# COMPARISON OF ASTROMETRIC CATALOGUES UCAC4, XPM, PPMXL

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ABSTRACT. We declare the first results of the representation of the differences between the proper motions UCAC4-PPMXL, XPM-UCAC4 and XPM-PPMXL by vector spherical harmonics in the 10 to 16 J mag range (2MASS photometric system). It was found that the PM systematic differences vary from - 15 to 15 mas/y. The proper motion XPM catalogue has the least systematic deviation from the PPMXL and UCAC4 for stars fainter J=13. The values of spin for UCAC4 on PPMXL are 5 times less than values for XPM on PPMXL and UCAC4. Magnitude equation is clearly seen through dependence of the decomposition coefficients and mutual spins on J magnitude. The influence of low order vector spherial harmonics on the determination of the Ogorodnikov-Milne coefficients is clarified.

#### 1. INTRODUCTION

The pre-GAIA modern astrometric catalogues UCAC4 (Zacharias, et al., 2013), PPMXL (Roeser, et al., 2010) and XPM (Fedorov, et al., 2009) with full coverage of the sky provide a qualitatively new material for investigations in various fields of astronomy. At present, the largest catalogue of positions and proper motions is PPMXL. It contains about 900 million objects and is full from the brightest stars down to magnitude V=20 with absolute proper motions in the ICRS reference frame. The mean errors of the proper motions range from 4 mas/y to 10 mas/y. The accuracies of positions are estimated to be 80-120 mas (at epoch 2000.0). The UCAC4 is an all-sky catalogue containing about 113 million stars covering the 8 to 16 magnitude range in a single bandpass between V and R. The positional accuracy of stars in UCAC4 at mean epoch is about 15-100 mas per coordinate, the formal errors in PMs range from about 1 to 10 mas/v depending on magnitude. Systematic errors in PMs are estimated to be about 1-4 mas/y. The UCAC4 may be considered complete to R=16. It contains accurate positions and proper motions on the ICRS at a mean epoch 2000. The XPM catalogue (2009) combines data from the 2MASS and USNO-A2.0 catalogues in order to derive the absolute proper motions of about 280 million stars distributed all over the sky excluding a small region near the galactic centre, in the magnitude range  $12^m < B < 19^m$ . The proper motions were derived from the 2MASS Point Sources and USNO-A2.0 catalogue positions with a mean epoch difference of about 45 years for the Northern hemisphere and about 17 years for the Southern one. The generated catalogue contains the ISRS positions of stars for the J2000 epoch, original absolute proper motions, as well as B, R, J, H and K magnitudes. The proper motion errors vary from 3 to 10 mas/y, depending on a specific field. The zero-point of the absolute proper motion frame (the absolute calibration) was specified with more than 1 million galaxies from 2MASS and USNO-A2.0

The PPMXL and UCAC4 realize the reference frames which do not rotate with respect to the quasars, whereas the XPM frame is claimed to be free of rotation with respect to galaxies. Theoretically, both quasars and galaxies form the quasi-inertial reference systems but due to different techniques of measurement the resulting reference frames may differ systematically. The main goal of this paper is to calculate the systematic differences in proper motions and to evaluate the mutual rotation of the frames under consideration.

## 2. MODELING AND ANALYSIS OF THE SYSTEMATIC DIFFERENCES

For the first time, the representation of systematic differences in positions and proper motions of stars by orthogonal functions was proposed by Brosche (1966). The modification of this approach based on functions "Legendre-Hermite-Fourier" (Bien, et al., 1978) became the standard tool for the comparison the RA and DEC systems of astrometric catalogues prior to Hipparcos. The two-dimensional vector spherical harmonics (henceforth VHS) were proposed by Mignard (Mignard and Morando, 1990; Mignard and Froeschle, 2000) to derive the systematic differences between Hipparcos and FK5. Further extensive study of this technique aiming at its application in the GAIA project may be found in (Mignard and Klioner, 2012). The three-dimensional kinematic study of proper motions and radial velocities with vector spherical harmonics was developed by Vityazev and Tsvetkov (2014).

In this paper we calculate the systematic differences using the notations of our works on kinematic study of the proper motions with vector spherical harmonics (Vityazev and Tsvetkov, 2009, 2014). The following steps have been done. First of all, by the cross identification of stars within J photometric band (2MASS photometric system) the list of 41 316 676 common stars in the J 10-16 range have been compiled for our catalogues. After that the differences  $\Delta \mu_l \cos b$  and  $\Delta \mu_b$  for stars belonging to each of 1200 HealPix (Gorski, et al., 2005) areas have been calculated and their means were assigned to the centers of the areas. In this way the differences PPMXL-UCAC4, XPM-UCAC4 and XPM-PPMXL were formed in the J magnitude bins 10-12, 12-14 and 14-16. The ranges of the mean values  $\Delta \mu_l \cos b$  and  $\Delta \mu_b$  are shown in Fig. 1 from which we may conclude that for stars fainter 13<sup>m</sup> the differences of all the catalogues vary within almost the same range, whereas for brighter stars the deviation of the XPM from PPMXL and UCAC4 exceeds the range of PPMXL–UCAC4 by factor 2 or 3.



Figure 1: Range of the systematic differences  $\Delta \mu_l \cos b$  (left) and  $\Delta \mu_b$  (right) as function of magnitude. Solid line – ((XPM-UCAC4)+(XPM-PPMXL))/2; dots – (PPMXL-UCAC4). Units mas/y.

At the second step the representation of the pixel's mean values with vector spherical harmonics was done according to equation

$$\Delta \mu_l \cos b \mathbf{e}_l + \Delta \mu_b \mathbf{e}_b, = \sum_{nkp} t_{nkp} \mathbf{T}_{nkp} + \sum_{nkp} s_{nkp} \mathbf{S}_{nkp}, \tag{1}$$

where  $\mathbf{e}_l$  and  $\mathbf{e}_b$  are the unit vectors in the directions of the longitude and latitude in a plane tangential to the sphere. Consequently, the toroidal (magnetic)  $\mathbf{T}_{nkp}$  and spheroidal (electric)  $\mathbf{S}_{nkp}$  vector harmonics are derived from the scalar vector harmonic  $K_{nkp}(l, b)$  according to formulae

$$\mathbf{T}_{nkp} = \frac{1}{\sqrt{n(n+1)}} \left( \frac{\partial K_{nkp}(l,b)}{\partial b} \mathbf{e}_l - \frac{1}{\cos b} \frac{\partial K_{nkp}(l,b)}{\partial l} \mathbf{e}_b \right),\tag{2}$$

$$\mathbf{S}_{nkp} = \frac{1}{\sqrt{n(n+1)}} \left( \frac{1}{\cos b} \frac{\partial K_{nkp}(l,b)}{\partial l} \mathbf{e}_l + \frac{\partial K_{nkp}(l,b)}{\partial b} \mathbf{e}_b \right).$$
(3)

The expansion coefficients  $t_{nkp}$  and  $s_{nkp}$  and their root-mean-square errors (rmse) can be derived from the equation (1) by the standard least-squares procedure. The total number of decomposition terms can be chosen with statistical criteria (Brosche, 1966; Mignard and Klioner, 2012). The full description of the VSH notations may be found in (Mignard and Morando, 1990).

We calculated the expansion coefficients for the PM differences in J mag bins 10-12, 12-14, 14-16. The results up to n=2 are shown in Table1 for PPMXL-UCAC4 and XPM-PPMXL. Obviously, to compare XPM and UCAC4 one may calculate (XPM-UCAC4)=(XPM-PPMXL)+(PPMXL-UCAC4).

To see how different the proper motions tied to quasars or galaxies may be, we introduce the mutual spin  $\Omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$ , where the components of the spin  $\omega_x, \omega_y, \omega_z$ , of one reference frame on another are connected with the first order coefficients of their systematic differences expansion on VSH by the relations (Mignard and Morando, 1990; Vityazev and Tsvetkov, 2009):  $t_{101} = 2.89\omega_z$ ,  $t_{110} = 2.89\omega_y$ ,  $t_{111} = 2.89\omega_x$ . The dependence of the coefficients  $t_{101}, t_{110}, t_{111}$  on magnitude is shown in Fig. 2. Here



Figure 2: Toroidal coefficients via magnitude. Left to right:  $t_{101}$ ,  $t_{110}$ ,  $t_{111}$ . Solid line – (PPMXL-UCAC4); dots – (XPM-UCAC4); dashes – (XPM-PPMXL). Units: mas/y.

	$10 - 12^m$	$10 - 12^m$	$12 - 14^m$	$12 - 14^m$	$14 - 16^m$	$14 - 16^m$
	PPMXL-	XPM-	PPMXL-	XPM-	PPMXL-	XPM-
	-UCAC4	-PPMXL	-UCAC4	-PPMXL	-UCAC4	-PPMXL
$t_{101}$	-1, 13	3,09	-1,37	2,48	-0,98	1,02
$t_{110}$	-0,07	4,85	-1,29	4,47	-0,72	2,53
$t_{111}$	-0,25	-2,58	0, 39	-1,67	-0, 18	-0,41
$t_{201}$	-0,32	0,08	0, 23	-0,37	0, 38	-0,29
$t_{210}$	-0,35	-0,03	-1,40	0, 89	-1,54	0, 38
$t_{211}$	0, 21	0,64	0, 56	0, 15	0,01	0, 51
$t_{220}$	1, 39	-0,88	2,38	-1,81	2, 39	-1,20
$t_{221}$	0, 19	-0,72	0,76	-0,62	0,65	-0,40
$s_{101}$	-1,26	2,15	-1,44	1,10	-2,29	1,37
$s_{110}$	-2,58	0,64	-2,90	0,96	-4, 17	1,94
$s_{111}$	-0,89	0,72	-1,05	1,57	-0, 18	1,35
$s_{201}$	-0,22	-0,66	0, 45	-0,66	0,44	-0,79
$s_{210}$	-0,67	1,42	-1, 32	1,67	-1, 31	1,62
$s_{211}$	-0,50	0,57	-0,78	0, 37	-0,63	0, 12
$s_{220}$	0, 51	-1, 39	0,81	-1,40	0,94	-1,49
$s_{221}$	0,19	-0,26	0,88	-0,88	1,24	-0,98
σ	$\pm 0, 10$	$\pm 0,21$	$\pm 0, 12$	$\pm 0, 14$	$\pm 0,14$	$\pm 0,15$

Table 1: Toroidal and spheroidal coefficients of the proper motion systematic differences representation on VSH. Units: mas/y. The last row - rmse of the coefficients.

	10-12	12-14	14-16
PPMXL-UCAC4	$0.40\pm0.04$	$0.67\pm0.04$	$0.43\pm0.05$
XPM-PPMXL	$2.18\pm0.07$	$1.86\pm0.05$	$0.95\pm0.05$
XPM-UCAC4	$2.04\pm0.07$	$1.25\pm0.04$	$0.66\pm0.05$

Table 2: Spin of mutual rotation. Units: mas/y.

one can see that the XPM does rotate on the PPMXL and UCAC4 faster than the UCAC4 rotates on the PPMXL. Moreover, using our VSH coefficients we can estimate the value of mutual spin of our catalogues (Table 2). From this Table we see that small mutual spin may be found for UCAC4 and PPMXL. In other words, the proper motions of both the catalogues being tied to quasars have but small mutual rotation. Quite opposite situation we see for XPM with respect to PPMXL and UCAC4 since the mutual spins XPM-PPMXL and XPM-UCAC4 are sufficiently large. Naturally, this is a consequance of the different obervational techniques used to derive the absolute proper motions by tying them to galaxies and quasars.

#### **3. STELLAR KINEMATICS**

The connection of low order VSH coefficients of the PMs decomposition with the Ogorodnikov-Milne coefficients is clarified in (Vityazev and Tsvetkov, 2009). In particular, the coefficients  $s_{210}$ ,  $s_{211}$ ,  $s_{220}$  (Fig. 3) are connected with the elements of the deformation tensor by expressions  $s_{210} = 2.24M_{23}^+$ ,



Figure 3: Spheroidal coefficients via magnitude. Left to right:  $s_{210}$ ,  $s_{211}$ ,  $s_{220}$ . Solid line – (PPMXL-UCAC4); dots – (XPM-UCAC4); dashes – (XPM-PPMXL). Units: mas/y.

 $s_{211} = 2.24M_{13}^+$ ,  $s_{220} = 2.24M_{12}^+$ . For this reason, the VSH coefficients of the PM systematic differences may be used for directly reducing a kinematic parameter from the system of one catalogue to the system of another catalogue. For example, the mean PPMXL-UCAC4 coefficients  $t_{101} = -1.16\pm0.07$  and  $s_{220} =$  $0.75\pm0.07$  mas/y yield the differences of the Oort constants  $\Delta B = -1.90\pm0.11$  and  $\Delta A = 1.59\pm0.15$ km/s/kpc, which is confirmed by direct evaluation of these parameters provided the same list of stars was used.

### 4. MAGNITUDE EQUATION

General analysis of the VSH coefficients in proper motions reveals rather strong magnitude equation. It may be seen, for example, in Figs. 1–3. The rigorous description of the magnitude equation requires introduction of additional basic functions which will be done elsewhere.

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