NEW HIGH-PRECISION EARTH AND MOON ROTATION SERIES AT LONG TIME INTERVALS

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ABSTRACT. Dynamics of the rotational motion of the Earth and Moon is investigated numerically at a long time intervals. In our previous studies (Pashkevich, 2013), (Pashkevich and Eroshkin, 2011) the high-precision Rigid Earth Rotation Series (designated RERS2013) and Moon Rotation Series (designated MRS2011) were constructed. RERS2013 are dynamically adequate to the JPL DE422/LE422 (Folkner, 2011) ephemeris over 2000 and 6000 years and include about 4113 periodical terms (without attempt to estimate new sub-diurnal and diurnal periodical terms). MRS2011 are dynamically adequate to the JPL DE406/LE406 (Standish, 1998) ephemeris over 418, 2000 and 6000 years and include about 1520 periodical terms. The main aims of present research are improvement of the Rigid Earth Rotation Series RERS2013 and Moon Rotation Series MRS2011, and as a result of the construction of the new high-precision Rigid Earth Rotation Series RERS2014 dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 years and Moon Rotation Series MRS2014 dynamically adequate to the JPL DE422/LE422 ephemeris over 6000 years. The elaboration of RERS2013 is carried out by means recalculation of sub-diurnal and diurnal periodical terms. Improve the accuracy of the series MRS2011 is obtained by using the JPL DE422/LE422 ephemeris.

1. INTRODUCTION

The aims of the present research are construction of the improved high-precision Rigid Earth Rotation Series RERS2014 (with including new sub-diurnal and diurnal periodical terms) and the high-precision Moon Rotation Series MRS2014 dynamically adequate to the JPL DE422/LE422 ephemeris, over 2000 and 6000 years, respectively. The numerical solution of the problem is obtained by solving the Lagrange differential equations of the second kind for the rigid Earth rotation with respect to the fixed ecliptic and equinox of epoch J2000 (Pashkevich, 2013). The orbital motions of the disturbing celestial bodies are defined by the DE422/LE422 ephemeris. The mathematical model of the problem is described in detail in the papers (Eroshkin and Pashkevich, 1997), (Pashkevich and Eroshkin, 2011), (Eroshkin, Pashkevich and Brzeziński, 2002) and (Pashkevich, 2013). The discrepancies between the high-precision numerical solutions and the semi-analytical solutions for the rigid Earth and the Moon rotation problems with respect to the fixed ecliptic of epoch J2000 are investigated by the iterative algorithm, which used the least-squares method and the spectral analysis methods (Pashkevich and Eroshkin, 2005, 2010). Comparison of the new Series RERS2014 and MRS2014 with the previous solution RERS2013 (Pashkevich, 2013) and MRS2011 (Pashkevich and Eroshkin, 2011), respectively is carried out. The rigid Earth rotation problem is solved for the relativistic (kinematical) case in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account. Investigation of the Moon rotation problem is carried out only for the newtonian (dynamical) case.

2. ALGORITHMS AND RESULTS

The results of the numerical solutions of the problem are compared with the semi-analytical solutions of the studied body rotation (RERS2013 for the Earth or MRS2011 for the Moon). The residuals of these comparison are studied by means *the iterative algorithm:*

1. Numerical solution of the studied body rotation (the rigid Earth or Moon) is implemented with the quadruple precision of calculations. The initial conditions are computed by the semi-analytical solution of the studied body rotation (RERS2013 for the Earth or MRS2011 for the Moon). Discrepancies between the numerical solution and the semi-analytical solution are obtained over all investigation time interval

with 0.1 day spacing in Euler angles (1) for the Earth case or with 1 day spacing in the perturbing terms of the physical librations (2) for the Moon case. The expressions for these discrepancies are as follows

$$\left.\begin{aligned}\Delta\psi &= \sum_{k=0}^{8}\psi_k t^k + \sum_{j=1}^{4113}\sum_{k=0}^{4}[\psi_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\psi_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\\ \Delta\theta &= \sum_{k=0}^{8}\theta_k t^k + \sum_{j=1}^{4113}\sum_{k=0}^{4}[\theta_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\theta_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\\ \Delta\phi &= \sum_{k=0}^{8}\phi_k t^k + \sum_{j=1}^{4113}\sum_{k=0}^{4}[\phi_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\phi_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\end{aligned}\right\}, \quad (1)$$

$$\left.\begin{aligned}\Delta\tau &= \sum_{j=1}^{1520}\sum_{k=0}^{3}[\tau_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\tau_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\\ \Delta\varrho &= \sum_{j=1}^{1520}\sum_{k=0}^{3}[\varrho_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\varrho_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\\ \Delta I\sigma &= I\sum_{j=1}^{1520}\sum_{k=0}^{3}[\sigma_{Sjk}\sin(\nu_{j0}+\nu_{j1}t)+\sigma_{Cjk}\cos(\nu_{j0}+\nu_{j1}t)]t^k\end{aligned}\right\}, \quad (2)$$

where ψ is the longitude of the ascending node of the Earth's dynamical equator on the fixed ecliptic J2000; θ is the angle of the inclination of the Earth's dynamical equator to the fixed ecliptic J2000; ϕ is the proper rotation angle of the Earth between the ascending node of the Earth's dynamical equator and the principal axis of the minimum moment of inertia; τ, ϱ and σ are the perturbing terms of the physical librations of the Moon for the fixed ecliptic of epoch J2000 in the longitude, in the inclination and in the node longitude, respectively; ν_{j0}, ν_{j1} are the phases and the frequencies of the corresponding semi-analytical solutions, respectively; t is the time in the Julian days; ψ_k, θ_k, ϕ_k are the coefficients of the secular terms; $\psi_{Sjk}, \theta_{Sjk}, \phi_{Sjk}, \psi_{Cjk}, \theta_{Cjk}, \phi_{Cjk}, \tau_{Sjk}, \varrho_{Sjk}, \sigma_{Sjk}, \tau_{Cjk}, \varrho_{Cjk}, \sigma_{Cjk}$ are the coefficients of the periodic and Poisson terms.

2. Investigation of the discrepancies is carried out by the least squares method (LSQ) and by the spectral analysis (SA) method (Pashkevich and Eroshkin, 2005, 2010). The sets of the frequencies of the semi-analytical solutions are used without change. Only the coefficients of the systematical terms, the coefficients of the periodical terms and the coefficients of the Poisson terms are improved. The systematic, periodic and Poisson terms representing the new high-precision studied body rotation series (RERS2014$_i$ for the Earth or MRS2014$_i$ for the Moon (where i is the number of iteration)) are determined:

$$\left.\begin{aligned}\psi_{\mathrm{RERS2014}_i} &= \Delta\psi_{i-1} + \psi_{\mathrm{RERS2014}_{i-1}}\\ \theta_{\mathrm{RERS2014}_i} &= \Delta\theta_{i-1} + \theta_{\mathrm{RERS2014}_{i-1}}\\ \phi_{\mathrm{RERS2014}_i} &= \Delta\phi_{i-1} + \phi_{\mathrm{RERS2014}_{i-1}}\end{aligned}\right\}, \quad \left.\begin{aligned}\tau_{\mathrm{MRS2014}_i} &= \Delta\tau_{i-1} + \tau_{\mathrm{MRS2014}_{i-1}}\\ \varrho_{\mathrm{MRS2014}_i} &= \Delta\varrho_{i-1} + \varrho_{\mathrm{MRS2014}_{i-1}}\\ \sigma_{\mathrm{MRS2014}_i} &= \Delta\sigma_{i-1} + \sigma_{\mathrm{MRS2014}_{i-1}}\end{aligned}\right\}, \quad (3)$$

where $\psi_{\mathrm{RERS2014}_0} = \psi_{\mathrm{RERS2013}}$, $\theta_{\mathrm{RERS2014}_0} = \theta_{\mathrm{RERS2013}}$, $\phi_{\mathrm{RERS2014}_0} = \phi_{\mathrm{RERS2013}}$, $\tau_{\mathrm{MRS2014}_0} = \tau_{\mathrm{MRS2011}}$, $\varrho_{\mathrm{MRS2014}_0} = \varrho_{\mathrm{MRS2011}}$ and $\sigma_{\mathrm{MRS2014}_0} = \sigma_{\mathrm{MRS2011}}$.

3. Numerical solution of the studied body rotation is constructed anew with the new initial conditions, which are calculated by RERS2014$_i$ (for the Earth) or MRS2014$_i$ (for the Moon).

4. Steps 2 and 3 are repeated till the assumed convergence level of the discrepancies between the new numerical solution and the new semi-analytical solution (RERS2014i for the Earth or MRS2014i for the Moon) has been achieved.

The investigation of the rigid Earth rotation over 2000 years time interval is carried out with used DE422/LE422 ephemeris. The discrepancies between the numerical solutions and semi-analytical series RERS2013 are depicted in Fig. 1 black color. The convergence level was achieved after application of the second iteration of the iterative algorithm. So, the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series RERS2014 was constructed, which include new recalculated the sub-diurnal and diurnal periodical terms and is dynamically adequate to the DE422/LE422 ephemeris over 2000 years. The discrepancies between the new numerical solutions and the semi-analytical solutions of RERS2014-2 do not surpass 3 μas over 2000 years for $\Delta\psi$ and $\Delta\phi$ and do not surpass 1 μas over 2000 years for $\Delta\theta$ (presented in Fig. 1 grey color).

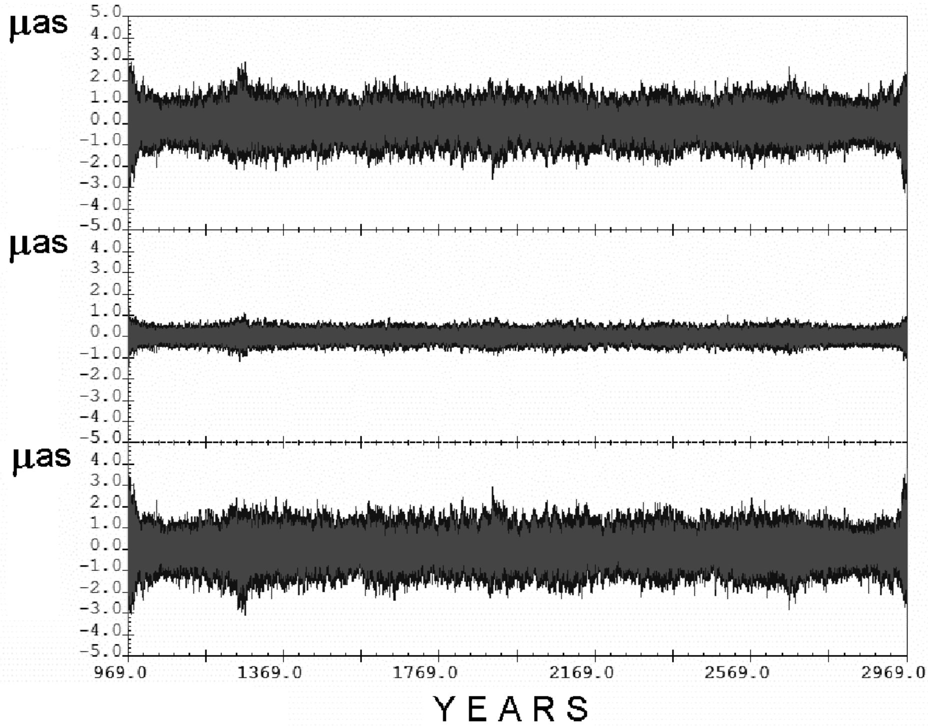

Figure 1: Discrepancies between the numerical and RERS2013 solutions of the Earth rotation (black) and between new numerical and RERS2014-2 solutions of the Earth rotation (grey).

The investigation of the Moon rotation over 6000 years time interval is carried out with used DE422/LE422 ephemeris. The discrepancies between the numerical solutions and semi-analytical series MRS2011 (dynamically adequate to the DE406/LE406 ephemeris) are depicted in Fig. 2 black color. The convergence level was achieved after application of the first iteration of the iterative algorithm. So, the process of the iterative algorithm was finished at this step. As a result, the Moon Rotation Series MRS2014 was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 6000 years. The discrepancies between the new numerical solutions and the semi-analytical solutions of MRS2014-1 do not surpass 8 arc seconds over 6000 years for $\Delta I\sigma$, do not surpass 4 arc seconds over 6000 years for $\Delta\varrho$ and do not surpass 0.6 arc seconds over 6000 years for $\Delta\tau$ (presented in Fig. 2 grey color).

Thus, the result of the comparison on 2000 years for the Earth case and 6000 years for the Moon case demonstrates a good consistency of new RERS2014 and MRS2014 series, respectively, with the DE422/LE422 ephemeris.

3. CONCLUSION

The new improved high-precision Rigid Earth Rotation Series RERS2014 (with including new sub-diurnal and diurnal periodical terms) dynamically adequate to the DE422/LE422 ephemeris over 2000 years have been constructed. RERS2014 include about 4113 periodical terms. The residuals between the numerical solution and RERS2014 do not surpass 3 μas over 2000 years.

The new high-precision Moon Rotation Series MRS2014 dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed. MRS2014 include about 1520 periodical terms. The residuals between the numerical solution and MRS2014 do not surpass 8 arc seconds over 6000 years, It means a good consistency of the MRS2014 series with the DE422/LE422 ephemeris.

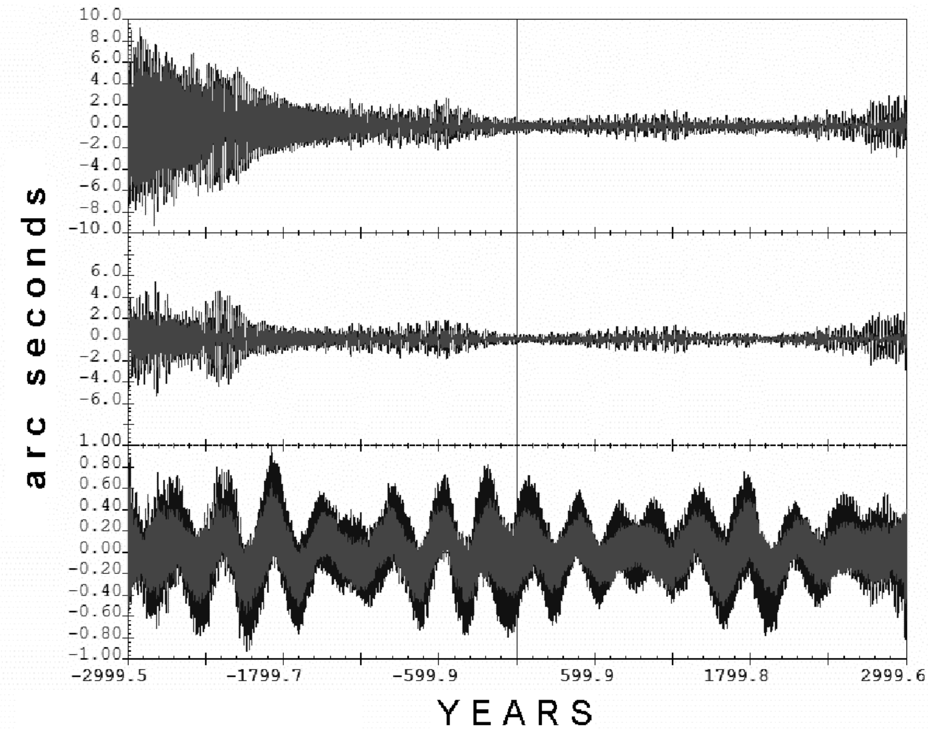

Figure 2: Discrepancies between the numerical and MRS2011 solutions of the Moon rotation (black) and between new numerical and MRS2014-1 solutions of the Moon rotation (grey).

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