STATISTICAL INVERSION METHOD FOR BINARY ASTEROIDS ORBIT DETERMINATION

I.D. KOVALENKO[1], D. HESTROFFER[1], A. DORESSOUNDIRAM[2], N.V. EMELYANOV[3]
R.S. STOICA[1,4]
[1] IMCCE, Observatoire de Paris, CNRS/UPMC
77 Av. Denfert Rochereau, F-75014 Paris, France
e-mail: ikovalenko@imcce.fr
[2] LESIA, Observatoire de Paris, CNRS
5, place Jules Janssen, F-92195 Meudon
e-mail: alain.doressoundiram@obspm.fr
[3] Sternberg Astronomical Institute
13 Universitetskij prospect, 119899 Moscow, Russia
e-mail: emelia@sai.msu.su
[4] Université Lille 1, Laboratoire Paul Painlevé
59655 Villeneuve d'Ascq Cedex, France
e-mail: radu.stoica@math.univ-lille1.fr

ABSTRACT. We focus on the study of binary asteroids, which are common in the Solar system from its inner to its outer regions. These objects provide fundamental physical parameters such as mass and density, and hence clues about the early Solar System. The present method of orbit calculation for resolved binaries is based on Markov Chain Monte-Carlo statistical inversion technique. In particular, we use the Metropolis-Hastings algorithm combined the Thiele-Innes equation for sampling orbital elements through the sampling of observations. The method requires a minimum of four observations, made at the same tangent plane; it is of particular interest for initial orbit determination. The observations are sampled within their observational errors with an assumed distribution. The sampling predicts the whole region of possible orbits, including the one that is most probable.

1. STATISTICAL INVERSION PROBLEM

The statistical ranging method using Markov-Chain Monte-Carlo for asteroid heliocentric orbit determination has been investigated by, for example, Oszkiewicz et al. (2009) and Virtanen et al. (2001). We use the similar approach for binary asteroids to determine relative orbit.

The N astrometric observations at times $t = (t_1, ..., t_N)$ are related to the theoretical positions through the observational equation:

$$\varphi = \psi(X) + \varepsilon\,,$$

where $\varphi = (\rho_1, \theta_1; ...; \rho_N, \theta_N)$ is a set of N observations, presented by relative distance and angle, $\psi(X)$ is a computed sky-plane positions, $X = (a, e, i, \Omega, \omega, T, P)$ is the vector of orbital elements (semi-major axis, eccentricity, inclination, longitude if the ascending node, argument of periapcsis, the time of perihelion passage) and the period respectively, and $\varepsilon = (\varepsilon_{\rho 1}, \varepsilon_{\theta 1}; ...; \varepsilon_{\rho N}, \varepsilon_{\theta N})$ is the vector of observational errors.

Applying the Bayesian statistics the *a posteriori* probability density of the binary asteroid orbital parameters can be estimated from the *a priori* and the noise probability density. Using the Bayes' theorem

$$p(X|\varphi) = \frac{p(X)p(\varphi|X)}{p(\varphi)}$$

and following the statistical inversion theory, the *a posteriori* probability density of the orbit elements is related to the *a priori* and noise probability densities:

$$p(X|\varphi) \propto p(X)p_\varepsilon\,,$$

where the likelihood function coincides with the noise probability density $p_\varepsilon = p(\varphi|X) = exp(-\frac{1}{2}\varepsilon^T\Lambda^{-1}\varepsilon)$, $\varepsilon = \varphi - \psi(X)$, and Λ is the covariance matrix $2N \times 2N$ for the observational errors.

The *a priori* probability density can be expressed as $p(X) = exp[-U(X)]$, where $U(X)$ includes the distributions of each parameter. The final *a posteriori* orbital parameters probability density function:

$$p(X|\varphi) \propto exp[-\frac{1}{2}\varepsilon^T \Lambda^{-1}\varepsilon - U(X)]\,.$$

2. MARKOV CHAIN MONTE-CARLO METHOD

The Metropolis-Hastings algorithm, based on the Markov Chain Monte-Carlo method, has been used for sampling parameters X. First, from the whole set of N observations we select four observations on the same tangent plane; this is necessary and sufficient for binary asteroid relative orbit determination using the Thiele-Innes method. Then, the corresponding relative distance ρ and angle θ which sampled. For each iteration we introduce the proposal densities for the set of four observations, correspond to the Gaussian distribution of observational errors. They are centred around the last accepted sampling. We denote the proposal set of 4 positions S' and last accepted S_t.

We generate proposal positions $(\rho'_i; \theta'_i), i = 1, 2, 3, 4$ for the four chosen observation data with the proposal densities

$$(\rho'_i \propto G(\rho^t_i; \sigma(\rho_i); \theta' \propto G(\theta^t_i; \sigma(\theta_i))\,.$$

Then from the four positions sampled we calculate an orbit X' using Thiele-Innes method, and check the solution to all observations. The new candidate orbital elements and system mass are rejected or accepted according to acceptance criteria:

$$a = \frac{p(X'|\varphi)p(S_t, S')J_t}{p(X_t|\varphi)p(S', S_t)J'}\,,$$

where J' and J_t are the determinants of the Jacobian matrix from (ρ, θ) coordinates to parameters X for the candidate and the last accepted sample. The Jacobians are defined as (Oszkiewicz et al., 2009):

$$J = \left|\frac{\partial S}{\partial X}\right|\,.$$

We use the symmetric p.d.f.s $p(S_t, S')$ and $p(S', S_t)$, therefore

$$a = \frac{p(X'|\varphi)J_t}{p(X_t|\varphi)J'}\,.$$

If $a \geq 1$ we accept the candidate elements, then $X_{t+1} = X'$. If $a < 1$ we accept the candidate elements with a probability equal to a, or $X_{t+1} = X_t$ with a probability equal to $1 - a$. This process is repeated until the stationary a posteriori density is reached. The algorithm is run for a large number of iterations until the entire possible orbital-element space is mapped.

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3. REFERENCES

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