RELATIVISTIC PRECESSION MODEL OF THE EARTH FOR A LONG TIME INTERVAL

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ABSTRACT. Tang et al. (2015) provided a numerical solution of the Earth's precession in the relativistic framework for a long time span. The motion of the solar system is calculated in the BCRS by numerical integration with a symplectic integrator. The part of Earth's rotation is obtained in the GCRS by integrating the post-Newtonian equations of motion published by Klioner et al. (2003). All the main relativistic effects are included following Klioner et al. (2010), especially we considered several relativistic reference systems with corresponding time scales, scaled constants and parameters. Now we improve this work to give new parameters to represent the precession of the equator, in order to avoid the problem from the calculation of a moving ecliptic in relativity. The results are still consistent with other long-term precession theories. The relativistic influences are obtained and analyzed here.

1. INTRODUCTION

The long-term precession expressions of the Earth have been developed by Vondrák et al. (2011) in the Newtonian framework. They provided an extension of the IAU 2006 (Capitaine et al., 2003) to scales of several thousand centuries. Later Tang et al. (2015) improved this work and gave a long time span relativistic precession model of the Earth. This model has very small discrepancies with respect to the IAU 2006 precession around J2000.0, with differences being only several arcseconds, and is also consistent with other long-term precession theories. However this work used the general precession p_{A} and the obliquity ϵ_{A} as the precession parameters for the equator. It's known that these two parameters mix the motion of the the equator in the Geocentric Celestial Reference System (GCRS) and the motion of the ecliptic of date, and the calculation of a moving ecliptic would present a serious problem in the GCRS. Here we give other parameters to represent the precession of the equator to avoid this problem.

The purpose of this paper is to provide a development for the precession of the equator, while the calculation of the precession of the ecliptic is the same as Tang et al. (2015). We use our new integrator to calculate the motion of the Earth's spin axis, and to obtain the luni-solar precession in longitude ψ_{A}, the inclination of moving equator on a fixed ecliptic ω_A directly, which are the orientation parameters of the mean equator of date in the mean ecliptic frame at epoch (Lieske et al., 1977). Details about the precession of the ecliptic can be found in Tang et al. (2015). Here we only give our result about the precession of the equator.

2. THE PRECESSION OF THE EQUATOR

The model of Earth's rotation which is used here is referred to Klioner et al. (2010). The Earth's rotation is modelled in the GCRS which is kinematically non-rotating with respect to Barycentric Celestial Reference System (BCRS). The model of the Earth's gravity field is defined in the terrestrial reference system that is obtained by rotating the GCRS spatial coordinates with a time-dependent matrix. After integrating the post-Newtonian equation of Earth's rotation given by Klioner et al. (2003), the motion of the Earth's spin axis is obtained. The post-Newtonian equations of motion are numerically integrated by the Runge-Kutta-Fehlberg (RKF) 7(8) method (Fehlberg 1968). All the initial conditions and constants are the same as in Tang et al. (2015).

By numerical analysis (Laskar et al., 1992), the basic quantities for the precession of the equator ψ_{A} and ω_{A} can be derived from the Euler angles directly (Bretagnon et al., 1997). The approximations for

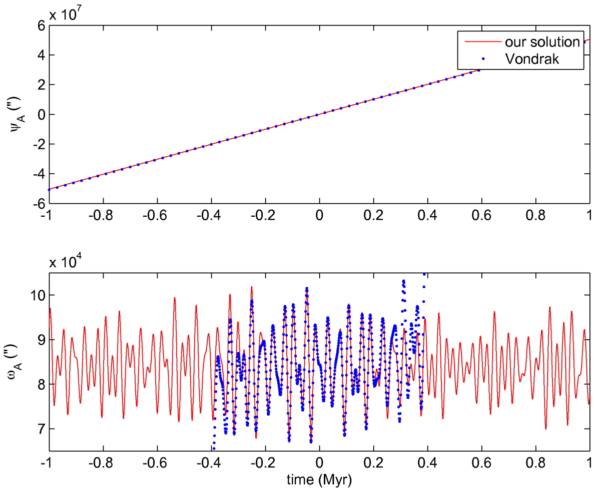

Figure 1: Comparison of our solution (solid line) and the Vondrák's solution (dotted line) for ψ_A (top), ω_A (bottom).

the precession of the equator read:

$$\begin{aligned}\psi_A &= 5773'' + 50''.4476T \\ &\quad + \sum_{i=1}^{30} C_i \cos(2\pi T/P_i) + S_i \sin(2\pi T/P_i), \\ \omega_A &= 83922'' - 1''.07 \times 10^{-4} T \\ &\quad + \sum_{i=1}^{30} C_i \cos(2\pi T/P_i) + S_i \sin(2\pi T/P_i), \quad (1)\end{aligned}$$

where T is in TT year from J2000.0, and the periods P_i with the amplitudes C_i, S_i are given in Table. 1. In the second column, the name of some special frequencies s_i and p are from Laskar (1985) and Laskar et al. (2004). The comparisons of our long-term model of the precession of the equator and the Vondrák's model are shown in Fig. 1. The difference are less than one degree within the interval ±200 millennia from J2000.0.

		ψ_A		ω_A			
i	Term	$C_i['']$	$S_i['']$	C_i $['']$	S_i $['']$	$P[yr]$	$f_i[''/yr]$
1	$p+s_3$	1829	-6033	-15571	-4737	40930	31.663632
2	p	1867	5362	12475	-4340	25691	50.444711
3	$p+s_4$	-541	-3209	-8199	1402	39799	32.563959
4	$p+s_1$	-2791	780	1840	6631	28839	44.938522
5	$p+s_6$	722	898	2532	-2027	53778	24.099094
6	$p+s_2$	-672	-974	-2346	1602	29649	43.711841
7		431	460	1193	-1087	41509	31.221854
8		151	-95	3445	-2438	1309223	0.9899
9		-61	26	-2992	1824	994480	1.303194
10		31	-31	2130	-619	718968	1.802584
11		-521	268	677	1320	42165	30.736352
12		29	-14	827	-911	417797	3.101988
13		-138	-363	-943	349	38904	33.313065
14		-223	-3	2	1091	15787	82.095307
15		286	234	542	-680	27332	47.41778
16		120	-330	-796	-288	30165	42.963876
17		-23	17	-803	173	556286	2.329735
18		207	-17	-15	-510	42839	30.252966
19		14	-119	-579	-66	16925	76.57474
20		-225	90	199	517	26037	49.774741
21		-96	50	254	467	15613	83.009872
22		97	76	369	-433	20466	63.323731
23		123	17	107	-557	20168	64.25881
24		53	84	415	-262	13587	95.382033
25		-24	32	-652	-150	372318	3.480894
26		-307	-75	-227	782	40303	32.156499
27		156	-49	-101	-358	28556	45.384613
28		13	-29	395	19	325726	3.978803
29		-24	100	260	71	29207	44.3731
30		-24	-46	-212	134	16729	77.47157

Table 1: The Periodic Terms in ψ_A, ω_A.

3. RELATIVISTIC EFFECTS

The relativistic effects on the precession can be obtained from our program which calculates the precession both for the Newtonian and the post-Newtonian case. The relativistic influences on the precession of the ecliptic were discussed and published by Tang et al. (2015). Here we discuss the relativistic effects on the precession of the equator.

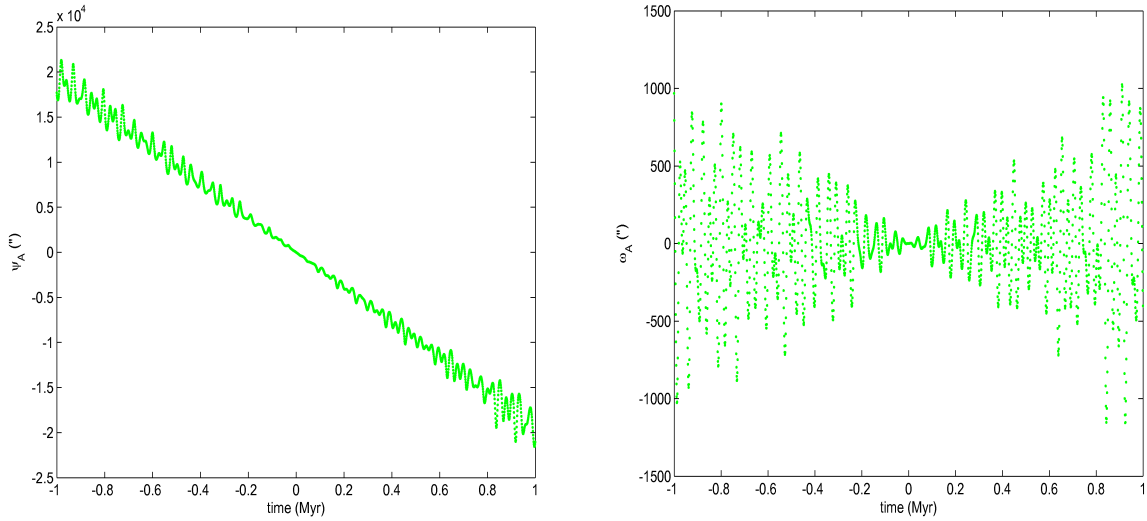

Figure 2: The effects of the geodetic precession on the precession parameter ψ_{A} (left) and ω_{A} (right) from -1 Myr to 1 Myr.

For the rotation of the Earth, the geodetic precession is the most important one and considered by all previous works. The traditional way to account for geodetic precession is to add a precomputed geodetic precession to a purely Newtonian solution which has been already shown to be incorrect (Klioner et al. 2010). Whereas our result is integrated in a more rigorous relativistic framework, with Klioner et al. (2010). The relativistic features considered by our work are: (1) rigorous treatment of geodetic precession/nutation as an additional torque in the equations of motion, (2) four time scales, TDB, TCB, TT, TCG, which are all evaluated at the geocenter, (3) correct relativistic scaling of constants and parameters. Fig. 2 shows the relativistic effects on the precession of the equator parameters ψ_{A} and ω_{A} due to the geodetic precession. The slope of the curve in Fig. 2 (left) related with the geodetic precession amounts to the well-known $2''$ per century. The influences accumulate with time and reach about $25\,000''$ and $1\,000''$ in ±1 Myr respectively. The influence on ω_{A} leads to large obvious periodical parts, and the main period is about 25 920 yr.

The effects of the post-Newtonian inertial torque, the relativistic scaling and time scales (except for the geodetic precession) are depicted in Fig. 3. All these relativistic effects are increasing with time, but they are still too small to be considered in most cases within ±1 Myr. The amplitude of these effects for the precession parameters ψ_{A} and ω_{A} is less than three arcsecond over this time span.

4. CONCLUSIONS

The model of the Earth's long-term precession is given above. It is consistent with the relativistic framework. The part of the precession of the ecliptic is discussed in Tang et al. (2015). The precession of the equator in the interval $\pm$ 1 Myr is calculated by using the RKF7(8) integrator, and the approximations for the precession parameters ψ_{A} and ω_{A} are provided. Our solutions have small discrepancies with respect to the IAU 2006 precession near J2000.0, and display good consistency with other long-term precession theories.

Our model of the Earth's precession is obtained in a relativistic framework. For the precession of the equator, we consider the relativistic features including: (1) the geodetic precession/nutation, (2) the post-Newtonian inertial torque, (3) several relativistic reference systems with corresponding times scales and relativistic scaling of parameters. The relativistic effects on the precession parameters ψ_{A} and ω_{A} are obtained and discussed.

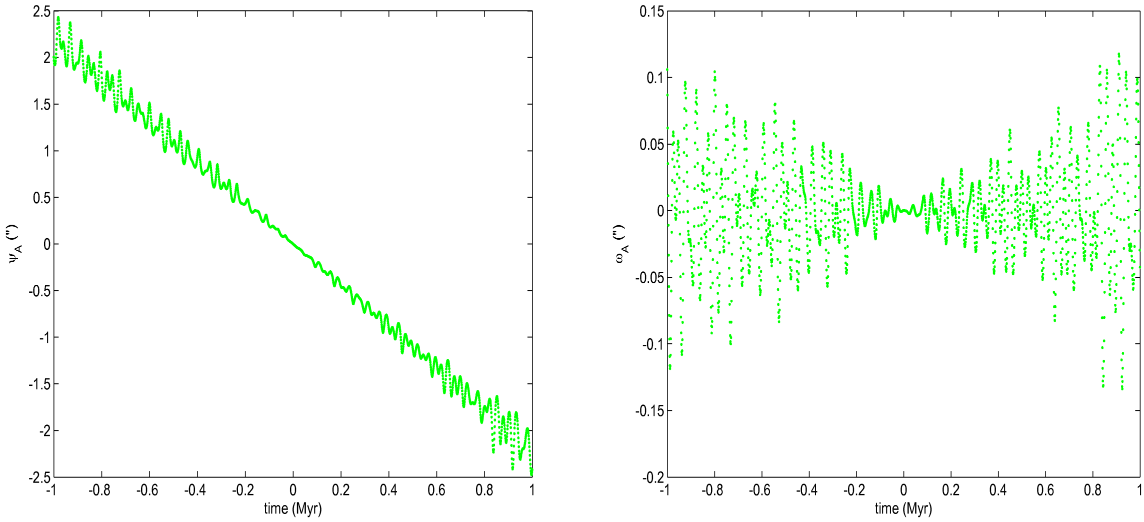

Figure 3: Other relativistic effects (except for the geodetic precession) on the precession parameter ψ_{A} (left) and ω_{A} (right) from $-1\,\mathrm{Myr}$ to $1\,\mathrm{Myr}$.

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