# POSSIBLE IMPROVEMENT OF THE IAU 2006 PRECESSION BASED ON RECENT PROGRESS

J.-C.  $LIU^1$ , N. CAPITAINE<sup>2</sup>

 <sup>1</sup> School of Astronomy and Space Science, Nanjing University 163 Xianlin Avenue, 210046 Nanjing, China
 e-mail: jcliu@nju.edu.cn
 <sup>2</sup> SYRTE, Observatoire de Paris, CNRS, UPMC
 61 avenue de l'Observatoire, 75014 Paris, France
 e-mail: n.capitaine@obspm.fr

ABSTRACT. We aim to investigate the possibility of improving the IAU 2006 precession model after more than 10 years since its publication based on new solutions of the Earth-Moon Barycenter (EMB) motion, new theoretical contribution to the precession rates, and the revised  $J_2$  long-term variation obtained from the Satellite Laser Ranging (SLR). We use these upgraded models and follow the same procedure as that followed by Capitaine et al. (2003) to provide the IAU 2006 precession expressions. The revised precession expressions for the ecliptic are derived by fitting the new analytical planetary theory VSOP2013 to the JPL numerical ephemerides DE422. For solving the precession of the equator, more realistic Earth model including the  $J_2$  quadratic variation and precession rate at initial epoch are applied in the integration of equations. The quadratic and cubic terms in the revised precession quantity  $\psi_A$  differs from IAU 2006 quite significantly. The statistics of the VLBI celestial pole offsets (1979–2014) and least squares fits with different empirical models show that the revised precession is slightly more consistent with the VLBI observations but the improvement relative to the IAU model is not convincing.

## 1. INTRODUCTION

The current precession model is the IAU 2006 model (Capitaine et al. 2003). The precession of the ecliptic was derived by fitting the analytical ephemerides VSOP87 to the long term ephemerides DE406 over 2000 years. The IAU 2006 precession of the equator is a dynamically consistent solution. The basic precession quantities  $\psi_A$  and  $\omega_A$  were derived by solving the dynamical equations using improved ecliptic precession, integration constants provided by IAU 2000 with a careful consideration of the perturbing effects, and the best non-rigid Earth model available at that time. The linear change in  $J_2$  was considered, which contributes -14 mas t in the theoretical precession rate in longitude, and is responsible for about -7 mas cy<sup>-2</sup> in the final polynomial expression of  $\psi_A$ . The uncertainty of  $J_2$  rate, which is expected to be of about 20%, is the main limiting factor for the accuracy of the precession in longitude.

This paper reports on our effort to develop upgraded precession solutions with application of new scientific progresses during the last ten years. The methods used are mainly based on Capitaine et al. (2003) and our results are compared with IAU 2006 model and VLBI observations

## 2. IMPROVING THE PRECESSION OF THE ECLIPTIC

The precession of the ecliptic is defined as the secular part of the ecliptic pole motion in the initial reference system, which is described by the parameters  $P_A$  and  $Q_A$ . We use the new analytical planetary solution VSOP2013 developed by Simon et al. (2013) to improve the precession of the ecliptic. VSOP2013 solution provides the elliptic elements, including p and q (equivalent to  $P_A$  and  $Q_A$ ), for the eight planets in the form of Poisson series, the secular parts of p and q for EMB representing the precession of the ecliptic. It is more accurate by a factor of 5 with respect to VSOP2000. On the other hand the improved DE422 numerical ephemerides are used as observational material to confine the secular motion of the ecliptic as provided by VSOP2013. Figure 1 shows the difference between DE422 and VSOP2013 for the EMB motion represented by  $P_A$  and  $Q_A$  in the dynamical ecliptic frame over 20 centuries.

The 250-day sampling series of  $(t, \Delta p, \Delta q)$  in sense of [DE422–VSOP2013] between J1000 and J3000 are fitted to the fifth order polynomials. The resulting coefficients of constant terms  $p_0$  and  $q_0$  are used



Figure 1: The difference  $\Delta P_A$  and  $\Delta Q_A$ , for  $P_A$  and  $Q_A$ , in sense of [DE422-VSOP2013] over 20 centuries.

to improve the rotation angles while the coefficients of  $t^1 - t^5$  terms are added to corresponding secular terms given by VSOP2013. Table 1 gives the final ecliptic precession quantities  $P_A$  and  $Q_A$  derived from VSOP2013 fitted to DE422. The major discrepancies between the revised and the IAU 2006 precession of the ecliptic are at the order of several tens of microarcseconds per century in the linear terms , while the second-order term differs less than  $20 \,\mu \text{as cy}^{-2}$ , which can be considered as negligible.

	unit	$t^1$	$t^2$	$t^3$	$t^4$	$t^5$
$P_{\rm A}$	//	4.19903	0.19401	$-2.23533 \times 10^{-4}$	$-1.03944 \times 10^{-6}$	$2.15694 \times 10^{-9}$
$\Delta P_{\rm A}$	$\mu as$	-65	18	1	-0.1	0.01
$Q_{\mathrm{A}}$	//	-46.81099	0.05102	$5.21368 \times 10^{-4}$	$-5.5808 \times 10^{-7}$	$-1.2059 \times 10^{-9}$
$\Delta Q_{\rm A}$	$\mu as$	28	-11	-3	0.1	0.02

Table 1: Precession quantities of the ecliptic derived from VSOP2013 and DE422 ephemerides and comparison with the IAU 2006 precession model.  $\Delta P_{\rm A}$  and  $\Delta Q_{\rm A}$  are calculated in sense of [revised – IAU 2006].

# 3. IMPROVING THE PRECESSION OF THE EQUATOR BASED ON RECENT PROGRESS

The solutions for the precession of the equator are derived by solving the differential equations. The classical 7(8)-degree Runge-Kutta-Fehlberg method was used to derive the discrete points (250-day step over 2000 years centered on J2000.0) for basic quantities  $\psi_A$ ,  $\omega_A$  and secondary quantities  $\epsilon_A$ ,  $\chi_A$  and  $p_A$ , then the polynomial expressions are obtained from least squares fit.

The Earth model used in precession computation is reflected in the theoretical expressions for the precession rates  $r_{\psi}$  and  $r_{\epsilon}$  with respect to the moving ecliptic where the complete list for the theoretical contributions are provided in Table 3 of Capitaine et al. (2003). The progress in precession rates, within our knowledge, include following terms: 1. Revised nonlinear terms in longitude  $-960 \,\mu \text{as cy}^{-1}$  and in obliquity  $+340 \,\mu \text{as cy}^{-1}$  (Capitaine et al. 2005); 2. Determination of the  $J_2$  long-term variation based on satellite laser ranging (SLR, Cheng et al. 2013). This will be discussed in detail in the following. 3. The contribution of tidal Poisson terms on non-rigid Earth rotation (Folgueira et al. 2007). This contributes  $88 \,\mu \text{as cy}^{-1}$  to the precession rate in obliquity; 4. The effect of second-order torque on precession rate in obliquity (Lambert & Mathews 2008). The value was found to be  $-1840 \,\mu \text{as cy}^{-1}$ ; 5. The effect from Galactic aberration (Liu et al. 2012). The systematic effect in precession rates caused by Galactic aberration is at the order of  $10 \,\mu \text{as cy}^{-1}$ .

Generally the long-term trend in  $J_2$  has been approximated by a negative linear drift. Cheng & Tapley (2004) has found from 28-year SLR observational data (1976-2004) a secular decrease of  $J_2$  with a rate  $-2.75 \times 10^{-9}$  cy<sup>-1</sup>, which is close to the value used in the IAU model. More recently, Cheng et al. (2013) reported the updated feature in  $J_2$  based on the time series of 30-day SLR estimate of  $J_2$  between 1976 and 2012. Figure 2 shows the variation of  $J_2$ . Straight lines and parabola are used as empirical models to interpret the long-term variations in the observations. The estimated linear trend with the data earlier than 1996 (green solid line) is  $-3.04\pm0.32\times10^{-9}$  cy<sup>-1</sup>, but a much smaller value  $-0.67\pm0.19\times10^{-9}$  cy<sup>-1</sup> can be found if more recent data between 1996 and 2012 are involved (red solid line). This shows that

the deceleration in  $J_2$  variation is significant; therefore the  $J_2$  variation can be described by a sum of a linear term and a parabola fitted to LSR data (black curve):

$$J_2 = 1.08263582 \times 10^{-3} - 0.53191 \times 10^{-9} t + 1.08490 \times 10^{-8} t^2.$$
(1)

Consequently the contribution of  $J_2$  variation to the precession rate in longitude is  $-2.482 \,\mu \text{as} t^2 + 50629 \,\mu \text{as} t^3$ .



Figure 2: 30-day estimates of  $J_2$  values from SLR and its long term variation. The constant  $\bar{J}_2$  is the mean value for  $J_2$ , which equals 0.0010826359797. The original data is provided by Cheng et al. (2013).

The integration constants  $r_0$  and  $u_0$  at J2000.0 for precession rates in longitude and obliquity are crucial for solving the precessional equations. The precession corrections that are consistent with the updated non-linear terms are given by Capitaine et al. (2005). Taking the spurious contributions (Capitaine et al. 2003) into consideration, we obtain the integration constants  $r_0$  and  $u_0$ :

$$r_0 = 5038''.482040; \quad u_0 = -0''.025754.$$
 (2)

By using (1) the updated ecliptic precession in Table 1, (2) additional theoretical contributions to the precession rates, and (3) integration constants in Eq. (2), we obtain the precession of the equator by solving differential equations. The basic precession quantities are:

$$\psi_{A} = 5038''.482040 t - 1''.0732414 t^{2} + 0''.01573401 t^{3} + 0''.000127135 t^{4} - 0''.0000001020 t^{5}$$
  

$$\omega_{A} = \epsilon_{0} - 0''.025754 t + 0''.0512625 t^{2} - 0''.0077249 t^{3} - 0''.000000267 t^{4} + 0''.000000267 t^{5}, \quad (3)$$

with  $\epsilon_0 = 84381''.406$ . The differences between the revised solution and IAU 2006 precession of the equator are:

$$\Delta\psi_{\rm A} = 532\,t + 5765\,t^2 + 16874\,t^3 - 6\,t^4 - 0.01\,t^5; \quad \Delta\omega_{\rm A} = -1\,t + 0.3\,t^2 + 0.1\,t^3 + 0.1\,t^4 - 0.07\,t^5, \quad (4)$$

where the units of the coefficients are  $\mu$ as and t is in Julian centuries from J2000.0. The largest difference in the quadratic and cubic terms for  $\psi_A$  are induced by using the updated empirical model for  $J_2$  variation.

## 4. COMPARISON WITH VLBI CELESTIAL POLE OFFSETS

The observed differences with respect to the IAU-model-predicted CIP positions are reported as "celestial pole offsets" dX and dY. We try to investigate the accuracy of the revised precession model using the best available VLBI data over 1979-2013. The time series for celestial pole offsets are derived with respect to the revised and IAU 2006 precession respectively. The free core nutation has been removed with the empirical model. For each dX and dY time series we calculated the Weighted Mean (WM) value and the Weighted Root Mean Square (WRMS) which can be used to indicate the overall consistency between the theoretical predictions and observations. Table 2 shows that the smaller WM and WRMS of the offsets can be found when the revised precession model has been applied to calculate the CIP location.

For the dX component, the revised precession appears to be more consistent with VLBI observations than the current IAU 2006 precession as the WM decrease about 72% refer to the IAU one. Regarding the dY component, the WM and WRMS relative to the revised solution are close to the value for the IAU model.

		IAU 2006	revised			IAU 2006	revised
WM	$\mathrm{d}X$	0.0467	0.0130	WRMS	$\mathrm{d}X$	0.1349	0.1261
	$\mathrm{d}Y$	-0.0565	-0.0561		$\mathrm{d}Y$	0.1442	0.1441

Table 2: Weight Mean (WM) and Weighted Mean Root Square (WRMS) of the celestial pole offsets related to the revised and the IAU 2006 precession models. The unit is mas.

To interpret the residuals between VLBI observations and two precession solutions, we have used parabola and straight line plus 18.6-year nutation for the least squares fit. The results show that the longer time span of VLBI data reduced the coefficients of the quadratic model especially for the  $t^2$  term compared to the results in Capitaine et al. (2009). However it is difficult to discriminate which model is more appropriate to interpret the physical reason for the overall residuals because the WRMS relative to both IAU and the revised precession are reduced by approximately the same level and the coupling between linear/quadratic and linear/18.6-year terms are significant.

### 5. DISCUSSION

In this work we have investigated the possibility of improving the IAU 2006 precession (Capitaine et al. 2003) model with recent progress in the last decade. The revised solution developed in this paper are based on recent improvements in EMB motion and theories in precession rates. However we recommend to retain the current IAU model for the following reasons: (1) The changes in the precession of the ecliptic is negligible; (2) The  $J_2$  variation can still be approximated by empirical model but not predicted by geophysical theories; (3) Improvement of the revised solution is not very convincing from the comparison with VLBI; (4) The precession model itself is a secular phenomenon over thousands of years: 10 years of progress seems not sufficient to change the standard of the model. More detailed analysis will be carried out by the authors in the near future.

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