# TOWARDS NEW NUTATION THEORY

V.E. ZHAROV

Lomonosov Moscow State University, Sternberg State Astronomical Institute Universitetskij prospekt, 13, Moscow 119991 e-mail: vladzh2007@yandex.ru

ABSTRACT. Time series of the Earth orientation parameters (EOP) were calculated. The ARIADNA software was used to analyze the corrections of the nutation angles. Main feature is un-modeled motion of the CIP in the GCRS that is known as the free core nutation (FCN). In contrast from theory the FCN motion is complex motion. Hypothesis of reason of this complex motion is based on amplitude modulation of the excitation that is connected with the atmospheric tide  $\psi_1$ .

## 1. INTRODUCTION

The extremely high precision with which the orientation of the Earth in space can now be measured by the space geodetic methods gives an opportunity to test new hypothesis to improve the conventional nutation theory. Nutation that is the motion of the Celestial Intermediate Pole (CIP) in the inertial reference frame is excited by the nearly diurnal forcing. The nutation is retrograde motion of CIP, and the retrograde terms have frequencies which are less than -1 cycles per siderial day.

The main reason of nutation motion is the lunisolar gravitational torque on the Earth's equatorial bulge. The effect of the atmosphere in diurnal frequency band is much smaller. Nevertheless the atmosphere has a non negligible effect on nutation, in particular on the prograde and retrograde annual nutation (Zharov, Gambis, 1996; Zharov, 1997).

According the IERS Conventions (2010) (Petit, Luzum, 2010) the transformation to relate the International Terrestrial Reference System (ITRS) and the Geocentric Celestial Reference System (GCRS) at the date of the observation include the precession-nutation theory IAU2000/2006 (Capitaine, Wallace, 2006). The theory determines the coordinates of the CIP in the GCRS or "celestial pole offsets" denoted as X and Y. Corrections  $\delta X$  and  $\delta Y$  to the X and Y coordinates are estimated from the VLBI observations and include mainly the contribution of the Free Core Nutation (FCN).

In this work we calculate the corrections to the celestial pole offsets and formulate the hypothesis to explain the FCN term by amplitude modulation of the atmospheric tide  $\psi_1$ .

#### 2. SOLUTION DESCRIPTION

Very Long Baseline Interferometry (VLBI) technique is used by the IERS for production of the Earth orientation parameters (EOP) that include corrections  $\delta X$  and  $\delta Y$  to the X and Y coordinates.

In this work the software ARIADNA was used for estimation of the EOP for period 1984–2013. Method that used is based on the equinox-based transformation matrix for precession-nutation. The matrix Q(t) that transforms from the true equinox and equator of date system to the GCRS composed of the classical nutation matrix, the precession matrix including four rotations, and a separate rotation matrix for the frame biases. New series of the nutation angles  $\delta X$  and  $\delta Y$  were calculated for preparation of our suggestion to improve the nutation theory.

Corrections  $\delta X$  and  $\delta Y$  are shown on Fig. 1. To emphasize effect of the FCN the original values were smoothed with period 430 days. Solution gsf2014a was used to compare our results.

According theory the FCN is a free retrograde diurnal motion of the Earth's rotation axis with respect to the Earth. It is caused by the interaction of the mantle and the fluid, ellipsoidal core as it rotates. Frequency of the FCN is determined by the mantle and the core parameters and equal to f = -1.002324(corresponding period is P = -430.23 days). Due to unknown time-varying excitation and damping, a FCN model was not included in the IAU 2000A nutation model. As a result the FCN as un-modeled motion of the CIP in the GCRS at the 0.1–0.3 mas level still exists after the IAU 2006/2000A model has been taken into account.



Figure 1: Corrections  $\delta X$  and  $\delta Y$  (black line). Smoothed values  $\delta X$ ,  $\delta Y$  from my (red line) and from gsf2014a solutions (yellow line).

Spectral density of the complex value  $\delta X + i\delta Y$  is shown on Fig. 2. Main features of the spectrum are the wide peak corresponding the FCN with maximum value of term with period equal to - 440 days and peaks with periods in range from -3200 to -8200 days and from 5200 to 8200 days. Short length of time series of  $\delta X + i\delta Y$  can not allow to reach more higher spectral resolution and can explain unsufficient accuracy of the IAU 2006/2000A model for low frequencies.



Figure 2: Spectral density of the complex value  $\delta X + i\delta Y$ .

But explanation of the FCN peak is not obvious. According theory only narrow line with the FCN frequency has to be in spectrum of  $\delta X + i\delta Y$ . Excitation of the FCN is connected with the atmospheric tide  $\psi_1$  that is one of tidal terms and results from semi-annual modulation of the thermal  $S_1$  tide (Zharov, 1997) (Fig. 3).

Broadening of the FCN spectral line can be connected with amplitude modulation of the tidal term  $\psi_1$ . If amplitude A of an signal

$$a(t) = A\cos(\omega_0 t + \varphi)$$

is function of time:

$$A(t) = A_0 + \Delta A \cos(\Omega t + \gamma)$$

then

$$a(t) = A_0 \cos(\omega_0 t + \varphi) + \frac{MA_0}{2} \cos[(\omega_0 + \Omega)t + (\varphi + \gamma)] + \frac{MA_0}{2} \cos[(\omega_0 - \Omega)t + (\varphi - \gamma)], \quad M = \frac{\Delta A}{A_0},$$



Figure 3: Spectral density of atmospheric pressure term around the FCN frequency.

or amplitude modulation of sinusoidal signal leads to appearance of two additional spectral terms with the same amplitude. If frequency of modulation  $\Omega$  much less then frequency  $\omega_0$  ( $\Omega \ll \omega_0$ ) then we can observe broadening of the spectral line  $\omega_0$  (Fig. 4) if spectral resolution is not enough.



Figure 4: Broadening of the spectral line due to amplitude modulation of sinusoidal signal.

From Fig. 2 we have  $f_{FCN} = -1.002273(P = -440)$  and sideband frequencies are -1.002439 and -1.002127). It means that frequency modulation is in the range (0.000146 ÷ 0.000166) or period modulation is close to 18.5 years.

To test this result the corrections  $\delta X$  and  $\delta Y$  were written as sum of four terms:

$$a(t) = \sum_{i=1}^{4} [D_i \cos(\omega_i t) + F_i \sin(\omega_i t)] = \sum_{i=1}^{4} A_i \cos(\omega_i t + \gamma_i), \quad \omega_i = 2\pi f_i.$$

Then function a(t) was fitted at first to  $\delta X$  and then to  $\delta Y$  by variation both amplitudes of sine and cosine terms and their frequencies. Initial values of amplitudes and frequencies are necessary to start procedure of fitting. Results are shown on Fig. 5.

In contrast to accepted value of the FCN period (P = -430.23 days) the fitting procedure gives period 417.3 and 415.8 days for  $\delta X$  and  $\delta Y$  and two sideband frequencies. Low frequency term is close to main nutation harmonic with period 18.6 years.

#### 3. CONCLUSIONS

Series of the celestial pole offsets corrections  $\delta X$  and  $\delta Y$  of 30-years duration were obtained and used for comparison with the IAU 2000/2006 nutation series. Spectral density of the complex value  $\delta X + i\delta Y$ was calculated. Main feature of the spectrum is the wide peak corresponding the FCN frequency. One of possible explanation of broadening of the FCN spectral line is amplitude modulation of the tidal term  $\psi_1$  that can excite the FCN. Period of modulation signal is close to 18.5 years. Question that can be asked "Is the FCN frequency splitting arise due to modulation by main nutation harmonic with period 18.6 years?" is nevetheless open.



Figure 5: Smoothed corrections  $\delta X$  and  $\delta Y$  (black line) and model of the FCN (red line).

Acknowledgements. This work was supported by the RFBR grant 14-02-00735.

### 4. REFERENCES

- Capitaine, N., Wallace, P.T., 2006, "High precision methods for locating the celestial intermediate pole and origin", A&A, 450, pp. 855–872, doi:10.1051/0004-6361:20054550.
- Petit, G., Luzum, B. (eds.), 2010, IERS Conventions (2010), IERS Technical Note 36, Frankfurt am Main, Germany: Verlag des Bundesamtes für Kartographie und Geodäsie.
- Zharov, V.E., Gambis, D., 1996, "Atmospheric tides and rotation of the Earth", J. of Geodesy, 70, pp. 321–326.
- Zharov, V.E., 1997, "Rotation of the Earth and atmospheric tides", Solar System Research, 31, pp. 501–506.