Impact of direct-digital-synthesizer finite resolution on atom gravimeters

R. Karcher, F. Pereira Dos Santos, and S. Merlet
LNE-SYRTE, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, 61 Avenue de l’Observatoire, F-75014 Paris, France

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We report on the study of the impact of the finite resolution of the chirp rate applied on the frequency difference between the Raman lasers beamsplitters onto the phase of a free fall atom gravimeter. This chirp induces a phase shift that compensates the one due to gravity acceleration, allowing for its precise determination in terms of frequencies. In practice, it is most often generated by a direct digital synthesizer (DDS). Besides the effect of eventual rounding errors, we evaluate here the bias on the \( g \) measurement due to the finite time and frequency resolution of the chirp generated by the DDS, and show that it can compromise the measurement accuracy. However, this effect can be mitigated by an adequate choice of the DDS chirp parameters resulting from a trade-off between interferometer phase resolution and induced bias.

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I. INTRODUCTION

Light pulse atom interferometry is now a mature technique, specially fit to design inertial sensors [1,2] which compete with state-of-the-art classical sensors [3–6]. Among these, cold atom gravimeters based on a three pulse Mach-Zehnder sequence [2] are absolute gravimeters which have demonstrated the ability to perform continuous measurements with better short-term sensitivities [3,7,8] and long-term stabilities than their classical counterparts [4]. They have also reached comparable or better accuracies by the careful evaluation of their systematic effects. The control of these systematic effects crucially depends on the control of the phase difference between the light beamsplitters, which get imprinted onto the atomic wave packets at each pulse during the interferometer. Both the spatial and temporal laser phase fluctuations, which have been extensively studied in the past, such as in [9–14], need to be perfectly controlled to ensure the accuracy of the measurement of the local gravity acceleration \( g \).

In gravimeters based on atom interferometry, the measurement of \( g \) is derived from the determination of the Doppler frequency chirp induced by the free fall of the atoms onto the lasers. In practice, the laser frequency difference can be adjusted at each pulse by switching between three fixed frequencies, a different one for each pulse, using an agile and stable synthesizer, such as in [9]. However, the finite frequency resolution of the synthesizer of 0.23 Hz allowed therefore a coarse step size of 0.23 rad when scanning the interferometer phase. Thus more than three frequencies were finally used to better control the interferometer phase. In addition, a frequency sweep (over a limited frequency range) was repeatedly applied during the pulses, as a chirp was required to ensure the efficiency of the velocity preselection, which relied on a 380-\( \mu \)s-long Raman pulse. This resulted in a complicated frequency control system, with four different synthesizers. An alternative allowed by current technology consists in applying a continuous frequency chirp over the whole interferometer duration, using a single synthesizer. Nevertheless, any parasitic frequency or phase shifts with respect to the ideal chirp will thus induce errors on the \( g \) measurement, such as, for instance, the bias due to frequency-dependent radio-frequency delays in the electronics studied in [9], which can be rejected by a proper symmetrization of the chirp with respect to the mid pulse between both directions of the effective wave vector.

In the following, we investigate the impact of the finite frequency resolution of the oscillator used to chirp the lasers frequency difference during the interferometer. This chirp being synthesized with discrete steps out of a clock reference signal which we consider here as perfectly stable, produces time-periodic frequency errors, leading to time-periodic errors on the interferometer phase. After a brief description of our measurement method of \( g \), we first calculate the biases in the \( g \) measurements induced by rounding and frequency errors arising from the finite resolution of the direct digital synthesizer (DDS) we use. Then, we performed measurements of these biases, which we find in agreement with numerical calculations. This analysis allows us to accurately evaluate the impact of these effects on our absolute cold atom gravimeter.

II. NUMERICAL SIMULATIONS: FREQUENCY ERROR OF THE CHIRP

A. \( g \) measurement protocol

In the most mature atom gravimeters, \( g \) measurements are performed via Raman interferometry on free falling atoms. There, the interferometer is realized thanks to the sequence of three two-photon stimulated Raman transitions of duration \( \tau-2\tau-\tau \) separated by a free evolution time \( T \), which respectively split, redirect, and finally recombine the atomic wave packets. As mentioned above, during the free fall, the Doppler shift modifies the resonance condition which has to be compensated for by applying a frequency chirp to the frequency difference between the Raman lasers. This chirp
rate $\alpha$ adds a phase shift ($\alpha T^2$) to the interferometer that, when properly tuned ($\alpha = \alpha_0 = k g$), exactly compensates the phase shift induced by the gravity acceleration ($-k g T^2$), $k$ being the effective wave vector of the Raman transition. Remarkably, this leads to a dark fringe in the fringe pattern obtained when scanning $\alpha$, whose position does not depend on the interferometer duration [15]. The measurement of the gravity acceleration, and its fluctuations, can thus finally be obtained from the precise tracking of this dark fringe and from the determination of the corresponding value of the chirp rate $\alpha_0$. Many of the systematic effects, such as related to one-photon light shifts or the quadratic Zeeman effect, are rejected by averaging $g$ measurements over opposite directions of the effective wave vector $k$ [10,16]. Indeed, unlike the gravity phase shift, these systematics do not depend on the direction of $k$ [9,10]. However, the efficiency of this rejection depends on the superposition of the two trajectories for the two interferometer configurations. In practice, the difference in the momenta imparted to the atoms leads to small differences between the trajectories, of about a few millimeters. These are much smaller than the few tens of centimeters of the free fall distance, which guarantees the efficiency of this rejection technique [17,18].

B. Finite resolution of the DDS: Rounding error

In our experiment, we generate the chirp thanks to a direct digital synthesizer (AD9852, from Analog Devices). It is a 48-bit DDS, clocked at 300 MHz, which corresponds to a frequency resolution of about 300 MHz/2$^{48} \approx 1$ $\mu$Hz. Within its chirped mode of operation, we can control both the temporal step $\delta t$ and the frequency step $\delta f$, which can be tuned to design the desired chirp with a rate $\alpha$ (inset, Fig. 1). The temporal step $\delta t$ is defined by a 20-bit word, and can thus be varied in between 1 and $2^{20}$ cycles of the master clock, i.e., between 3.3 ns and 3.5 ms. In our case, the frequency chirp required to compensate for gravity, of about 25 MHz/s, is realized with frequency steps of 125 Hz every 10 $\mu$s. Indeed, the DDS is compared in a phase frequency detector to the beat note between the Raman lasers (after a down conversion, realized by mixing the beat note with a 7 GHz oscillator, and a subsequent division by a factor 2). At last, this results in a frequency chirp resolution of about $2 \times 1$ $\mu$Hz/10 $\mu$s $\simeq$ 0.2 Hz/s. The corresponding resolution in terms of gravity acceleration is $8.3$ $\mu$Gal (1 $\mu$Gal = 10 nm/s$^2$), which is smaller than the shot to shot noise on the $g$ measurement (peak to peak of $\sim$100 $\mu$Gal). This resolution is thus not a limit in our measurement protocol, which consists of a digital integrator that steers the chirp rate onto the central fringe [19]. However, the correction applied by the lock system onto the chirp rate is impacted by this resolution, as there is a difference between the corrections we calculate at each measurement cycle, and the ones that are actually applied, due to rounding errors. These errors, which amount on average to $\pm 1/2$ bit (or $\pm \sim 4.15$ $\mu$Gal), could in principle be eliminated by recording the applied changes instead of the requested ones. On the other hand, and quite remarkably, these errors cancel when averaging over the two opposite $k$ directions, as they lead to underestimating the chirp rate when the frequency is ramped up and overestimating it when ramped down.

Finally, more important than the effect of these roundings, the DDS cannot exactly produce change of the frequency/phase of the Raman phase difference because of its stepwise character. This leads to systematic errors, which we discuss and evaluate in the following section.

C. Finite resolution of the DDS: frequency error

As shown in Fig. 1, the chirp is realized by the DDS by incrementing the laser frequency by $\delta f$ every $\delta t$, leading to a chirp rate $\alpha$. The deviation from a perfect linear frequency chirp induces a periodic frequency error represented by the sawtooth function displayed in Fig. 1. This function is determined by the two parameters $\delta f$ and $\delta t$. Also, we name $\Delta f$ the time difference between the beginning of the chirp and the beginning of the interferometer sequence. Note that for the chirp displayed in Fig. 1, $\Delta f = 0$.

To calculate the bias on the interferometer phase, we use the sensitivity function $g_s$ [20], which describes the impact of frequency fluctuations $\Delta f(t)$ onto the interferometer phase $\Delta \Phi$:

$$\Delta \Phi = \int_{-\infty}^{+\infty} g_s(t) 2 \pi \Delta f(t) dt.$$  

(1)

It is an odd function: $g_s(-t) = -g_s(t)$. It is given, for $t > 0$, by

$$g_s(t) = \begin{cases} 
\sin(\Omega_R t) & \text{for } 0 < t < \tau \\
1 & \text{for } \tau < t < T + \tau \\
-\sin[\Omega_R(T - t)] & \text{for } T + \tau < t < T + 2\tau \\
0 & \text{for } t > T + 2\tau.
\end{cases}$$

(2)

From now on and throughout the rest of the paper, we take, without loss of generality, the time origin $\tau = 0$ at the center of the middle $\pi$ pulse. $\Omega_R$ is the Rabi frequency, given by

FIG. 1. Frequency chirp applied to the Raman lasers frequency difference. The perfect linear case is the straight line represented in black. The actual chirp is composed of frequency steps of $\delta f = 125$ Hz with temporal steps $\delta t = 10 \mu$s and a null delay $\Delta t$. The inset shows the parameters of the DDS ($\delta t$, $\delta f$, $\Delta t$). Bottom: residual or frequency difference $\Delta f$ between the ideal and actual chirps, represented as a sawtooth function.
FIG. 2. Calculated bias $g_\delta t$ as a function of the temporal step of the chirp $\delta t$ for $T = 80$ ms, $\tau = 16$ $\mu$s, and $\Delta t = 0$ $\mu$s. (a) $T$ fixed. (b) $T$ adjusted to the nearest integer multiple of $\delta t$.

$\Omega_R = \pi / 2 \tau$. To evaluate the bias on the $g$ measurement, we thus simply integrate Eq. (1) modulated by the sawtooth function over the duration of the interferometer.

Figure 2(a) shows the calculated bias for $\delta t$ ranging from 1 to 100 $\mu$s for our typical interferometer parameters ($\tau = 16$ $\mu$s, $T = 80$ ms), and for a delay $\Delta t = 0$ $\mu$s. The bias is found to exhibit rapid variations, inside an envelop that is growing with $\delta t$. A smoother behavior is actually found when considering the particular case where the free evolution time $T$ is a multiple of $\delta t$. There, the contributions in the above integral from the two free evolution phases compensate. For $T = 80$ ms exactly, this corresponds to specific values of $\delta t$ only, for which the corresponding biases are displayed as red circles in Figs. 2(a) and 2(b). This condition also holds if adjusting, only very slightly, $T$ for each $\delta t$. The corresponding biases are displayed as the solid red line in Fig. 2(b).

For $\delta t = 10$ $\mu$s the expected bias (0.06 $\mu$Gal) is found to be negligible, but it tends to increase as a function of $\delta t$, leading to a bias as large as $\sim 100$ $\mu$Gal for $\delta t = 100$ $\mu$s.

For a given $\delta t$, two parameters can be easily modified to modulate the impact of frequency errors:

1. First, one can vary the delay $\Delta t$ between the beginning of the interferometer with respect to the beginning of the chirp. This parameter is varied between 0 and 100 $\mu$s in Fig. 3, which displays the results of the calculated bias as a function of $\Delta t$ for $\delta t = 10$, 20, 50, and 100 $\mu$s.

2. Second, one can change the Raman pulse durations. Figure 4 is the result of the calculated bias as a function of $\tau$ for $\delta t = 50$ and 100 $\mu$s. To illustrate the effect, we choose $\delta t$ for which the effect is large. In this calculation, we use realistic durations of $\tau$ of the order of a few tenths of microseconds and there is no delay ($\Delta t = 0$). The result is a periodic function which dampens for increasing values of $\tau$. This illustrates the fact that when $\tau \gg \delta t$, the bias averages down to zero.

In a different manner than the rounding error which is rejected by the $k$-reversal algorithm, this bias can be canceled with a proper choice of the Raman pulse duration. A simple choice would be to use $\tau = \delta t / 2$, for which this effect is calculated to be null.

III. MEASUREMENTS AND COMPARISON WITH NUMERICAL CALCULATIONS

In this section, we report on the experimental study of the impact of the DDS finite resolution on the measurements of...
our cold atom gravimeter, which we compare to the calculations described in the previous section.

A. Experimental setup

Here we briefly present the experimental setup we have used to perform the measurements. A more detailed description can be found in [10]. We start by trapping a sample of $^{87}$Rb atoms in a three-dimensional MOT, and further cool it down to $2\mu K$ with far detuned molasses. About $10^8$ atoms are released in free fall for 200 ms and prepared in the $|m_F=0\rangle$ magnetic state in a narrow vertical velocity distribution. We then apply the interferometer sequence which lasts a total time of $2T=160$ ms. As discussed earlier, the Raman lasers are kept resonant with the atoms during the free fall, by actually chirping one of the two Raman lasers [21].

The phase difference in this two-wave interferometer modulates the population $N_1$ and $N_2$ at the two output ports, which are measured at the bottom of the chamber by an internal-state-selective fluorescence detection, using the state labeling property of Raman transitions [22]. From these measurements we derive the transition probability $P$, which is given by $P = N_1/(N_1 + N_2) = \frac{1}{2} [1 + C \cos(\Delta \Phi)]$, where $C$ is the contrast of the interferometer and $\Delta \Phi = -kgT^2 + \alpha T^2$ the total interferometer phase shift. Gravity measurements are performed by steering $\alpha$ towards $kg$, so as to nullify $\Delta \Phi$. The measurement repetition rate is about 3 Hz.

In the following, the measurements are performed in a differential way by alternating gravity measurements with different sets of parameters $S = [\delta t, \Delta t, \tau]$ and directions of the effective wave vector. Our conventional parameters are $S_{\text{ref}} = [10\mu s, 0\mu s, 16\mu s]$.

In practice, we compare a first pair of configurations, $g(k_1, S_{\text{ref}})$ and $g(k_{\downarrow}, S_{\text{ref}})$, with opposite effective wave vectors and reference sets of parameters, with other pairs of configurations $g(k_1, S)$ and $g(k_{\downarrow}, S)$. In most of the measurements presented below, the quantity of interest is the difference between the average measurements of the pairs:

\[
\Delta g = \frac{g(k_1, S) + g(k_{\downarrow}, S)}{2} - \frac{g(k_1, S_{\text{ref}}) + g(k_{\downarrow}, S_{\text{ref}})}{2}.
\]

B. Evaluation of the rounding error

At first, we tried to evaluate the rounding error. As it is rejected by the average over opposite directions of $k$, we consider here the differences between single-$k$ directions such as

\[
\Delta g_{\text{RE}} = \frac{[g(k_1, S) - g(k_{\downarrow}, S_{\text{ref}})] - [g(k_{\downarrow}, S) - g(k_1, S_{\text{ref}})]}{2}.
\]

Results are presented in Fig. 5 (taking as a reference $\Delta g_{\text{RE}}^{\mu s=10 \mu s}$ for which the bias is expected to amount to $\approx -4.15$ $\mu$Gal), together with the calculated errors in red.

These differential measurements are found to converge towards $4.15$ $\mu$Gal, corresponding in fact to a null bias for asymptotically large values of $\delta t$. The measurements agree with the calculated values, within their uncertainties, except for $\delta t = 1$ $\mu s$.

C. Measurement of the impact of the finite resolution of the DDS

More interesting, we performed differential $g$ measurements [as in Eq. (3)] as a function of $\delta t$, using $\delta t_{\text{ref}} = 10$ $\mu s$.
as a reference parameter, in order to probe the influence of
the finite duration of the time step of the chirp. The results
are displayed in Fig. 6. They show a good agreement with
the results of the calculation, which are represented in red. The
residues, also plotted in Fig. 6, are lower than 2 μGal even for
the largest biases.

As discussed in Sec. II C, we then studied the possibility
to modulate this effect by either modifying the offset Δt or
by changing the duration of the Raman pulses τ. We started
by varying the offset Δt, from 0 to δt, for two different
values of δt = 50 and 100 μs. These two examples were
chosen because of the large biases they are expected to induce.
Indeed, according to the calculations performed above, this
bias is expected to oscillate with a maximum amplitude of
only 0.5 μGal for δt = 10 μs, which would be hard to resolve.
The measurements were performed in a differential manner
with δt = 50 μs (100 μs, respectively) and Δtref = 0 as a
reference. The results of these measurements are plotted as
blue circles in Figs. 7(a) (δt = 50 μs) and 7(b) (δt = 100 μs),
together with the results of the calculation, displayed as
continuous red lines. Once again the measurements agree with
the numerical calculations.

Finally, we tried to cancel this bias for the particular case
of δt = 50 μs, by using a Raman sequence of τ-2τ-τ =
25 μs-50 μs-25 μs. We use here the classical sequence
τ-2τ-τ = 16 μs-32 μs-16 μs and δt = 50 μs as a reference.
We obtain a difference of 37.3(1) μGal close to the expected
bias of 37.1 μGal. The measured bias on the difference being
comparable to the expected bias obtained with the simulation,
we are confident that the bias is indeed null for the (25-50-25)
configuration.

IV. CONCLUSION

We have evaluated the impact of the finite temporal resolu-
tion of the direct digital synthesizer we use to chirp the Raman
laser frequency difference during the interferometer in our
cold atom gravimeter. This chirp is essential to maintain the
resonance condition of the Raman lasers during the free fall
of the atoms. We have discussed and measured the rounding
error, of 1/2 bit on average, arising from the finite frequency
resolution of the DDS. Increasing the duration of the time step
δt decreases the amplitude of the rounding, and improves the
resolution on the interferometer phase, which could be useful
in a very low noise environment.

However, this results in larger frequency deviations with
respect to the ideal linear chirp, inducing potentially large
biases on the interferometer phase. This effect can in principle
be compensated for by a proper choice of either the duration
of the Raman pulses or the time delay between the chirp
sequence and the interferometer sequence. Our typical mea-
surement condition δt = 10 μs results from a compromise be-
tween the resolution and the bias on the interferometer phase.
The rounding error, of 4.15 μGal, is rejected by the k-reversal
algorithm, whereas the bias arising from the frequency error

FIG. 6. Measurement of Δg as a function of δt, the time step of
the chirp (blue circles). The continuous line in red is the calculated
bias, which is described in the previous section. The residues be-
tween theory and measurements are displayed at the bottom as blue
open circles.

FIG. 7. Measurement of Δg as a function of the offset Δt between the starts of the chirp and the interferometer (in blue circles). (a) The
reference is δt = 50 μs and (b) δt = 100 μs.
is as small as 0.06 μGal, well below the current accuracy of the instrument, of the order of 2 μGal [14]. But, other choices for the time step can lead to much larger biases, from which g measurements would need to be corrected.

We finally stress that the calculations have been performed with an instrument transfer function which corresponds to square-shaped Raman laser pulses. As a follow-up study, one could investigate the impact of either more realistic or deliberately shaped pulses [23] onto the errors related to the finite resolution of the frequency chirp.

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