NUMERICAL–ANALYTICAL MODELING OF THE EARTH'S POLE OSCILLATIONS

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ABSTRACT. For the purpose of more accurate forecasting the oscillatory process of the Earth pole in time periods with significant anomalies (irregular deviations) a numerical-analytical approach is presented for the combined modeling of the interdependent dynamical processes - the oscillatory-rotational motion of the Earth and the time dependant coefficients of the geopotential. The oscillations of the inertia tensor components of the Earth depend on various factors such as mechanical and physical parameters of the planet, the motions of the tide-generating bodies and observed large scale natural events. Time variations of these and some other factors affect the Earth orientation parameters. The generalization of the previously researched mathematical model of Chandler and annual oscillations of the Earth pole is being held with the use of celestial mechanics methods and the mathematical description of the Earth gravitational field's temporal variations. The latter makes possible to improve the forecast precision of the Earth pole trajectory. Also the more precise model is to have small number of parameters and to agree with the previously developed one (to have the same structural features and to have a correspondence between the averaged dynamical parameters and the parameters of the basic model).

1. INTRODUCTION

To achieve the characteristics of a high-accuracy forecast of oscillations of the Earth's pole, interdependent dynamic processes are considered, namely, rotary-oscillatory motions of the Earth and time-varying coefficients of the planetary geopotential. Oscillations of the Earth's inertia tensor components depend on many factors, e.g., the mechanical and physical parameters of the planet, the motion of tide-generating bodies, and observed large-scale natural phenomena. Time variations in these and other factors have an effect on the parameters of the Earth's rotation. In connection with this, joint simulation of the oscillatory motion of the Earth's pole and time variations in geopotential coefficients having an effect on parameters of the rotating geoid is of scientific and practical interest.

We described the rotational motions of the deformable Earth and the oscillations of the Earth's pole using a simplified mechanical model for the viscoelastic rigid body of the Earth. To take into account gravitational-tidal effects, we assumed the Earth to be axially symmetric and two-layered, i.e., consisting of a rigid (spherical) core and a viscoelastic mantle. We could have used some more complex model. However, employing anymore complex figure for the Earth is not justified, since we cannot determine the geometrical and physical parameters of the Earth with the required accuracy and completeness via a statistical processing of indirect data from seismic measurements. We adhere to the idea that the complexity of a model must strictly correspond to the problem formulated and to the accuracy of the data used. To construct a model for the polar oscillations, we can determine a small number of some mean (integrated) characteristics of the inertia tensor. Comparison with measurements and further analysis indicate that our simplifications are justified (Akulenko, et al., 2012).

2. MATHEMATICAL MODEL OF THE EARTH'S POLE MOTION

It is convenient to represent the trajectory of the Earth's pole as an ensemble of an irregular trend (drift containing secular and low-frequency component with periods of six years and longer) and polhode (trajectory of the pole motion around the middle position) expressed in terms of the amplitude a and phase ψ of the pole motion. Then, the pole coordinates have the form

$$x_p = c_x + a\cos\psi, \quad y_p = c_y + a\sin\psi. \tag{1}$$

When moving around the middle position, the pole describes a helical curve that is obtained as the sum of two main components: the Chandler wobble with a period of $2\pi/N \simeq 433$ days and the annual nutation. The choice of the parameters a and ψ turns out to be more convenient for describing the fluctuations of the main components of the modulation motion of the pole.

It is well known (Akulenko, et al., 2007) that the amplitude and phase of the Chandler component of the oscillatory process of the pole are very sensitive to different disturbing factors, in particular, to those possessing irregular properties (gravitational, oceanic, atmospheric, and, probably, others). It is natural to associate the mechanism of these actions with weak perturbations of the inertia tensor. The Earth's figure is a dynamic geoid figure due to variations in the inertia tensor; at the same time, it creates an additional time-dependent perturbing potential $\delta W(t)$. The largest summand from the expansion of the potential δW is the perturbation from the second harmonic δW_2:

$$\delta W_2 = \frac{f m_E R_E^2}{r^3} \Delta \bar{Y}_2(\theta, \varphi), \tag{2}$$

$$\Delta \bar{Y}_2 = \delta c_{20} \bar{P}_{20}(\cos\theta) + [\delta c_{21}\cos\varphi + \delta s_{21}\sin\varphi]\bar{P}_{21}(\cos\theta) + [\delta c_{22}\cos 2\varphi + \delta s_{22}\sin 2\varphi]\bar{P}_{22}(\cos\theta)$$

where θ, φ and r are spherical coordinates; R_E is the average radius of the Earth ($R_E \simeq 6.38\times10^6$ m); and $fm_E = 3.98600442 \times 10^{14} m^3 s^{-2}$. The change in the normalized spherical function $\Delta\bar{Y}_2(\theta,\varphi)$ is expressed in terms of second order coefficients of the geopotential expansion and $\bar{P}_{2m}(\cos\theta)$ are normalized adjoint Legendre functions.

Differential equations for the amplitude and phase of the modulation motion of the Earth's pole can be obtained from the dynamic Euler-Liouville equations of the Earth's motion with respect to the center of masses:

$$\begin{aligned} \dot{a} &= \frac{2m_E R_E^2}{A^*} r_0 \left[c_{22}^* \left(1 - \frac{C^*}{B^*}\right) + \delta c_{22} \right] a \sin 2\psi + \left[\mu_p \cos\psi + \mu_q \sin\psi\right], \\ \dot{\psi} &= -N_q \cos^2\psi - N_p \sin^2\psi + a^{-1}\left[\mu_q \cos\psi - \mu_p \sin\psi\right]. \end{aligned} \tag{3}$$

Here A^*, B^*, C^* are effective principal central moments of inertia with allowance for deformations of the "frozen" figure of the Earth; $c_{2m} = c_{2m}^* + \delta c_{2m}$, $s_{2m} = s_{2m}^* + \delta s_{2m}$ are second order coefficients of the potential expansion into a series in terms of spherical functions; r_0 is the average velocity of axial rotation of the Earth; and variable coefficients

$$N_p = \frac{C^* - B^* + \delta C - \delta B}{A^* + \delta A} r_0, \quad N_q = \frac{C^* - A^* + \delta C - \delta A}{B^* + \delta B} r_0$$

are close quantities determining the frequency of Chandler oscillations of the pole. The quantities μ_p and μ_q are determined by gravitation-tidal moments of forces from the Sun and the Moon. The average frequency of free nutation N^*, according to solution (3), is $\sqrt{N_p^* N_q^*}$. Variation in the frequency of Chandler oscillations (free nutation frequency) is a function of the dynamic compression of the geoid and variation in the axial moment of inertia:

$$N \cong N^* + \delta N, \quad \delta N = -F(\delta C, \delta c_{20}). \tag{4}$$

Then, for the amplitude a_{ch} and phase ψ_{ch} of the Chandler oscillation, we obtain the expressions

$$\begin{aligned} a_{ch} &= a_{ch}^0 + a_{ch}^{var}\left(t, \frac{\pi}{N}\right), \\ \psi_{ch} &= \psi_{ch}^0 - N^* t + \int F(\delta C, \delta c_{20}) dt + \psi_{ch}^{var}\left(t, \frac{\pi}{N}\right), \end{aligned} \tag{5}$$

where a_{ch}^0, ψ_{ch}^0 are the average value of the amplitude and constant phase shift and a_{ch}^{var}, ψ_{ch}^{var} are summands depending on the sectorial c_{22} and other coefficients; they express the ellipticity of the Chandler component trajectory with a very small eccentricity.

As follows from the results of the numerical simulation, the parameters of the perturbed Chandler oscillation can be found from variations in the geopotential coefficient c_{20}. As an example, Fig. 1 presents the variation in the perturbed Chandler oscillation frequency $\Delta\dot{\psi} - N^*$ and variations in the second zonal harmonic δc_{20} according to SLR (Satellite Laser Ranging) data (Cheng and Tapley, 2004).

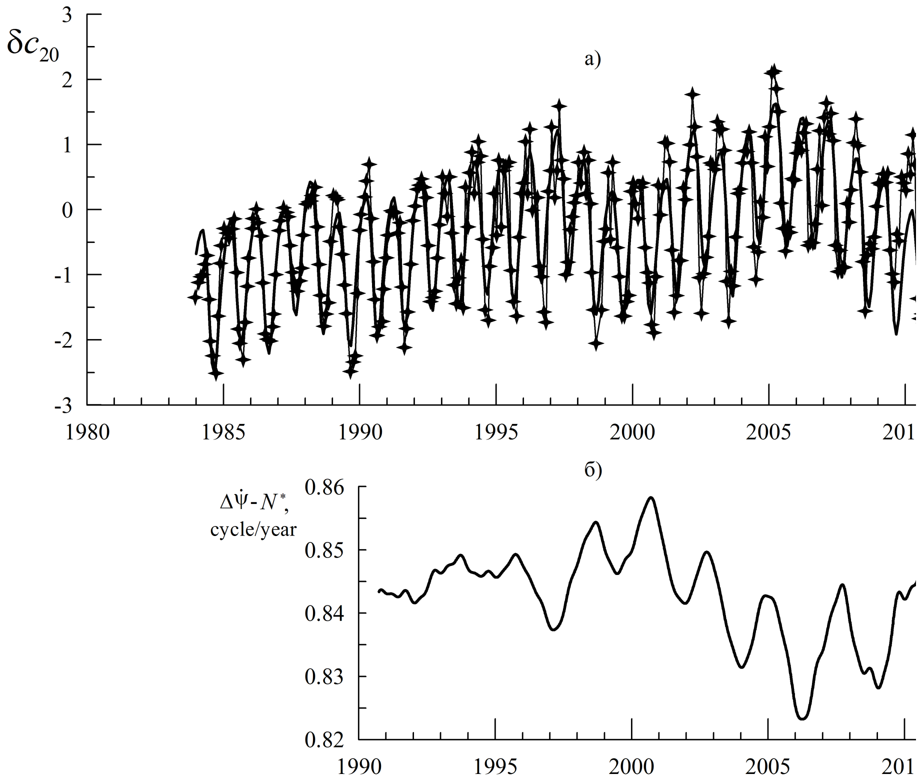

Figure 1: (a) Interpolation of variations in the second zonal harmonic δc_{20} of the geopotential on the time interval of 1984-2008 and a forecast for six years (2009-2014): the stars joined with a thin solid line are the SLR measurement data and the contrast solid line is the constructed curve. (b) Variation in the frequency $\Delta\dot{\psi}$ of the perturbed Chandler oscillation of the Earth's pole constructed in the course of the numerical simulation (1990-2014).

For coordinates of the Earth's pole (neglecting the difference in the amplitudes of the main components $\tilde{a}_{ch,h} \approx a^{p,q}_{ch,h}$), we obtain the final expressions:

$$\begin{aligned}
x_p &= c_x + \tilde{a}_{ch} \cos\left(\psi^0_{ch} - N^* t + \delta\psi + \Delta\psi\right) + a_h \cos(\psi^0_h + \nu_h t + \chi), \\
y_p &= c_y + \tilde{a}_{ch} \sin\left(\psi^0_{ch} - N^* t + \delta\psi + \Delta\psi + \varepsilon\right) + a_h \sin(\psi^0_h + \nu_h t), \\
\delta\psi &= \int F(\delta C, \delta c_{20}) dt.
\end{aligned} \tag{6}$$

Here, $\tilde{a}_{ch}$ is the resulting amplitude of the Chandler oscillation; ε and χ are the phase shifts in x_p and y_p for the Chandler and annual oscillations, respectively; and ν_h is the annual oscillation frequency.

Figure 2 presents the results of the numerical simulation of the Earth's pole motion according to the basic model (Akulenko, et al., 2012) and model (6). The plot shows an interpolation on a long time interval (from 1990 up to and including 2012) and a forecast for 2013 and 2014 for the oscillatory process in coordinates of the Earth's pole according to two models - the basic model and the refined one (6) in comparison with highly accurate IERS data.

In addition, Fig. 2 yields residuals between IERS data and theoretical curves. The corresponding root-mean-square deviations calculated on the interpolation interval for the basic model (σ^*_x, σ^*_y, σ^*_{xy}) and model (6) (σ_x, σ_y, σ_{xy}) are given in milli arcseconds:

$$\begin{aligned}
\sigma^*_x &= 44.30672865, \quad \sigma^*_y = 43.32902488, \quad \sigma^*_{xy} = 61.97169186, \\
\sigma_x &= 24.14765269, \quad \sigma_y = 20.25418818, \quad \sigma_{xy} = 31.51731698.
\end{aligned} \tag{7}$$

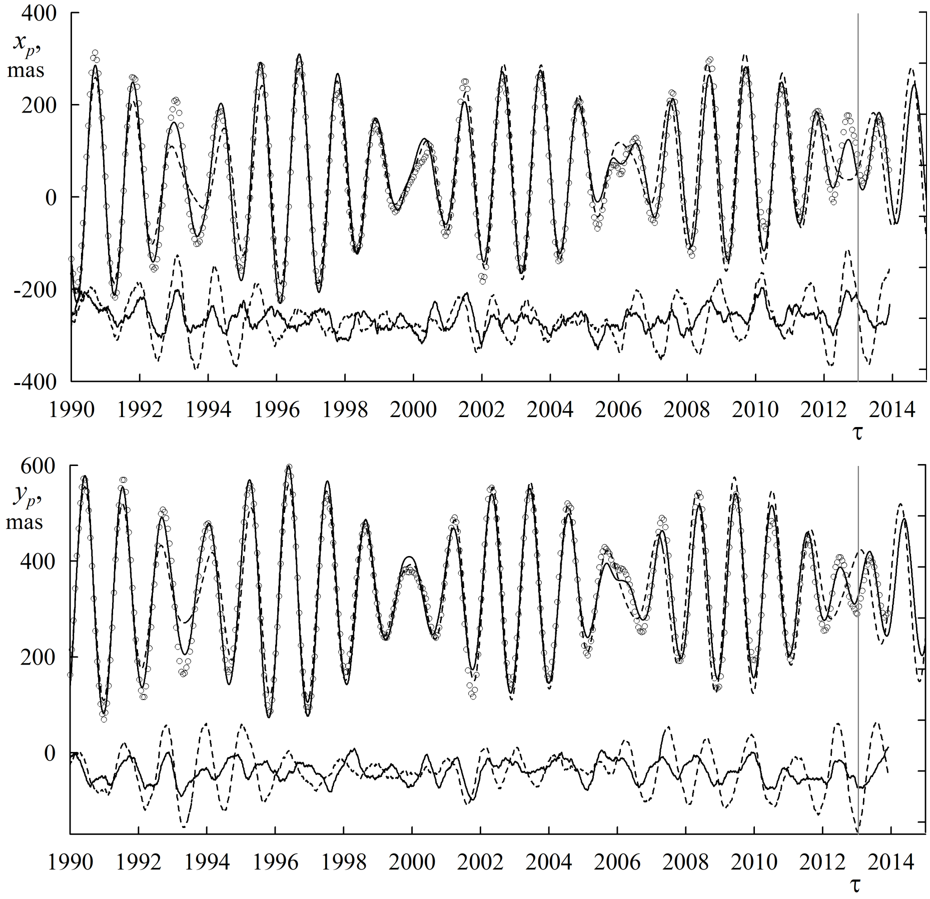

Figure 2: (a) Interpolation on the time interval from 1990 up to and including 2012 and forecast for 2013 and 2014 for the oscillatory process of the Earth's pole coordinates according to the basic (dashed line) and refined (solid curve) models in comparison with highly accurate IERS observation and measurement data (discrete points). Residuals (given below the basic plots), differences between IERS data and theoretical curves constructed according to the basic (dashed line) and refined (solid curve) models.

Based on the obtained interpolation results and forecast of pole oscillations, one can conclude that joint simulation of dynamic processes (taking into account time variations of the geopotential) allows one to refine the analytical model and improve the forecast for the pole motion trajectory.

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3. REFERENCES

Akulenko, L.D., Klimov, D.M., Markov, Yu.G., Perepelkin, V.V., 2012, "Oscillatory-rotational processes in the Earth motion about the center of mass: Interpolation and forecast", Mechancs of Solids, 47(6), pp. 601–621, doi: 10.3103/S0025654412060015.

Akulenko, L.D., Kumakshev, S.A., Markov, Yu.G., Rykhlova, L.V., 2007, "Analysis of multifrequency effects in oscillations of the Earth's pole", Astron. Rep., 51(5), pp. 421–427, doi: 10.1134/S1063772907050083.

Cheng, M., Tapley, B.D., 2004, "Variations in the Earth's oblateness during the past 28 years", J. Geophys. Res.(Solid Earth), 109, B09402, doi: 10.1029/2004JB003028.