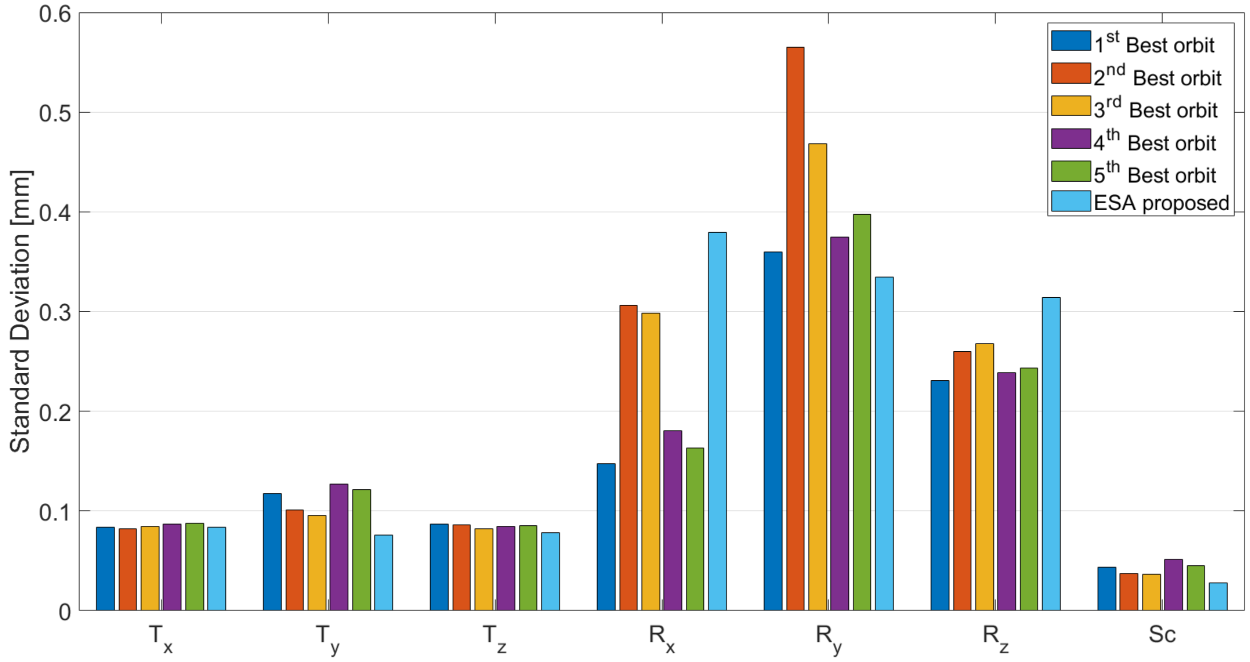


Impact of GENESIS orbit on the absolute orbit determination

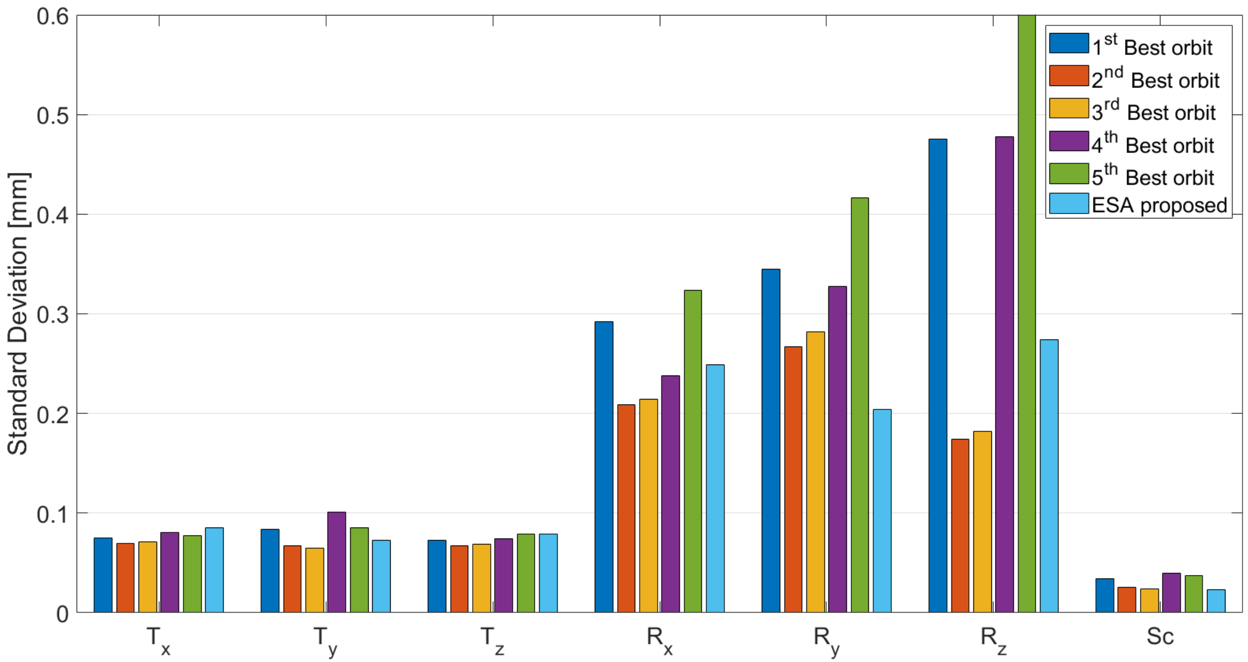
Assessment of estimated orbit at RF realization level

$$C_x = \hat{\sigma}_{apost}^2 (N + H^T W H)^{-1} \quad \Rightarrow \quad C_\theta = (EE^T)^{-1} \mathrm{E} C_x E^T (EE^T)^{-1}$$

GPS Constellation



Galileo Constellation



Absolute orbit determination: Impact of GENESIS orbit at the reference frame level.

Influence of Solar Radiation Pressure and satellite constellation geometry

Simulation of measurements: Box-wind model

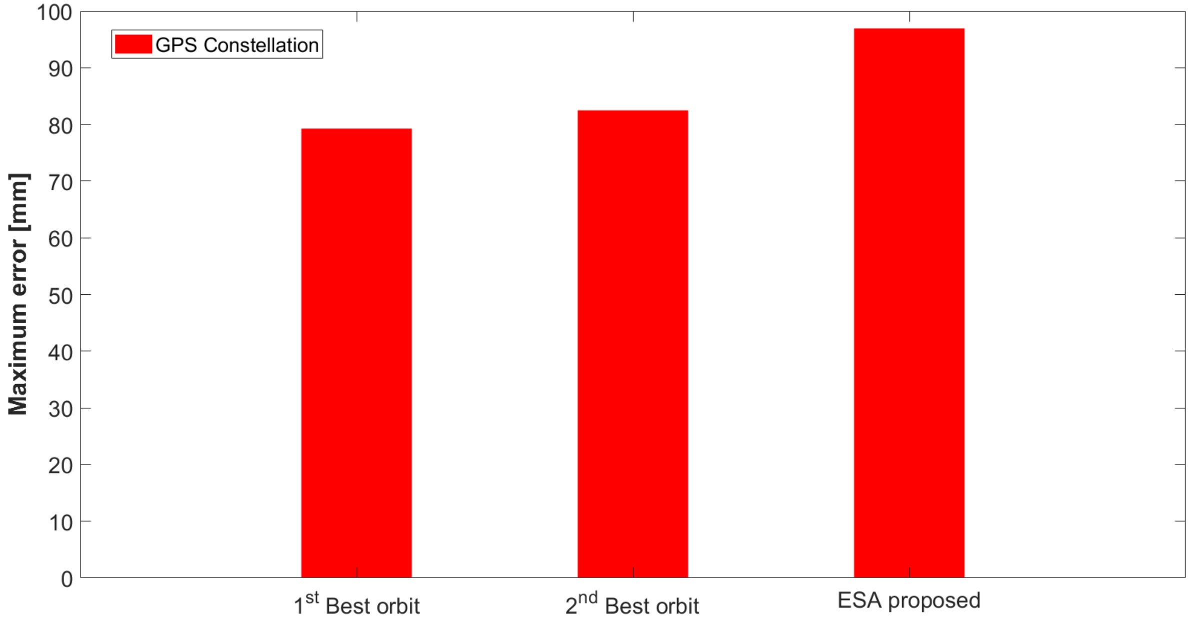
Orbit determination: Cannonball model

GENESIS Orbit: Altitude: 9000 km, Inclination: 64 deg
8250 km, 57 deg
6000 km, 95.5 deg

Reference frame: GCRS (inertial)
Time-period of data: 12 and 14 hours
Sampling rate: 120 sec
Datum definition strategy: Partial inner constraints on the rotation parameters

Maximum 2-norm of relative state error (estimated minus true orbit) over a fixed simulation period of 24 hours for the GENESIS satellite.

Scenario **3 cm**

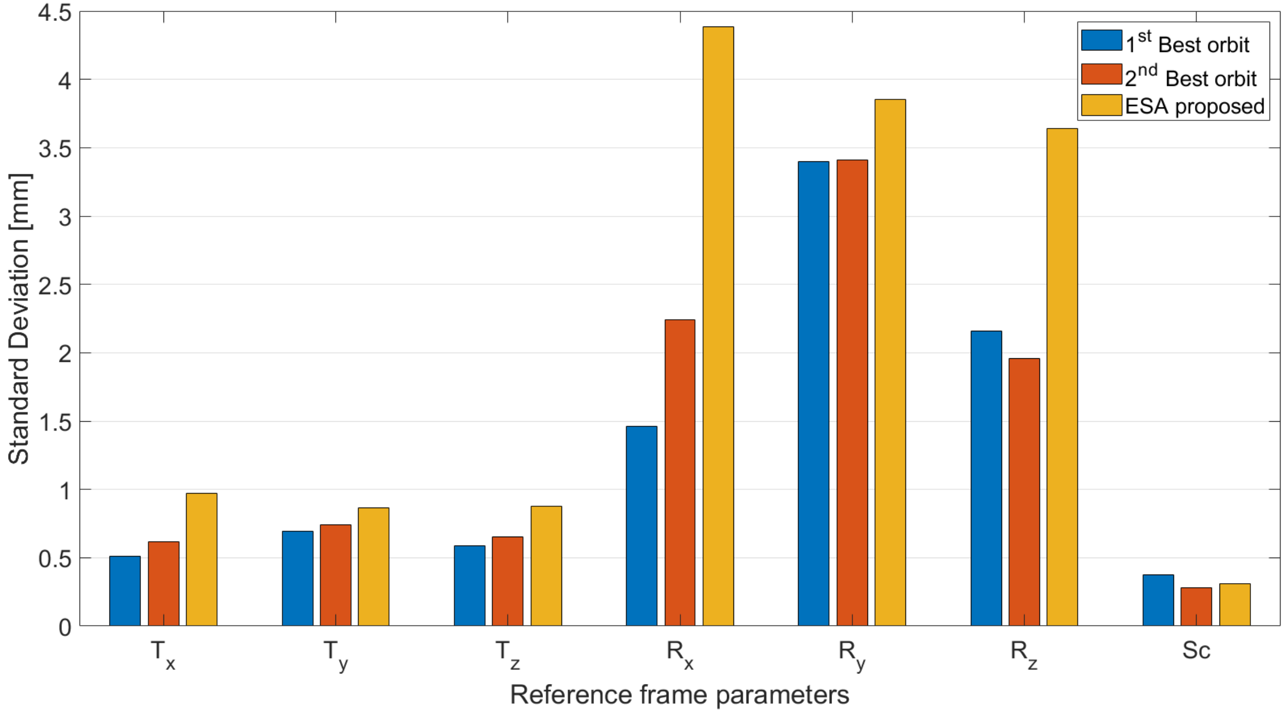


Impact of SRP on the GENESIS absolute orbit determination.

Assessment of estimated orbit at RF realization level

$$C_x = \hat{\sigma}^2_{apost}(N + H^T W H)^{-1} \quad \Rightarrow \quad C_\theta = (EE^T)^{-1} E C_x E^T (EE^T)^{-1}$$

GPS Constellation



GENESIS absolute orbit determination: Impact of SRP mismodeling at the reference frame level.

Summary

Relative orbit determination:

- The precision of the relative orbit determination problem depends on the assumptions made to derive the relevant analytical or not equations (shape of chief orbit and linearisation order).
- The rank deficiency of the relative orbit determination problem depends on the kinematic and dynamical model.

Absolute orbit determination:

- The rank deficiency of the absolute orbit determination problem is equal to 3 (3 orientation reference frame parameters) for both models.
- The datum definition strategy has one of the biggest impacts on the final solution (estimated parameters or internal precision).
- The partial inner constraints provide the best solution and they have the smalest dependency on the satellite constellation.
- The tight constraints (Cartesian or Keplerian model) alter the geometry of the constellation.

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Merci!