REFINEMENTS ON PRECESSION, NUTATION, AND WOBBLE OF THE EARTH.

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ABSTRACT Most of the essential elements of the theory of nutation of the nonrigid Earth have been presented in the IAU adopted model MHB2000 (Mathews et al., 2002) considering an ellipsoidal rotating Earth, with a solid inner core, a liquid outer core, and an ellipsoidal inelastic mantle, and with a magnetic field. However in the meantime, the observed nutation amplitudes have been redetermined with a better precision. A number of relatively small significant effects have to be taken into account before one can expect to have a theoretical framework that can yield numerical results matching the precession and nutation observations. The adopted model already accounts for the existence of a geomagnetic field passing through the mantle and the fluid core regions and beyond. The model MHB2000 considers an electromagnetic torque generated by this field when the core and the mantle are moving relative to each other, which can in turn affect some nutation amplitudes (both in phase and out-of-phase) to the extent of a few hundreds of microarcsecond (μas), playing thus a significant role. The paper revisits the last adopted model in order to incorporate potential additional coupling effects at the core-mantle boundary, that can be at an observable level, such as the existence of a non-hydrostatic core-mantle boundary topography, the viscosity of the liquid core, the existence of stratification in the core, the existence of boundary layers at both sides of the core-mantle boundary.

1. STARTING FROM OBSERVATIONS AND THE IAU2000 ADOPTED NUTATION MODEL

Nutation observations are performed using Very Long Baseline Interferometry (VLBI). The performance of the VLBI antenna networks used for these observations has increased during the recent years, allowing a higher-precision determination of the nutation amplitudes. Figure 1 shows the residuals in milliarcsecond (mas) as a function of time between the nutation observations and the theoretical nutation amplitude as

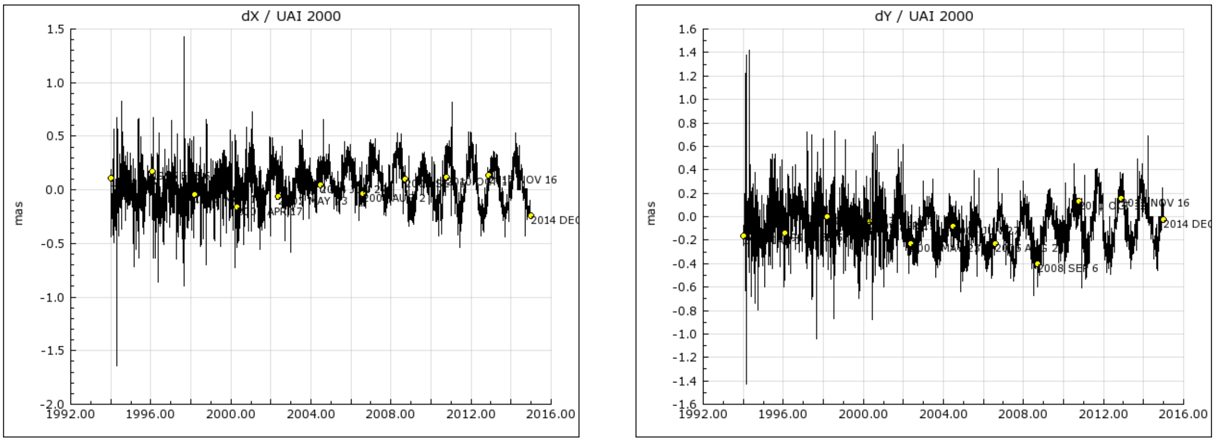

Figure 1: Residuals between nutation observations and the IAU2000 nutation model (dX,dY), as determined from the IERS EOP Product Center website http://hpiers.obspm.fr.

adopted by the IAU and IUGG in 2000 and 2003. These residuals are mainly due to the Free Core Nutation (FCN), a free mode excited by the atmosphere. The FCN amplitude cannot be precisely determined due to the poor knowledge of its excitation however an estimation can be obtained from the observations themselves. Even after subtraction of the effect of the FCN free mode contribution (as determined by the IERS) on the nutation in the time domain, there are still contributions at the level of a few tens of μas in the main nutation amplitudes. These residuals are presently not believed to be due to imperfection in the rigid Earth precession and nutation theory, accounting not only for the luni-solar direct effect on the Earth but as well for the direct and indirect of the planets. Starting from recent observed nutation series, Koot et al. (2010) have redetermined new estimations for the coupling constants at the CMB (core-mantle boundary) and at the ICB (inner core boundary) that are not explained by the presently adopted MHB2000 nutation model. This paper provides potential explanations considering additional coupling mechanisms at the CMB (core-mantle boundary).

2. IMPROVEMENT IN COUPLING MECHANISMS AT CORE-MANTLE BOUNDARY

There are several coupling mechanisms that have to be considered to explain the observed coupling constant at the CMB: (1) the classical ellipsoidal pressure-gravitational torque, already considered in MHB2000 nutation model, (2) the electromagnetic torque, also considered in MHB2000, (3) the viscous torque, and (4) the topographic torque. These coupling mechanisms are discussed below.

First we examine the constraints that we can use for further interpreting the coupling constants in terms of physics at the CMB. If we want to compute the electromagnetic torque acting at the CMB, we need to know the initial magnetic field and the outer core electrical conductivity as well as the lower mantle conductivity. The mantle is composed of silicates but a thin boundary layer at the bottom of the mantle (typically 200 m thickness) may still have a large conductivity possibly due to contamination with iron from the core. The electrical conductivity σ in that layer must of course be lower than the iron alloy conductivity (or equal to it in an extreme case), determined from laboratory experiments to be at the level of $\sigma = 5\ 10^{-5}\ Sm^{-1}$ (Stacey and Anderson, 2001). The electrical conductivity inside the mantle can be considered to be with typical values like $\sigma = 10\ Sm^{-1}$, $\sigma = 10^{-4}\ Sm^{-1}$, and $\sigma = 5\ 10^{-5}\ Sm^{-1}$. The poloidal magnetic field component at the CMB can be computed from downward continuation of the observed value at the Earth surface. This provides a typical value for the mean amplitude, the so-called RMS of the magnetic field, at the level of 0.3 mT, far below the amplitudes expected from the nutation data. Indeed, Koot et al. (2010) have deduced the coupling constants at the CMB from VLBI data as explained in the previous paragraph and have used these values in order to show that if one considers electromagnetic coupling only, the RMS of the radial magnetic field at the CMB must be 0.7 mT or larger, depending on the electrical conductivity considered for the bottom of the mantle.

In order to compute the viscous torque at the CMB we need to know the viscosity of the outer core fluid. Laboratory experiments and *ab initio* computations suggest that the molecular viscosity is at the level of $10^{-6}\ m^2s^{-1}$ and the eddy viscosity is at the level of $10^{-4}\ m^2s^{-1}$ (Buffett and Christensen, 2007). Koot et al. (2010) show that, in order to allow for lower values of the magnetic field at the CMB, in agreement with the value deduced from the surface magnetic field, we would need values for the viscosity of the core at the level of $10^{-2}\ m^2s^{-1}$, far too large with respect to the values that are admissible as mentioned above. The viscous coupling at the CMB is shown to be negligible for reasonable values of the core viscosity and other mechanisms must be considered to explain the observed coupling constant and to impose a decrease of the large magnetic field amplitude inferred when other coupling mechanisms are ignored.

The question is then how to explain this large electromagnetic coupling at the CMB if the viscous torque is disregarded? One explanation has been recently provided by Buffet (2012) considering the results of laboratory experiments published by Pozzo et al. (2012a, 2012b). These later authors have shown that the thermal conductivity of liquid iron under the conditions in the Earth's core is several times higher than previous estimates. This has the consequence that the heat carried by conduction in this layer is increased; less heat is thus available to drive convection in the core, which decreases the electrical resistance. In the induction equation for the induced field at the CMB, there is thus more generation than loss in the magnetic field balance equation (Buffett, 2012).

Another explanation can be found in considering that the only constraints on the core magnetic field that we have from the surface magnetic field observations are for the degrees lower than 13. But smaller scales contributions are unknown. In that consideration, nutation suggests that most of the energy of the magnetic field at the CMB comes from these.

Alternatively, the inclusion of topographic coupling may also reduce the need of a large electromagnetic field. We know from seismology that the core-mantle boundary topography is at the km level. The liquid pressure at the CMB on this topography induces a pressure torque able to transfer angular momentum from the core to the mantle. This phenomena is well known for explaining decadal variations of Earth rotation (Hide, 1977). At the nutation diurnal timescale, it is difficult and challenging to compute, but the topographic torque cannot be ruled out to explain the coupling constants determined from nutation observations. Wu and Wahr (1997) have used seismic value for the topography at the CMB and have computed the effect on nutations. They have shown that the effects on the retrograde annual nutation can be at the milliarcsecond level and that for some topography wavelength there are amplifications of the contributions. While Wu and Wahr (1997) used a numerical technique, Puica et al. (2014) examine the approach and equations and further study them using an analytical approach. Aiming at obtaining the torque and the associated effects on nutation, the following strategy must be used: (1) establish the motion equations and boundary conditions in the fluid; (2) compute analytically/numerically the solutions; (3) obtain the dynamic pressure as a function of the physical parameters; and (4) determine the topographic torque. With this strategy, Puica et al. (2014) show that the amplifications can exist due to resonances with inertial waves in the rotating fluid core and that some of the resonances as determined from their approach can be found near the main nutations. Though, these conclusions may change in the presence of an inner core.

Lastly, one can consider that there are chemical interactions between the core and the mantle (Buffett, 2010). In this approach, the core is considered to be stratified. The motions in the liquid core are then almost parallel to the constant density surfaces; there are only small changes in density; and the resulting buoyancy forces are weak. However, in the presence of a topography at the CMB, the vertical component of the motion in the fluid core can be important, the density field in a stratified fluid is disturbed and a buoyancy force arises, lowering the required strength of the radial magnetic field, as we wanted.

3. CONCLUSIONS

From our above discussion we can conclude that the existence of a topography of the CMB may provide a coupling mechanism between the core and the mantle for explaining nutation contributions and that contributions from some of the wavelenghts of the CMB topography may be larger than others due to resonance effects with inertial waves or due to large topography amplitudes. However other mechanisms can also be invoked such as the existence of a core stratification that enables buoyancy force to arise, lowering the required strength of the radial magnetic field in the electromagnetic coupling, or the existence of smaller scales in the magnetic field amplitude contributing largely to the electromagnetic torque, or even an increase of the electromagnetic torque arising from a decrease in electrical resistance consequently from the fact that the thermal conductivity of liquid iron under the conditions in Earth's core can be several times higher than previous estimates.

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