METHOD OF DETERMINING THE SMALL BODIES ORBITS IN THE SOLAR SYSTEM BASED ON AN EXHAUSTIVE SEARCH OF ORBITAL PLANES

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ABSTRACT. A universal method of determining the orbits of newly discovered small bodies in the Solar System using their positional observations has been developed. In this method we avoid determining the topocentric distances of an object by iterations. Instead the different orbital planes of object's motion are considered and the most appropriate one is chosen as a first approximation for the differential method of improving orbit. Criterion for choosing the most appropriate plane is the least rms of the observations. For each considered plane the topocentric distances are calculated and the two reference observations are chosen. The orbits for each plane are calculated using the method of determining orbital elements by two heliocentric positions and times.

1. INTRODUCTION

Newly discovered asteroids that have short observational arcs and few observations pose a special problem in orbit determination. Gauss developed his method for orbit determination about 2 ages ago but it uses only 3 observations. Nowadays generally the amount of asteroid's observations more than tree even at the first day of observing this object. Also the iterations used in this method sometimes can diverge of tend to inappropriate or strange result (e.g. topocentric distances less than zero).

Usually the problem of calculating small body orbits can be divided into two stages: determining the Keplerian imperturbation orbit; improving the orbit by differential method taking significant perturbations into account.

Let's there are $n \geq 3$ positional observations of a body: points in time t_j, right ascensions α_j and declinations δ_j $(j = \overline{(1, n)})$. Then, unit vectors $\underline{\mathbf{L}}_j$ pointing to the body in the topocentric equatorial coordinate system have the following form: $\underline{\mathbf{L}}_j = (\cos\alpha_j \cdot \cos\delta_j, \sin\alpha_j \cdot \cos\delta_j, \sin\delta_j), \quad (j = \overline{(1, n)})$

The relationship between the heliocentric and topocentric vectors of the celestial body positions is determined by the equations:

$$\underline{\mathbf{X}}_j = \rho_j \cdot \underline{\mathbf{L}}_j + \underline{\mathbf{E}}_j, \quad (j = \overline{(1, n)}) \tag{1}$$

where $\underline{\mathbf{X}}_j$ are the heliocentric vectors of the celestial body positions, ρ_j are the topocentric distances, and $\underline{\mathbf{E}}_j$ are the heliocentric vectors of the observer's position. Note that $\underline{\mathbf{E}}_j$ can be calculated by some planet ephemerid (e.g. DE431, INPOP13c or EPM 2013).

The unknown variables in the equation system (1) are topocentric distances ρ_j and 6 orbital parameters (vectors $\underline{\mathbf{X}}_j$ are functions of orbital parameters). Consequently we have $3n$ equations and $6 + n$ unknown variables. In order to find the orbit one should solve this nonlinear system. Generally this system is solved using iterations that can can diverge of tend to inappropriate or strange result especially in cases of short observation arc and few observations.

2. DESCRIPTION OF THE PROPOSED METHOD

In this method we want to avoid using iteration in the solving of system (1). Note that topocentric distances can be considered as functions of observation and only two orbital elements: inclination i and longitude of the ascending node Ω (as lengths of vector sections $\underline{\mathbf{L}}_j$ pointing from the observer to the object till the intersection with the plane).

Our main idea is find the first approximation for the differential method. We propose the following scheme. We do exhaustive search of orbital planes and for each plane we do the following (Bondarenko et al., 2014):

1. Calculate topocentric distances ρ_j
2. Take aberration corrections into account
3. Choose two reference observations (generally the first and the last ones)
4. Determine the orbit using the method of determining orbital elements based on two heliocentric positions and times
5. Calculate rms $\sigma = \sqrt{\frac{1}{2n}\sum_{j=0}^{n}(\alpha_j - \alpha_j^c)^2\cos^2\delta_j + (\delta_j - \delta_j^c)^2}$, where α_j^c and δ_j^c are the calculated equatorial coordinates of the celestial body.

Then we consider the orbit, which associated with the least σ, as the most appropriate one and use it as a first approximation of the orbit.

The advantage of this approach is that we always obtain some approximation that we can try to improve.

3. RESULTS

The efficiency of the technique was verified with 34 new celestial objects published in the Minor Planet Center circulars between September 17–29, 2010, and May 24 – June 3, 2011. This method found satisfying first approximations of orbits, which were improved by differential method, for all considered asteroids. On the other hand using the classical Gauss method, we failed to determine preliminary orbits for 11 asteroids that could be further improved using the differential method. In nine cases the epochs of observations were represented as two groups separated by a fairly long time interval. For one asteroid the accuracy of the mean observation was not well enough. And in one case a problem with the convergence of iterations in the determination of geocentric distances arose while calculating the orbit. The values of geocentric distances for this asteroid obtained using the Gauss method turned out to be negative.

4. REFERENCES

Bondarenko, Yu.S., Vavilov, D.E., Medvedev, Yu.D., 2014, "Method of Determining the Orbits of the Small Bodies in the Solar System Based on an Exhaustive Search of Orbital Planes", Solar System Research, 48(3), pp. 212–216.