DO WE NEED VARIOUS ASSUMPTIONS TO GET A GOOD FCN?

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ABSTRACT. Free core nutation (FCN) is a rotational modes of the Earth with fluid core. All traditional theoretical methods produce FCN period near 460 sidereal days with PREM Earth model, while precise observations (VLBI + SG tides) say it is approximately 430 days. In order to fill this big gap, astronomers and geophysicists give various assumptions, e.g., increasing core-mantle-boundary (CMB) flattening by about 5%, a strong coupling between nutation and geomagnetic field near CMB, viscous coupling, or topographical coupling cross CMB, etc. Do we really need these unproved assumptions? or is it only the problem of these traditional theoretical methods themselves? Earth models (e.g., PREM) provide accurate and robust profiles of physical parameters, like density and Lame parameters, but their radial derivatives, which are also used in all traditional methods to calculate normal modes (e.g., FCN), nutation and tides of non-rigid Earth theoretically, are not so trustable as the parameters themselves. Moreover, the truncation of the expansion series of displacement vector and stress tensor in traditional methods is also of question. A new stratified spectral method is proposed and applied to the computation of normal modes, to avoid these problems. Our primary result of the FCN period is 435 ± 3 sidereal days.

1. KEY QUESTION TO THE STUDY OF FCN

Free core nutation (FCN) is a rotational mode of the Earth with fluid outer core (FOC), as the rotating axes of the FOC and that of the mantle do not coincide. The period of FCN reflects and depends on the physical parameters and dynamics of the FOC, mantle and especially that near core-mantle-boundary (CMB). Moreover, FCN influences strongly the retrograde annual (−1yr.) nutation due to their resonance. Therefore, FCN is key parameter in the study of the Earth rotation and the physics of the Earth interior.

Almost all calculated FCN periods from traditional theoretical approaches are near 460 days (e.g., Dehant & Defraigne, 1997; Huang et al., 2001; Rogister, 2001; Mathews et al., 2002; Crossley & Rochester, 2014) with a one-dimensional Earth model like PREM (Dziewonski & Anderson, 1981), while modern precise observations from VLBI and superconductivity tidal gravimetry produce FCN period near 430 days with precision of less than one day (e.g., Malkin, 2007; Cui et al., 2014). Their difference of approximate 30 days is significant large comparing with the observation precision. Although some studies discover that the FCN period may vary in the past three decades, its variation is still less than 3 days and much smaller than the 30 days gap.

In order to fill this big gap, astronomers and geophysicists give various assumptions. The earliest and easiest assumption is to increase the flattening of core-mantle-boundary (CMB) by about 5% (see e.g., Newburg et al., 1990; Huang, 1999) from that calculated from hydro-static-equilibrium Earth by Clairaut equation and PREM Earth model, i.e., increasing the difference between the polar radii and the equatorial radii by about 500 meters.

Another mechanism proposes a strong coupling between nutation and geomagnetic field near CMB. In the original work of Rochester & Smylie (1965), the equatorial components of the electromagnetic torque act on the mantle is calculated rigorously and the electromagnetic damping of the Chandler wobble is first quantitatively investigated, and it is shown that the electromagnetic core-mantle coupling fails by several orders of magnitude either to generate or to damp the Chandler wobble. Following the same way of this work, Buffett et al. (2002) discuss the effect of this coupling on nutation and FCN and show that this coupling can explain the gap of FCN period. However, Huang et al. (2011) argue that, even using the same values of the electro-magnetic parameters near CMB as used by Buffett et al. (2002), the contribution of this coupling to FCN period is one order of magnitude smaller than required.

There are also some other assumptions proposed to interpret this big gap of FCN period. They are viscous coupling (e.g., Mathews & Guo, 2005; Buffett, 2010, and topographical coupling (Wu & Wahr, 1997; Dehant et al., 2014), etc.

Do we really need these unproved assumptions? or is it only the problem of these traditional theoretical methods themselves? We propose an independent approach here to study FCN and show that we do not need these assumptions and our primary result of FCN period is 435 ± 3 sidereal days, very close to the observation.

2. THEORETICAL APPROACH

As usual, we start from the dynamic equation for infinitesimal elastic-gravitational motion for a rotating, slightly elliptical Earth is given as, in a steadily rotating reference frame with constant speed Ω_0, (Smith, 1974; or Dahlen & Tromp (1998) for more information):

$$\rho D_t^2\mathbf{s} + 2\rho\mathbf{\Omega_0} \times D_t\mathbf{s} = -\rho\mathbf{\Omega_0} \times (\mathbf{\Omega_0} \times \mathbf{s}) + \nabla \cdot \mathbf{T}^e - \nabla(\gamma\nabla \cdot \mathbf{s}) - \rho\nabla\phi_1 - \rho\mathbf{s} \cdot \nabla\nabla\phi + \nabla \cdot [\gamma(\nabla\mathbf{s})^T], \quad (1)$$

where γ is the equilibrium pressure, and ϕ_1 is the incremental gravitational potential induced by the mass redistribution due to deformation. The stress tensor $\mathbf{T}^e$ is the incremental stress with respect to the reference stress, $\mathbf{T}^{ref.} = -\gamma\mathbf{I}$ where $\mathbf{I}$ is the identity tensor, and is related to the displacement field by two Lamé parameters (λ, μ) for an isotropic medium as

$$\mathbf{T}^e = \lambda(\nabla \cdot \mathbf{s})\mathbf{I} + \mu[\nabla\mathbf{s} + (\nabla\mathbf{s})^T] \quad (2)$$

where rigidity $\mu = 0$ in a liquid part of the Earth.

There is not magnetic field (or Lorentz torque) nor viscosity involved here. The boundary conditions cross various kinds of boundaries are the same as usual (Smith, 1974; or Huang, 2001; for more information) and not presented here.

In order to solve these sets of equations, one can use direct numerical integration approach or other approaches. We propose here another new stratified spectral method (SSM), as well as a linear operator method (LOM) instead of generalized spherical harmonics (GSH).

The main idea of this SSM is to divide the Earth model into several subdomains (for example, solid inner core, fluid outer core and mantle) and to apply spectral method (Galerkin method or collocation method) on each subdomain. We will discuss and one-dimension example to show this method.

Global spectral method uses a single representation of an unknown function $u(x)$ through out the whole domain via a truncated series expansion, for instance,

$$u(x) \approx u_N(x) = \sum_{n=0}^{N} c_n\phi_n(x), \quad (3)$$

where $\phi_n(x)$ are the basis functions and c_n are their coefficients. This series is then submitted into a differential (or integral) equation like Eq. (1) which is abbreviated as

$$L[\sum_{n=0}^{N} c_n\phi_n(x)] = d \quad (4)$$

L is a linear operator. This equation can be solved by Galerkin method, collocation method or other spectral method.

Using Galerkin method the above equation turns into a group equations

$$\int_V \psi_j(x)L[\sum_{n=0}^{N} c_n\phi_n(x)]dx = d \quad (5)$$

where $\psi_j(x)$ is the trial functions. For a complex Earth model, a global domain resolves into K subdomains. In the $k^{th.}$ subdomain, the unkown function $u^{(k)}(x)$ is

$$u^{(k)}(x) \approx \sum_{n=0}^{N} c_n^{(k)}\phi_n^{(k)}(x), \quad (6)$$

where $\phi_n^{(k)}(x)$ are the basis functions of the $k^{th.}$ subdomain and $c_n^{(k)}$ are their coefficients. We us polynomial functions of radii (r) as the basis functions, i.e. $r^1, r^2, r^3, ..., r^n$. So Eq. (5) turns into K groups of equations:

$$\int_{V^{(k)}} \psi_j^{(k)}(x) L^{(k)}[\sum_{n=0}^{N} c_n^{(k)} \phi_n^{(k)}(x)]dx = d^{(k)} \tag{7}$$

where $\psi_j^{(k)}(x)$ are the trial functions in the $k^{th.}$ subdomain, and $L^{(k)}$ is its corresponding linear operator. Eq. (7) will create a $K(N+1) \times K(N+1)$ matrix. Suppose that there are M boundary conditions:

$$B_i[\sum_{k=1}^{K} u^{(k)}(x)] = 0, \quad i = 1 \cdots M \tag{8}$$

We use Tau method to combine these boundary conditions with Eq. (7). The detail process of this SSM and Tau method will be presented in another paper.

3. EARTH MODEL AND ELLIPTICITY PROFILE

We adopt PREM as input Earth model. The Earth is divided by solid inner core (1 subdomain), fluid outer core (1 subdomain), and mantle (10 subdomains, including crust, but without ocean). The Earth is treated as hydro-static equilibrium. The ellipticity (ϵ) profile interior is derived from Clairaut equation with precision of $o(\epsilon^1)$ and plotted in Fig. 1, which is identical as what was used in Huang et al. (2001). The ellipticity of CMB $(r = 3480km)$ is $\epsilon_{cmb} = 0.00254656$.

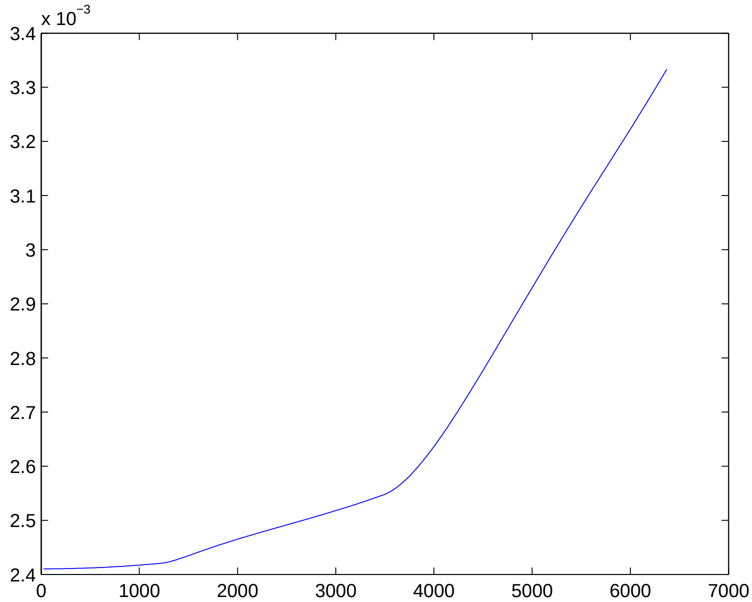

Figure 1: Ellipticity profile of hydro-static equilibrium Earth. x-axis is radii in km.

4. PRIMARY RESULT OF FCN AND DISCUSSION

The displacement field is truncated in following chain $\mathbf{s} \approx \mathbf{T}_1^1 + \mathbf{S}_2^1 + \mathbf{T}_3^1$. The polynomial (basis) functions of radii is truncated at $N = 4$ and the result is well converged.

Our preliminary result of the FCN period is 435 ± 3 sidereal days without any unproved assumptions (e.g., increasing core-mantle-boundary (CMB) flattening by about 5%, a strong coupling between nutation and geomagnetic field near CMB, viscous coupling, or topographical coupling etc.). It is very close to the observed one (about 430 days).

The following question is: How can this SSM get better result of FCN than other approaches? or, what is the essential difference among these approaches? or is it only the problem of these traditional theoretical approaches themselves? Earth models (e.g., PREM) provide accurate and robust profiles of physical parameters, like density and Lame parameters, but their radial derivatives, which are also used in all traditional methods to calculate normal modes (e.g., FCN), nutation and tides of non-rigid Earth theoretically, are not so trustable as the parameters themselves.

As in this SSM approach, physical parameters (density and Lame parameters) from a given Earth model are needed as input, however, their radial derivatives (like $\partial_r \rho$) which are also used in all other traditional approaches are not needed in this SSM approach. A numerical experiment to test the influence of the uncertainty of Earth model on nutation has been made (Huang & Zhang, 2014) and show that the uncertainty in the radial derivatives of the material density near CMB ($\partial_r \rho_{cmb}$) do have large influence on the calculated result of FCN period, although this experiment investigated only the factor of $\partial_r \rho_{cmb}$, and in this numerical experiment, the change of $\partial_r \rho_{cmb}$ is somewhat arbitrary or even not consistent with PREM model. This experiment, at least, provides us a hint that the uncertainty of $\partial_r \rho_{cmb}$ in an input Earth model may be a problem. Unfortunately, PREM model does not give the information of its precision nor accuracy.

Moreover, the truncation of the expansion series of displacement vector and stress tensor in traditional methods is also of question. All these possibilities need to be further studied.

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