LONG TIME DYNAMICAL EVOLUTION OF HIGHLY ELLIPTICAL SATELLITES ORBITS

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ABSTRACT. Dynamical evolution of objects near Molniya-type orbits is considered. Initial conditions correspond to highly elliptical satellite orbits with eccentricities 0.65 and a critical inclination 63.4°. Semi-major axis is varied near resonant value 26560 km in an interval 500 km. Variations were analyzed for positional orbital elements, an ascending node longitude and an argument of pericenter. Initial conditions determined when orbital elements variations are minimal. These regions can be used as orbits for safe stationing satellites which finish work on Molniya-type orbits. The study of dynamical evolution on long time intervals was performed on the basis of the results of numerical simulation. The model of disturbing forces taken into account the main perturbing factors. Time interval was up to 24 yr. Area-to-mass ratio varied from small values corresponding to satellites to big ones corresponding to space debris.

1. INTRODUCTION

Region of high-elliptical orbits (HEO) has a very complex dynamics. Both active and passive objects are moved on HEO. There is a problem of protecting active satellites from space debris. It requires high-accuracy propagation of HEO objects motion. These objects have a long-term evolution of eccentricities and inclinations due to the Lidov–Kozai resonance (Lidov, 1962; Kozai, 1962). There are secular perturbations of semi-major axes due to the atmospheric drag. The Poynting–Robertson effect also leads to secular perturbations of semi-major axes for objects with area-to-mass ratio (AMR) more than 1 m^2/kg (Kuznetsov et al., 2012). The dynamical evolution of high AMR objects in the Molniya-type orbits was studied by (Sun et al., 2013). In this paper, a vicinity of Molniya orbit is considered. A stochastic trajectory formation due to objects passage through high-order resonance zones was considered.

We present both analytical and a numerical results for locations and sizes of high-order resonance regions in the vicinity of Molniya-type orbits. Secular perturbations of the semi-major axes of the orbits are estimated in the vicinity of the resonance zones. A long-time orbital evolution is investigated for HEO orbits and orbits surrounding these regions. AMR values are variable. Capture and escape from resonance, as well as a passage through resonance, is considered to be an orbital evolution.

2. ANALYTICAL APPROXIMATION

The frequencies of the perturbations caused by the effect of sectoral and tesseral harmonics of the Earth's gravitational potential are a linear combinations of the mean motion of a satellite n_M, angular velocities of pericenter motion n_g and node motion n_Ω of it's orbit, and angular velocity of the Earth ω.

Following Allan (1967a, 1967b), we form the frequencies

$$\nu_1 = p(n_M + n_\Omega + n_g) - q\omega, \qquad \nu_2 = p(n_M + n_g) + q(n_\Omega - \omega), \qquad \nu_3 = pn_M + q(n_g + n_\Omega - \omega) \quad (1)$$

of three critical arguments

$$\Phi_1 = p(M+\Omega+g) - q\omega t = \nu_1 t, \quad \Phi_2 = p(M+g)+q(\Omega-\omega t) = \nu_2 t, \quad \Phi_3 = pM+q(g+\Omega-\omega t) = \nu_3 t, \ (2)$$

where M is the mean anomaly, Ω is the longitude of the ascending node, g is the argument of the pericenter, and p, q are an integers.

The condition $\nu_1 \approx 0$ corresponds to the resonance p:q between the satellite's mean motion n_M and the Earth's angular velocity ω. This condition represents the n-resonance. The condition $\nu_2 \approx 0$ corresponds to an i-resonance under which the position of the ascending node of the orbit repeats periodically in a rotating coordinate system. The condition $\nu_3 \approx 0$ corresponds to an e-resonance at which the position of the line of apsides is considered.

Analytical estimations were obtained for locations and sizes of resonance regions. Mean motions n_M, n_g, n_Ω were calculated taking into account the secular perturbations from the Earth's oblateness $\dot{M}_{J_2}$, $\dot{g}_{J_2}$, $\dot{\Omega}_{J_2}$ (Beutler, 2005), the Moon's attraction $\dot{M}_L$, $\dot{g}_L$, $\dot{\Omega}_L$, the Sun's attraction $\dot{M}_S$, $\dot{g}_S$, $\dot{\Omega}_S$ (Timoshkova and Kholshevnikov, 1974).

$$n_M = \sqrt{\frac{\varkappa^2}{a^3}} + \dot{M}_{J_2} + \dot{M}_L + \dot{M}_S, \qquad n_g = \dot{g}_{J_2} + \dot{g}_L + \dot{g}_S, \qquad n_\Omega = \dot{\Omega}_{J_2} + \dot{\Omega}_L + \dot{\Omega}_S. \tag{3}$$

Where $\varkappa^2$ is the Earth's gravitational parameter and a is a semi-major axis of an orbit,

$$\dot{\Omega}_{J_2} = -\frac{3}{2} J_2 n \left(\frac{r_e}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}, \qquad \dot{\Omega'} = -\frac{3}{16} n \frac{m'}{m_\oplus} \left(\frac{a}{a'}\right)^3 \frac{2+3e^2}{(1-e^2)^{1/2}} (2 - 3\sin^2 i') \cos i,$$

$$\dot{g}_{J_2} = \frac{3}{4} J_2 n \left(\frac{r_e}{a}\right)^2 \frac{5\cos^2 i - 1}{(1-e^2)^2}, \qquad \dot{g'} = \frac{3}{16} n \frac{m'}{m_\oplus} \left(\frac{a}{a'}\right)^3 \frac{4 - 5\sin^2 i + e^2}{(1-e^2)^{1/2}} (2 - 3\sin^2 i'),$$

$$\dot{M}_{J_2} = \frac{3}{4} J_2 n \left(\frac{r_e}{a}\right)^2 \frac{3\cos^2 i - 1}{(1-e^2)^{3/2}}, \qquad \dot{M'} = \frac{1}{16} \frac{m'}{m_\oplus} \left(\frac{a}{a'}\right)^3 (8 + 15e^2)(2 - 3\sin^2 i')(2 - 3\sin^2 i).$$

Where J_2 is the second zonal harmonic coefficient, n is the two-body mean motion, r_e is the mean equatorial radius of the Earth, i and e are the inclination and eccentricity of satellite's orbit, $m_\oplus$ is the Earth's mass, m', a' and i' are the mass, semi-major axis and inclination of perturbing body orbit (the Moon or the Sun).

Expansions of perturbing functions coincide for outer body attraction and solar radiation pressure. These expansions are differed by notations and limits of summation only. We used expansion for solar attraction to take into account solar radiation perturbations. The Sun's mass was reduced on

$$\mu = -\frac{1}{f} b \gamma P_0 r_S^2. \tag{4}$$

Where μ is the Sun's mass reduction (Polyakhova and Timoshkova, 1984), f is the gravitational constant, b is the reflection coefficient of the satellite surface, γ is AMR, $P_0 = 4.56 \cdot 10^{-6}$ kg m^{-1} s^{-2} is the solar pressure, r_S is the distance from the Earth to the Sun.

We estimated values of the semi-major axis corresponding to the n-, i- and e-resonances from the conditions $\nu_1 = 0$, $\nu_2 = 0$, and the $\nu_3 = 0$ in the vicinity of Molniya-type orbits. Initial conditions corresponded to high-elliptical orbits with an eccentricity 0.65 and critical inclination 63.4°. Semi-major axis values varied from 26000 km to 27100 km. There were 17 high-order resonance relations $p{:}q$ between mean motion of angular orbital elements and the Earth's angular velocity: $16 \leqslant |p| \leqslant 25$, $33 \leqslant |q| \leqslant 49$, orders of the resonances are $49 \leqslant |p| + |q| \leqslant 74$ in this region.

3. NUMERICAL SIMULATION

The study of orbital evolution on long time intervals was performed based on the results of numerical simulations conducted using "A Numerical Model of the Motion of Artificial Earth's Satellites" developed by the Research Institute of Applied Mathematics and Mechanics of the Tomsk State University (Bordovitsyna et al., 2007). The model of disturbing forces accounts the nonsphericity of the gravitational field of the Earth (model EGM96, harmonics up to the 27$^{\text{th}}$ order and degree inclusive), the attraction of the Moon and the Sun, the tides in the Earth's body, the direct radiation pressure, taking into account the shadow of the Earth (the reflection coefficient of the satellite surface $b = 1.44$), the Poynting–Robertson effect, and the atmospheric drag. The integration of motion equations was carried out using the Everhart's method of the 19$^{\text{th}}$ order.

Initial conditions as mentioned above correspond to high-elliptical orbits with an eccentricity $e_0 = 0.65$ and critical inclination $i_0 = 63.4°$. Initial semi-major axes a_0 values are consistent with a resonant conditions arisen from the analytical approximation. The initial value of the argument of the pericenter g_0 was 270°. The initial values of the longitude of the ascending node Ω_0 are 0°, 90°, 180°, and 270°.

This coincides with initial values of a solar angle $\varphi_0 = \Omega_0 + g_0 = 270°$, $0°$, $90°$, and $180°$. AMRs tried were equal to 0.02, 0.2, and 2 m^2/kg. Period of integration is 24 years.

4. DYNAMICAL EVOLUTION IN A REGION NEAR THE 22:45 RESONANCE

We present dynamical evolution using the example of the 22:45 resonance. Qualitative evolution for the rest 16 high-order resonances is the same.

Semi-major axis evolution depends on the solar angle weakly when AMR is 0.02 m^2/kg and it corresponds a satellite. The evolution of e, i, and g for Molniya-type orbits depend on the orientation of the orbital plane significantly. Maximal magnitudes of oscillations are reached when the initial solar angle $\varphi_0 = 0°$. The magnitudes of oscillations are minimal at $\varphi_0 = 180°$. The argument of the pericenter has a libration near the initial value of $g_0 = 270°$ due to the initial critical inclination $i_0 = 63.4°$.

Object has temporary captures into i- and e-resonance due to the long-term evolution of eccentricity and inclination of it's orbit. Libration of critical argument Φ corresponds to resonant motion. Object has capture into resonance and escape from resonance due to the long-term evolution of eccentricity and inclination of it's orbit when mean value of semi-major axis is saved almost constant. Secular perturbations of semi-major axis is approximately to -5 m/year due to the Pointing–Robertson effect.

When AMR is 2 m^2/kg, it corresponds a space debris. Increase of AMR leads to increase of magnitude of short-periodic perturbations. There are captures into n-resonance when mean value of semi-major axis is equal to resonant value one. After 12 years the mean value of the semi-major axis is became less the resonant value due to the Poynting–Robertson effect. Secular decrease in the semi-major axis, which, for a spherically symmetrical satellite with AMR = 2 m^2/kg near the 22:45 resonance region, equals approximately -0.5 km/year. Numerical simulation shows that this effect weakens slightly, in resonance regions. Under the Poynting–Robertson effect objects pass through the regions of high-order resonances.

5. STOCHASTIC TRAJECTORIES FORMATION

The Poynting–Robertson effect results in a secular decrease in the semi-major axis of a spherically symmetrical satellite (Smirnov et al., 2001). The secular perturbations of the semi-major axis lead to formation weak stochastic trajectories. We described the stochastic properties of the motion based on an analysis of the integrated autocorrelation function (IACF) $\mathcal{A}$ (Wytrzyszczak et al., 2007).

The IACF $\mathcal{A}$ asymptotically approaches unity for constant time series. For a uniform time series representing a periodic sine function, $\mathcal{A} = 0.5$. For other periodic and quasi-periodic time series, $\mathcal{A}$ approaches a finite value close to 0.5. For chaotic trajectories, $\mathcal{A}$ asymptotically approaches zero with a speed proportional to the inverse of the exponential decay time.

Figure 1 shows the IACF $\mathcal{A}$ for the semi-major axis a. Initial value of semi-major axis a_0 is 26162 km, AMR is 0.02 m^2/kg. The IACF $\mathcal{A}$ is asymptotically decreasing to 0.02 for all the solar angles. The dynamical evolution has chaotic properties for all initial values of the solar angle.

6. CONCLUSION

The Poynting–Robertson effect results in a secular decrease in the semi-major axis of a spherically symmetrical satellite. Secular decrease in the semi-major axis is approximately -0.5 km/year for an object near-resonance 22:45 region with AMR = 2 m^2/kg. In resonance regions the effect weakens slightly. Reliable estimates of secular perturbations of the semi-major axis were obtained from the numerical simulation. Under the Poynting–Robertson effect objects pass through the regions of high-order resonances. The Poynting–Robertson effect and secular perturbations of the semi-major axis lead to formation weak stochastic trajectories.

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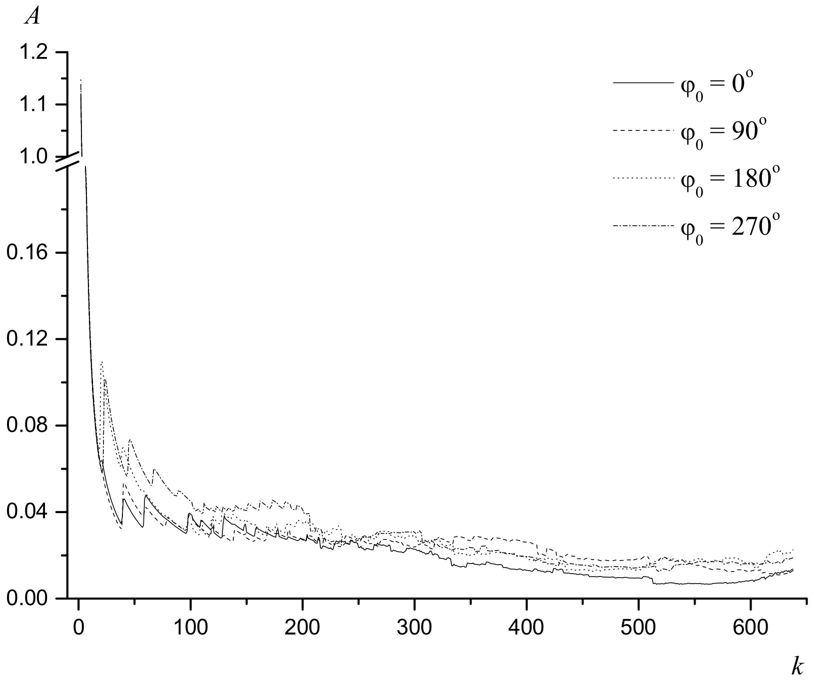

Figure 1: The integrated autocorrelation function $\mathcal{A}$ for the semi-major axis a.

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