PREDICTION OF THE CHANDLER WOBBLE

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ABSTRACT. Chandler wobble amplitude have been decreasing in 2010s as in 1930s. We try to predict its future behaviour through prediction of its complex envelope. The excitation of the Chandler wobble (ChW) reconstructed by Panteleev's filter was also analized. The equation for the complex envelope propagation through the Euler-Liouville equation was derived. Similarities with the climate change characteristics are discussed.

1. INTRODUCTION

Chandler wobble (ChW) is one of the crucial component of the Earth's polar motion (PM). It was discovered in 1891 by Seth Carlo Chandler, astronomer and economist, who manually processed more than 33000 astronomical observations in order to detect variations of latitude. The period of ChW, now estimated as 433 days (Vicente and Wilson, 1997; Liu et al., 2007), was a great surprise for astronomers, for they expected a free wobble of 305-day period corresponding to a rigid oblate Earth. The annual PM was also discovered by Chandler. Since, Newcomb and others modified the rigid Earth theory to take into account the non-rigidity of the Earth: mantle elasticity, presence of fluid parts like the oceans and the core. This allows to get a theoretical Chandler period consistent with observations. Actually the free wobble concept, where no forcing is required, does not exactly hold any more. Some small excitation is required to maintain resonant ChW, because of its damping having a relaxation time between 20 and 100 years. To model the pole path described in the geographic equatorial plane by the complex coordinate $p(t) = x(t) - iy(t)$, we use the linear Liouville equation (Munk, MacDonald, 1960; Lambeck, 1980)

$$\frac{i}{\sigma_c}\frac{dp(t)}{dt} + p(t) = \chi(t), \tag{1}$$

where the complex Chandler angular frequency $\sigma_c = 2\pi f_c(1 + i/2Q)$ depends on the real Chandler frequency $f_c = 0.8435$ yr^{-1} and the quality factor $40 < Q < 200$, which empirically determines the damping (in this work we use the value $Q = 100$); $\chi(t) = \chi_1(t) + i\chi_2(t)$ is the complex equatorial excitation function.

It is well known, that ChW amplitude has been changing in $\sim$ 150-years period of observations, and has even completely decayed in 1930s (Fig. 1, left). Several interpretations were proposed: the first one suggests two near-by frequencies of ChW (see Guo et al., 2005), which produce the beating effect. But this concept contradicts the only one resonant frequency f_c of equation (1). Another school proposes that random variations of hydro-atmospheric excitation are responsible for ChW changes (Gross, 2000; Brzeziński et al., 2012).

The analysis of atmospheric angular momentum AAM and oceanic angular momentum OAM for the recent 60 years proves that at f_c the changes of currents and ocean bottom pressure, from one side, and winds and atmospheric pressure, from another side, is sufficient to maintain ChW. However, AAM and OAM spectra do not show any prominent feature at Ch. freq. (Fig. 1, right) and behave like white noise there. Indeed the main AAM and OAM modes are at annual, ter-annual, semi-annual, and tidal frequencies (mainly diurnal and semi-diurnal). Since resonant motion does not require much forcing, sometimes it is also proposed that the side-lobe of the near-by annual mode, evident in AAM and OAM spectra, and causing 365-day PM, can also force 433-day ChW. It means that the current theory understands ChW and its amplitude changes as a random process at resonant frequency, given by linear equation (1) and maintained by small stochastic oscillations in the ocean and atmosphere (see Chao and Chung, 2012).

On the other hand, work of (Sidorenkov, 2009) suggests a non-linear interrelation between ChW and planetary oscillation modes, such as El Nino Southern Oscillation (ENSO) and quasi-biannual oscillations (QBO), for these latter present some super-harmonics of the ChW period.

In our work we analyse the complex ChW envelope within the framework of classical equation (1) in order to predict it for next decades. The method we use to extract ChW and its excitation is explained in (Zotov, Bizouard, 2012). The results are presented in the next section.

2. CHANDLER WOBBLE ENVELOPE

To extract ChW the Panteleev's filter and Complex Singular Spectrum Analysis (CSSA) can be used (Zotov, Bizouard, 2012). They allow to extract ChW component in a very narrow prograde frequency band. Both methods of processing give very similar results. The x-component of the obtained signal is presented in Fig. 1, left. The y-component of purely prograde ChW is similar, but shifted by $\pi/2$ (109 days). The red rectangle displays the region, where the filtering edge effects can be neglected. One of the most prominent feature of ChW envelope (obtained by Gabor transform) is its decrease around 1930s, at the beginning of the interval, 1840s, and at the end, 2010s (edge effects should be negligible for CSSA). Another variation of amplitude with a 40-year period and minima around 1890, 1930, 1970, is superimposed on the first one (see Nastula et al., 1993).

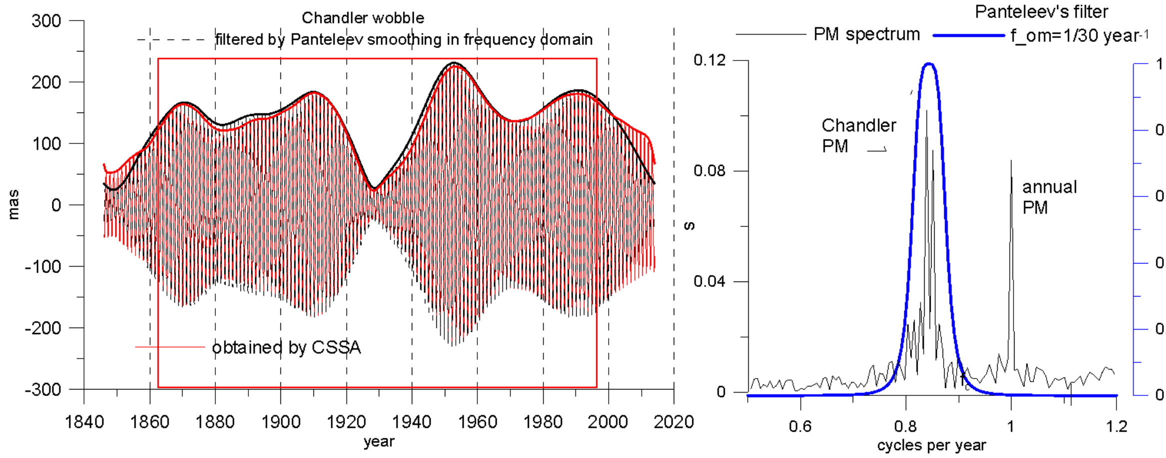

Figure 1: Chandler wobble obtained by CSSA and Panteleev's filter, and its envelope (left). Complex PM spectrum (module) around Chandler frequency and Panteleev's filter frequency response (right).

In Figure 1, right, the PM spectrum in the ChW frequency band and Panteleev's filter transfer function are represented. The filter does not allow annual and other non-chandlerian frequencies to pass. The ChW component is centred around f_c, but it is not just one line, corresponding to one harmonic. The ChW line is split into two frequency components, it also has side-lobes. It means that the oscillation is not purely harmonic, but has modulation (beating). Fourier analysis represents the signal as a set of harmonics of given amplitudes and phases with infinite time extent. At the same time, we can represent ChW by one harmonic with the circular frequency $\omega_c = 2\pi f_c$ and complex amplitude $C(t)$ according to

$$p(t) = C(t)\exp\{i\omega_c t\} = A(t)\exp\{i\phi(t)t\}\exp\{i\omega_c t\}, \tag{2}$$

or through $A(t) = |C(t)|$, $\phi(t) = Arg\{C(t)\}$ – real instantaneous amplitude and phase. They completely describe ChW behaviour, but in a different way from the spectrum in Fig. 1, right. The real envelope $E(t)$ and phase $\theta(t)$ of the excitation at Chandler frequency

$$\chi(t) = E(t)\exp\{i\theta(t)t\}\exp\{i\omega_c t\} \tag{3}$$

can be related to $A(t), \phi(t)$ through the expression

$$E(t)\frac{\exp\{i\theta(t)t\}}{\exp\{i\phi(t)t\}} = \frac{i}{\sigma_c}\left(\frac{dA(t)}{dt} + i\frac{d\phi(t)}{dt}A(t)\right) + \left(1 - \frac{\omega_c}{\sigma_c}\right)A(t), \tag{4}$$

obtained by substitution of representations (2), (3) into (1). For example, if $A(t) = \sin(2\pi t/T_{mod})$, where T_{mod} is the period of ChW modulation, let's say 40 years, the main term of excitation amplitude $E(t)$ would be proportional to $|\dot{A}(t)| = |\cos(2\pi t/T_{mod})|$ i.e. it would have period of 20 years.

We modelled the ChW envelope $A(t)$ from Fig. 1, obtained by Panteleev's filter, using non-linear least squares method (NLSM). The estimates for the mean and two harmonics are given in Table 1. Using this simple model of envelope we made a prediction until 2045, shown in Fig. 2, left, whereby the ChW amplitude reaches its minimum now (2015) and will start to increase soon.

	Period, years	Amplitude, mas	Phase (for epoch 1880), deg
80-year component	83.44	42.6	40.8
40-year component	42.0	54.6	−101.5
mean	–	134.8	–

Table 1: The components of ChW envelope, obtained by NLSM.

The phase of ChW $\phi(t)$ is shown in Fig. 2, right. It has a jump by π in 1930s. It was also modelled and predicted, but the phase prediction is much more uncertain than the amplitude's one. It is possible that the next phase jump will accompany the present-day ChW amplitude decrease.

We reconstructed the Chandler excitation by the corrective filtering method, presented in (Zotov, Bizouard, 2012). Its envelope $E(t)$ was compared to those one, reconstructed using eq. (4) from $C(t)$. It is very important to include ChW phase $\phi(t)$ information into this reconstruction. The results presented in Fig. 3 for both methods completely match each other.

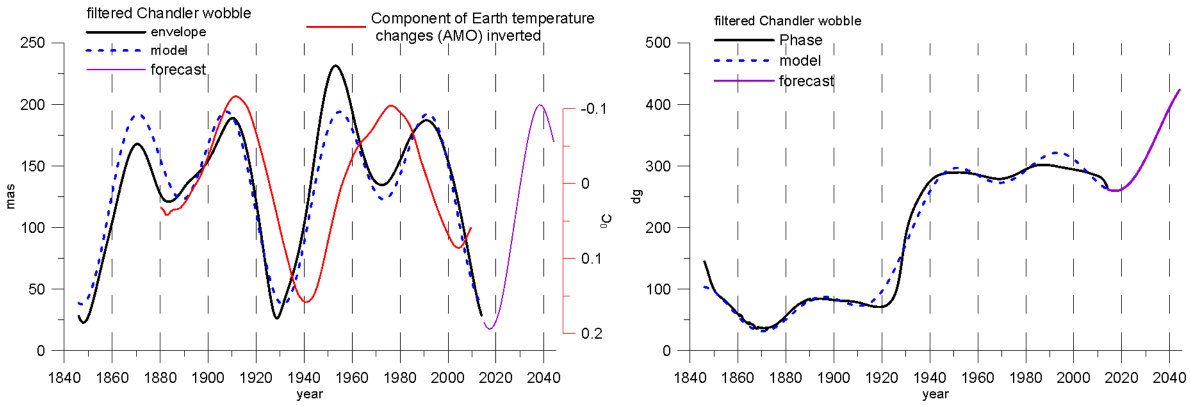

Figure 2: ChW envelope $A(t)$ (left) and phase $\phi(t)$ (right), their models, and forecasts, compared to the Earth temperature changes component.

3. DISCUSSION AND CONCLUSION

We modelled ChW amplitude and phase by a very simple harmonic model and predicted them for thirty years in the future. Our epoch is probably the one of the ChW amplitude minimum, as predicted by (Nastula et al., 1993), and reminiscent the minimum observed in 1930s. The complex ChW envelope $C(t)$ completely describes its amplitude and phase changes and is an alternative for the spectrum representation, given in Fig. 1, right. The equation (4) allows to obtain Chandler excitation envelope $E(t)$, given ChW's one $C(t)$.

In (Zotov, Bizouard, 2012) it was proposed, that 20-years modulation of ChW excitation could be caused by the Moon orbital precession. We prove this modulation of the envelope using eq. (4), Fig. 3.

In (Zotov, 2013) it was shown, that Earth climate, in particular global mean temperature GMT and global mean sea level GMSL have similar 20-years oscillations. In addition, the first principal component extracted from GMT by means of multichannel SSA, also shown in Fig. 2, has ~60-year period with maxima in 1880, 1940, and 2000s, coinciding with the minima of the Chandler wobble amplitude.

While the Chandler wobble is mostly produced by the hydro-atmospheric excitations, it has some non-random, periodic changes in amplitude and phase. In particular, ChW have minima in 1840s, 1930s, and 2010s, what is very similar to temperature changes on Earth usually attributed to the Multidecadal Atlantic Oscillation (MAO). If ChW amplitude changes could be related to the changes of Earth's climate characteristics, then such events, as Hiatus, pause of Global warming observed in 2000s, and present-day absence of El Nino/La Nina could be potentially predicted based on the Earth rotation.

Another possible explanation of ChW variations could be that its main excitation acts not exactly at the frequency f_c, but at one of the near-by frequencies $f_c \pm 1/T_{mod}$, providing ChW modulation with period T_{mod}. The phase jump by π (Fig. 2, right) could be related to the excitation frequency migration from one side of f_c to another. But this is a subject of another study.

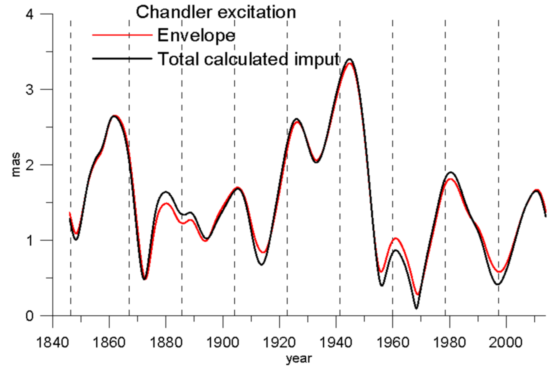

Figure 3: Comparison of ChW geodetic excitation derived in (Zotov, Bizouard, 2012) and through the equation (4).

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