PROBABILISTIC APPROACH TO DESCRIBING THE CHANDLER WOBBLE: THE ROLE OF THE OCEAN

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ABSTRACT. The atmospheric component of polar motion can be treated as the anisotropic Markov process with discrete time, and the torque exerted by the atmosphere on the solid Earth, as the white noise. The efficiency of the atmospheric mechanism in the excitation of the Chandler wobble (CW) is estimated in the context of the probabilistic model. It was shown, that one can interprets the oceanic perturbation as a stationary anisotropic random process characterized by the correlation time less than 100 day. The probabilistic approach to the description of the CW is expanded to the case of anisotropic random load. The polar motion is treated as a two-dimensional Markov process, i.e. the solution of the Liouville equation with discrete time. With a sufficiently large time step, the polar motion can be considered as an isotropic process irrespective of the particular ratio between the eigenvalues of the diffusion matrix. Thus, it is demonstrated that the observed variations in amplitude can be explained in the context of the probabilistic approach without hypothesizing the isotropy of the random load.

1. INTRODUCTION

The probabilistic approach to describing the Chandler wobble (CW) was suggested by Arato and Kolmogorov (Arato et al., 1962). The authors of the quoted paper assumed that the moment of forces causing CW is a stationary random process with a small (compared to the length T of the time series of the observations) correlation time τ_{cor}. Then, the CW itself can be considered as a diffusion Markov process with discrete time, in which the sampling interval should satisfy the condition $\Delta \gg \tau_{cor}$. In (Tsurkis et al., 2009) it was shown that the probabilistic model is consistent with the observations. Besides, the authors of the quoted work obtained the estimates for τ_{cor} and coefficient of diffusion d:

$$\tau_{cor} < 100 \text{ days}, \tag{1}$$

$$d = 1.1 \cdot 10^{-16} \dots 1.8 \cdot 10^{-16} \text{rad}^2/\text{day}. \tag{2}$$

Studying the processes that are responsible for CW is an equally important task. The polar motion is caused by a few factors, among which the impact exerted on the solid Earth by the ocean and atmosphere is perhaps most important (Gross et al., 2003). The analysis of the oceanic angular momentum data is carried out in (Tsurkis et al., 2012). The present communication relies on the results obtained in the quoted paper.

2. STATEMENT OF THE PROBLEM

The oceanic component of CW is described by the linearized Liouville equation::

$$\frac{d}{dt}x_1 + \frac{1}{2Q}\frac{d}{dt}x_2 + \omega x_2 = f_1, \quad \frac{d}{dt}x_2 - \frac{1}{2Q}\frac{d}{dt}x_1 - \omega x_1 = f_2,$$

Here $x_k, k = 1, 2$ are the dimensionless coordinates of the pole; Q is the Q-factor (at the frequencies of the order of the Chandler frequency $\omega \approx 0,0145$ day^{-1}), $f_k = M_k/(\Omega C)$, M_k are components of the torque that acts on the solid Earth from the ocean, Ω is the average frequency of the Earth's rotation, C is the axial moment of inertia of the Earth. In the probabilistic approach, f_k are random functions of time. We hypothesize that loading $\big(f_1(t), f_2(t)\big)$ is a stationary normal random process with correlation time τ_{cor}, which is small compared to the length of the time series of the observations. In other words,

$$M\big(f_1(t_1), f_1(t_2)\big) = F_{11}\delta(t_2 - t_1), \quad M\big(f_2(t_1), f_2(t_2)\big) = F_{22}\delta(t_2 - t_1);$$

$$M\big(f_1(t_1), f_2(t_2)\big) = F_{12}\delta(t_2 - t_1), \tag{3}$$

where F_{11}, F_{22}, F_{12} are the components of non-negative symmetric matrix.

$$\mathbf{F} = \begin{pmatrix} F_{11} & F_{12} \\ F_{12} & F_{22} \end{pmatrix}.$$

Our aim is to test the statistical hypothesis (3) and to estimate the correlation time τ_{cor} and the parameters characterizing matrix $\mathbf{F}$: the coefficient of diffusion a and anisotropy constant κ

$$a(\mathbf{F}) = \mathrm{Tr}\mathbf{F} = F_{11} + F_{12}, \qquad \kappa(\mathbf{F}) = 1 - F_2/F_1,$$

where F_1 and $F_2 \leq F_1$ are eigenvalues of matrix $\mathbf{F}$. We do not assume that $F_2 = F_1$.

3. DATA

We used the time series of the oceanic excitation functions $\chi_k(t)$, $k = 1, 2$ for the period from January 1, 1980 to March 27, 2003 provided by the IERS (http://www.iers.org). The IERS data are referred to the Cartesian coordinates whose axes and are located in the equatorial plane and axis x_1 is oriented along the projection of the Greenwich meridian onto this plane. Components M_1 and M_2 of the torque that acts on the solid Earth from the ocean are

$$M_1 = \omega C\big(\Omega - \chi_2 - \dot{\chi}_1\big), \qquad M_2 = -\omega C\left(\Omega\chi_1 - \dot{\chi}_2\right),$$

where $\Omega 2\pi$/day is the average frequency of the Earth's rotation, $C = 7.04 \times 10^{37}$ kg·m^2 is the axial moment of inertia of the Earth, $\omega \approx 0.0145$ day^{-1} is the frequency of free nutation (Chandler frequency). The method for calculating the excitation functions $\chi_1(t)$ and $\chi_2(t)$ is described in (Gross et al, 2003).

4. RESULTS

1. The data studied are in accordance with the main hypothesis; the estimation for the correlation time coincides with (1).

2. With probability $P > 0.92$, parameters a and κ belong to the intervals:

$$a = 1.3 \cdot 10^{-17} \ldots\ 2.2 \cdot 10^{-17}\mathrm{rad}^2/\mathrm{day}, \quad \kappa = 0.06 \ldots 0.65. \tag{4}$$

One can see that confidence interval for the anisotropy constant entirely lies in the positive area; therefore, the random load acting on the solid Earth from the ocean is anisotropic with probability > 0.92.

3. Comparing (4) and (1), we see that $a \sim 0.1d$; so if the polar motion were entirely excited by the ocean, the amplitude would be on average about $\sqrt{a/d} \sim 1/3$ of the observed value. But if we subtract the oceanic torque from the angular momentum acting on the Earth's rotation axis, the mathematical expectation of the CW amplitude insignificantly decreases (by about 5%). This is due to the fact that the average CW amplitude as a function of the diffusion coefficient is not linear but it scales as a square root function.

5. REFERENCES

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