Study of the prograde and retrograde excitation at the Chandler frequency

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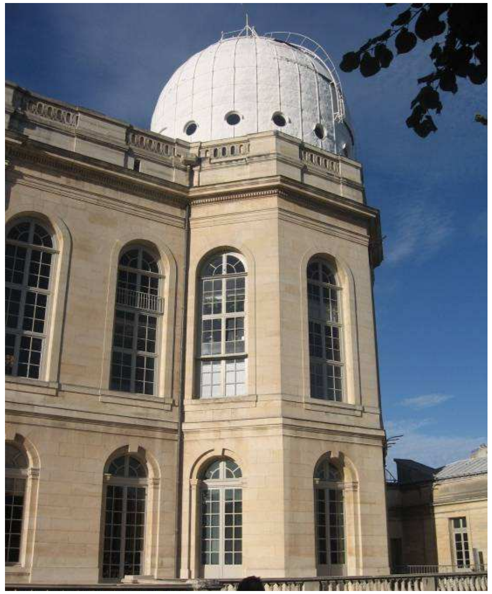


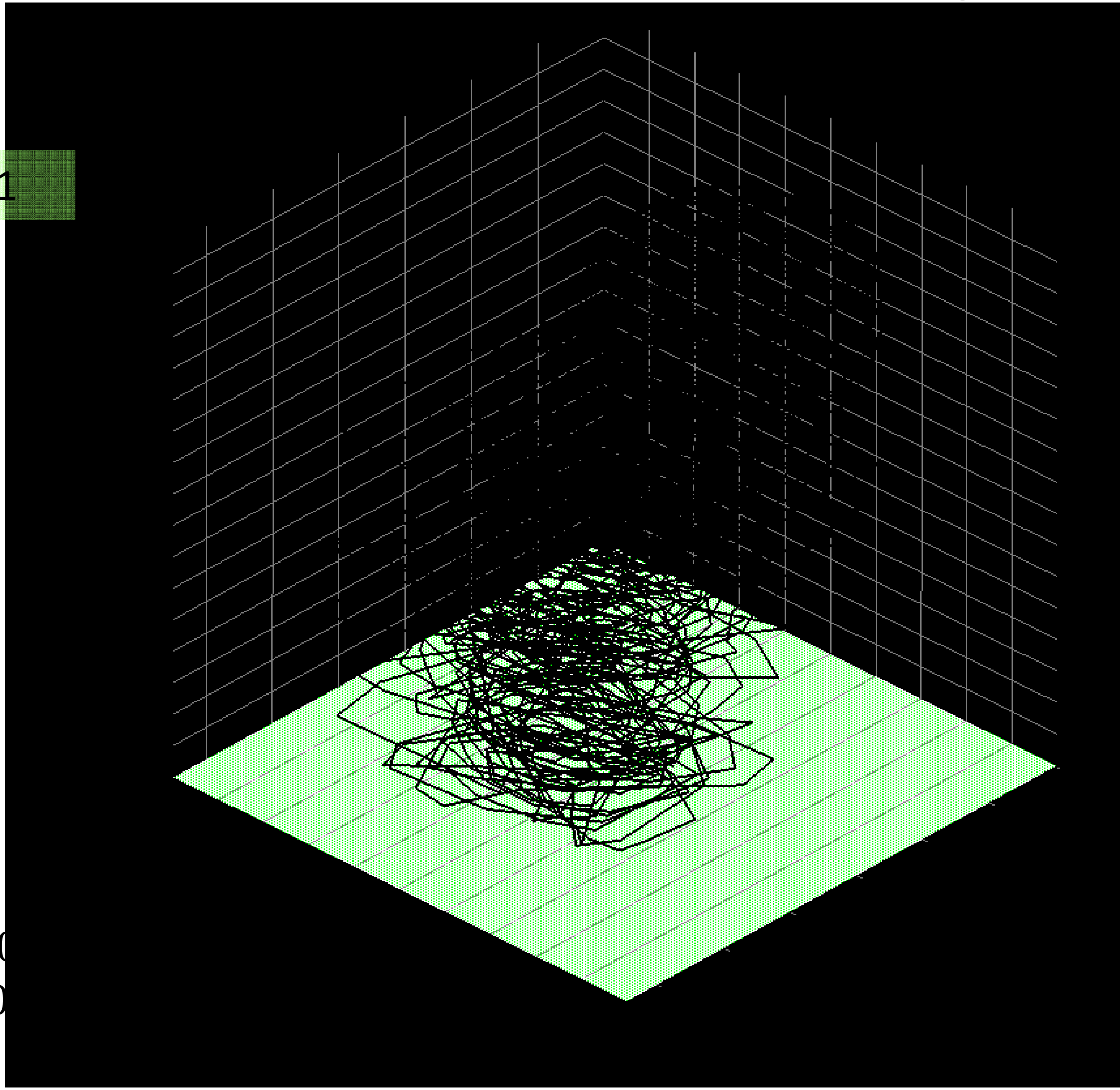
Journees 2013
17 Septembre
Paris

Plan of the talk

- Polar Motion components
- Panteleev filtering method
- Chandler wobble geodetic excitation
- AAM and OAM geophysical excitation
- Results for Generalized Euler-Liouville equation with asymmetric part
- AAM maps filtering example

Motion of the Earth's pole

EOP CO1



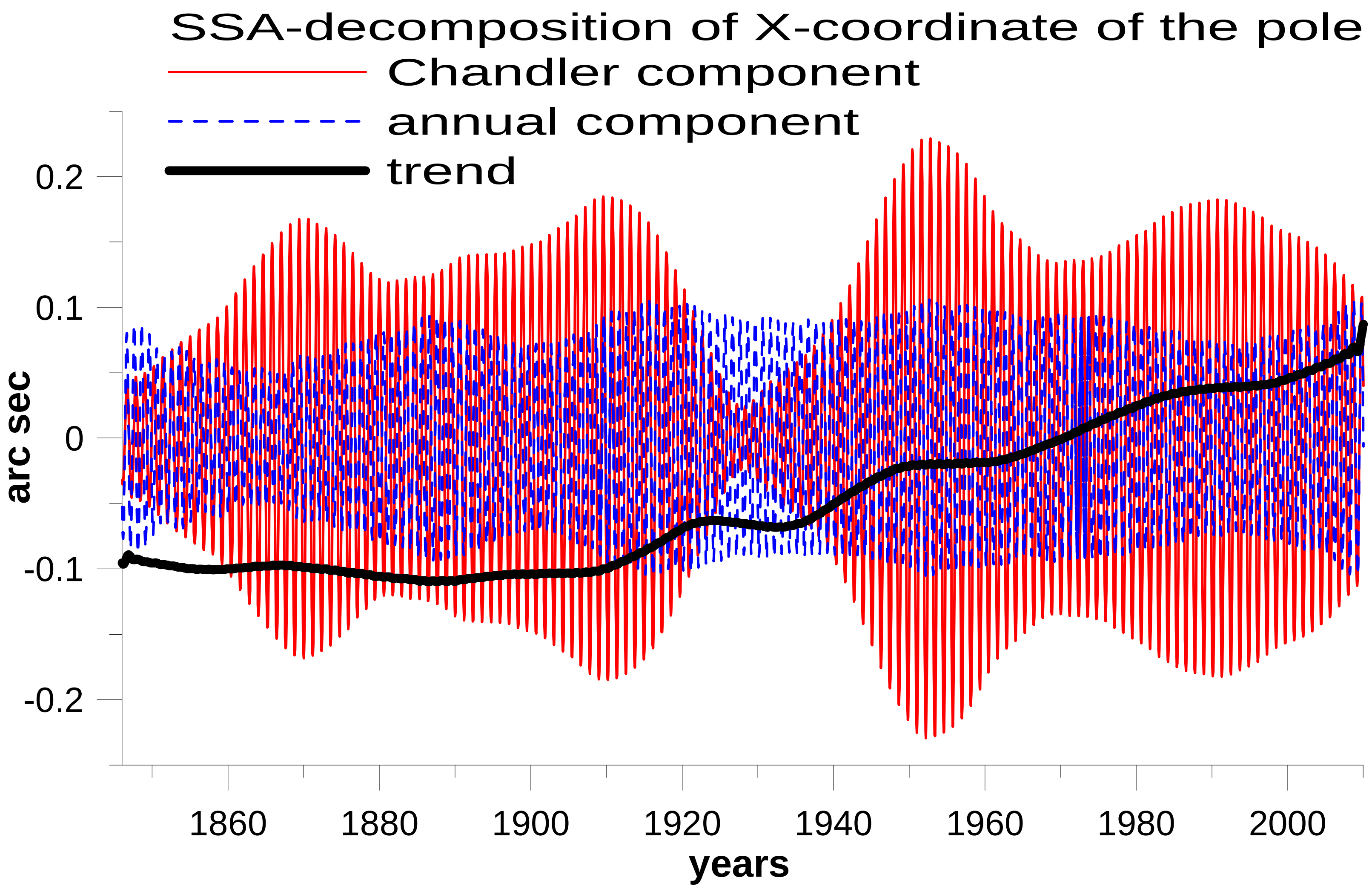
2D trajectory

$m(t) = x - iy$

1846-20

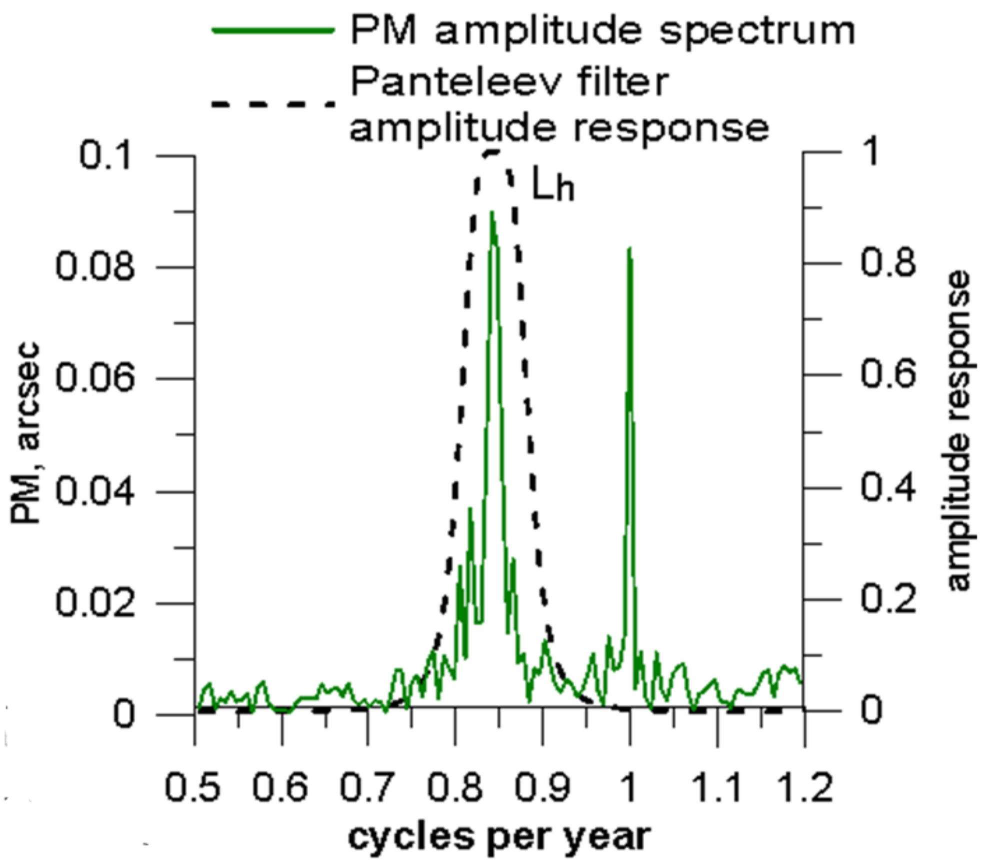
step 0.0

Singular Spectrum Analysis (SSA) - decomposition of the polar motion



PM spectrum and Panteleev's filtering

Filtering in time domain – by convolution



Filtering in frequency domain – by spectra multiplication

$$h(t) = \frac{\omega_0}{2\sqrt{2}} e^{-\left(\frac{\omega_0|t|}{\sqrt{2}} - i2\pi f_c t\right)} \left(\cos\frac{\omega_0 t}{\sqrt{2}} + \sin\frac{\omega_0 |t|}{\sqrt{2}}\right) \qquad \omega_0 = 2\pi f_0.$$

$$L_h(f) = \frac{f_0^4}{(f - f_c)^4 + f_0^4} \qquad f_0 = 0.04\ \mathrm{yr}^{-1}.$$

Dynamical model in time and frequency domain with resonance at the Chandler frequency

$$\frac{i}{\sigma_c}\frac{dm(t)}{dt} + m(t) = \chi(t)$$

$$\sigma_c = 2\pi f_c(1 + i/2Q)$$

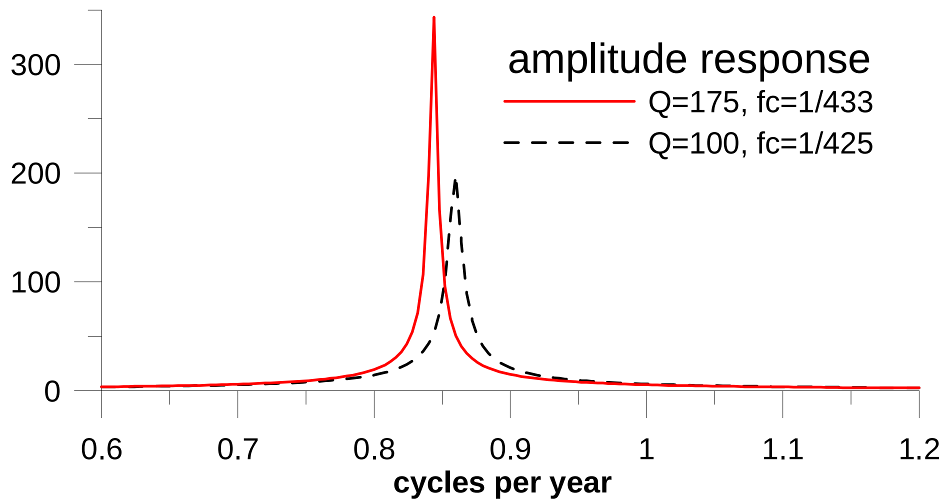
$$f_c = \frac{1}{433} \qquad Q = 175$$

$\hat{}$ - Fourier transform

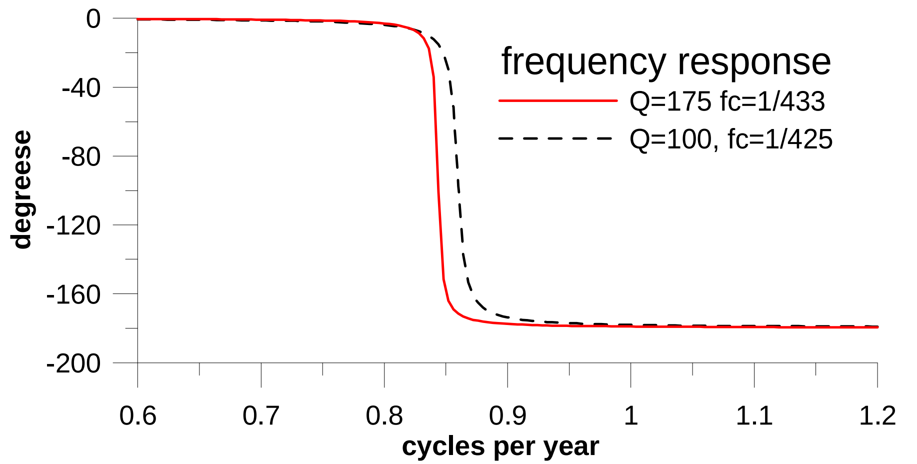
$$\hat{m}(\omega) = L(\omega)\cdot\hat{\chi}(\omega)$$

$$L(\omega) = \frac{\sigma_c}{\sigma_c - \omega}$$

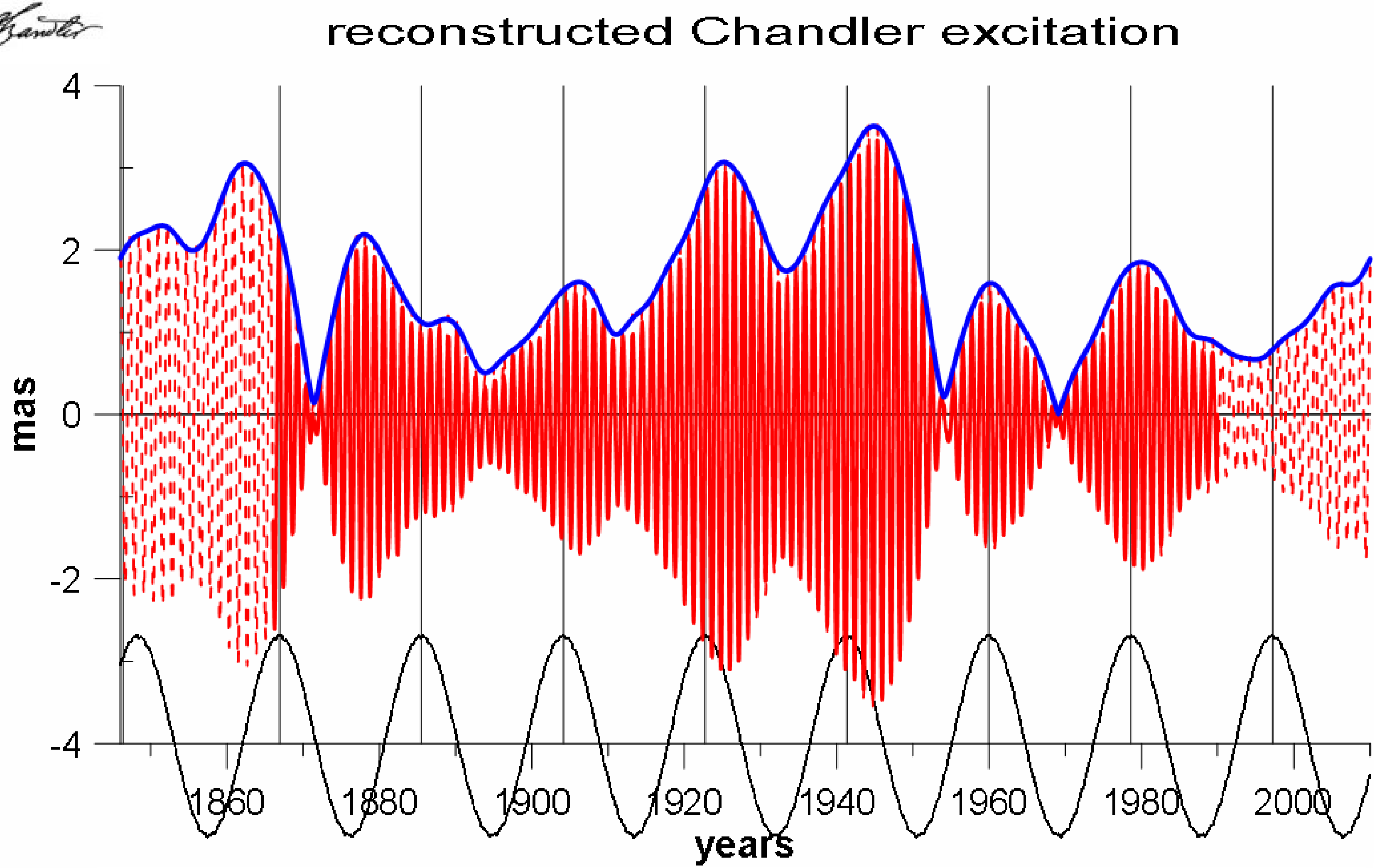
$|L(\omega)|$



$\arg(L(\omega))$

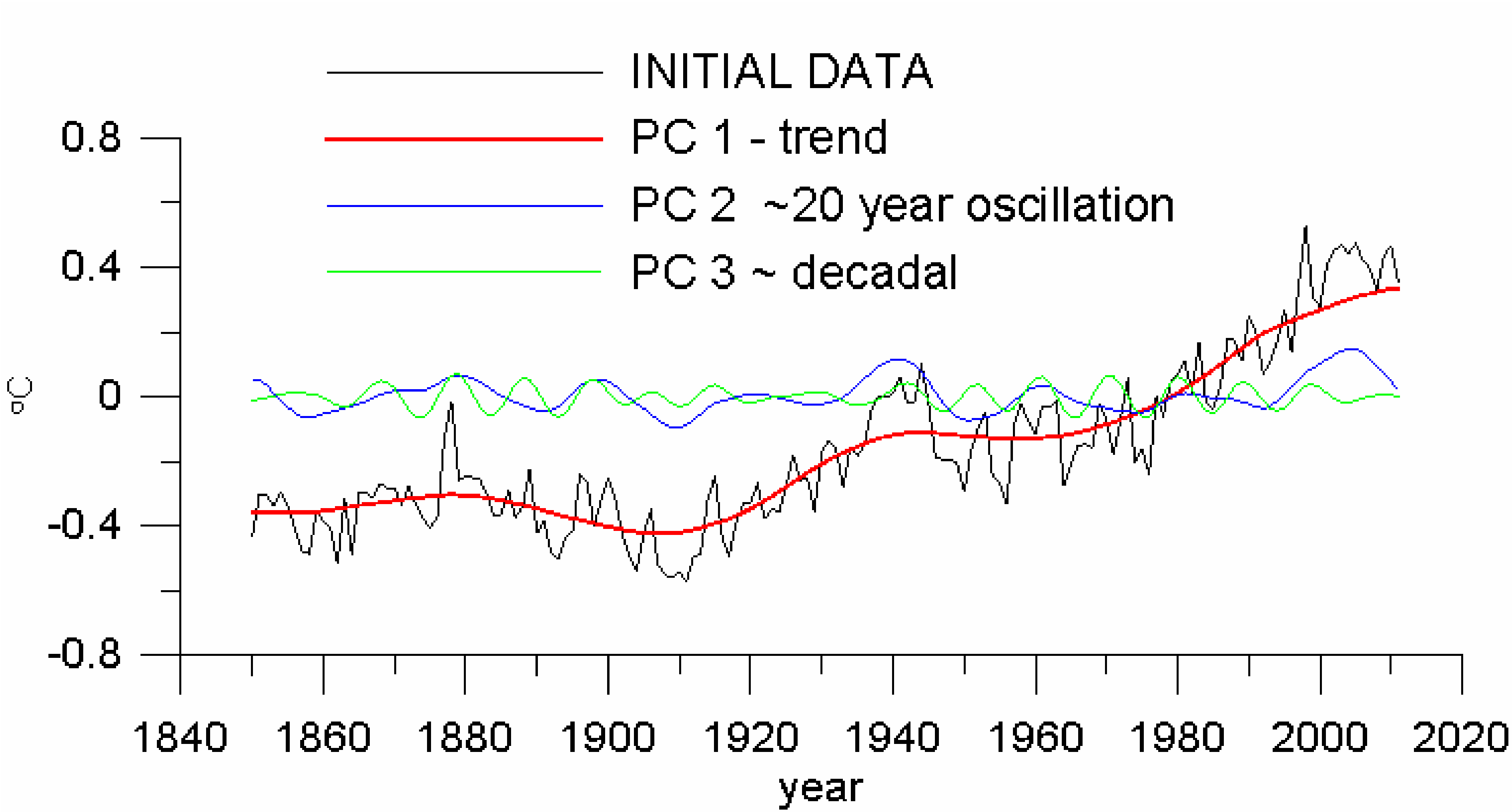


Chandler PM and its excitation



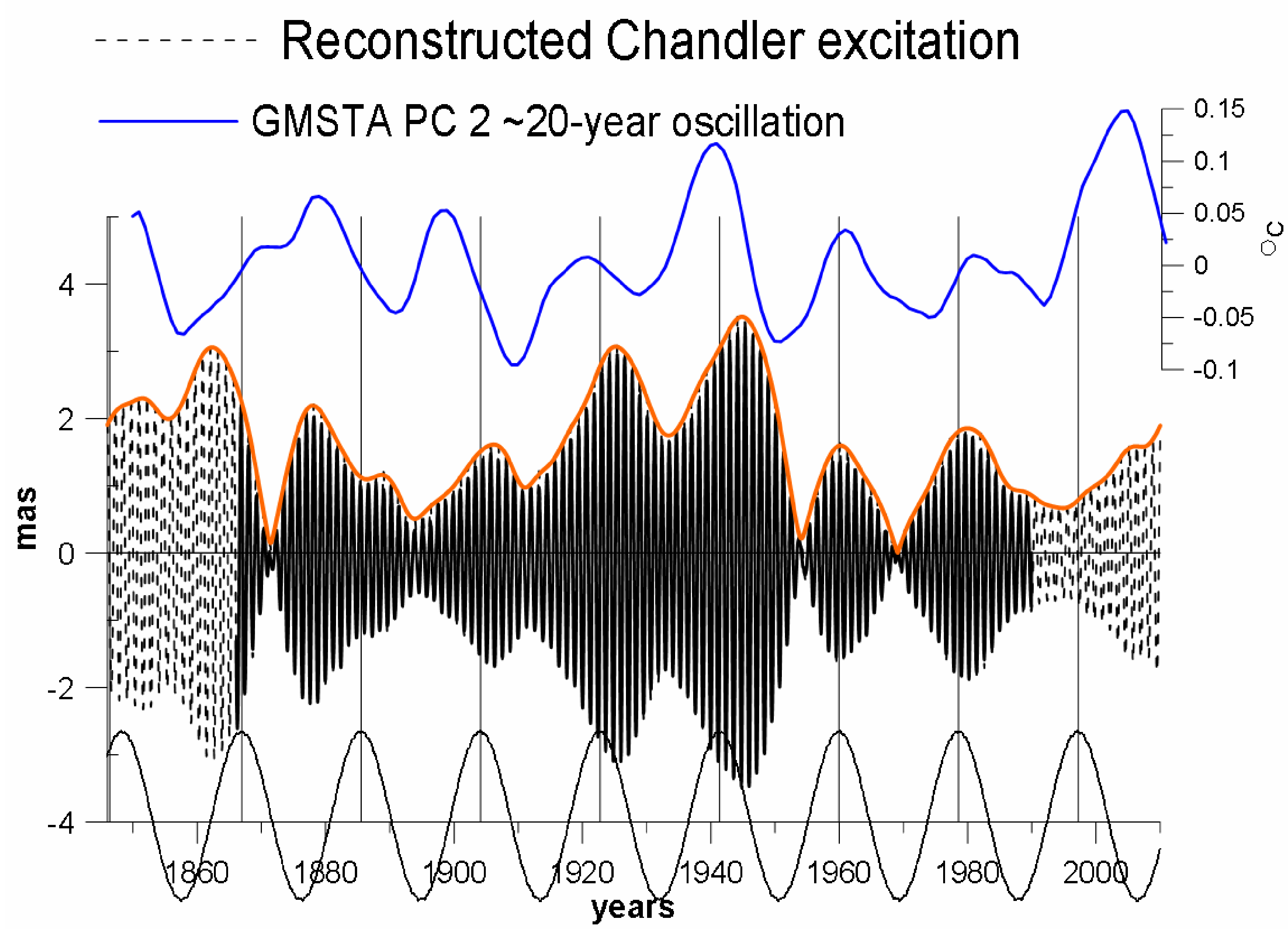
18.6-year harmonic taken from the tide model
Wilson filter used for reconstruction

Singular Spectrum Analysis of the global temperature data HadCRUT3



Lag=20 years

GMSTA oscillations and Chandler excitation



AAM and OAM data - equatorial components

Atmospheric Angular Momentum data with 6 hours resolution in time since 1948 yr for wind and pressure terms from NCEP/NCAR reanalysis

Pressure term

$$\chi^{\mathbf{P}} = \chi_1^P + i\chi_2^P = \frac{-1.098R^4}{(C-A)g} \int\int p_s \sin\phi \cos^2\phi e^{i\lambda} d\lambda d\phi$$

Motion term

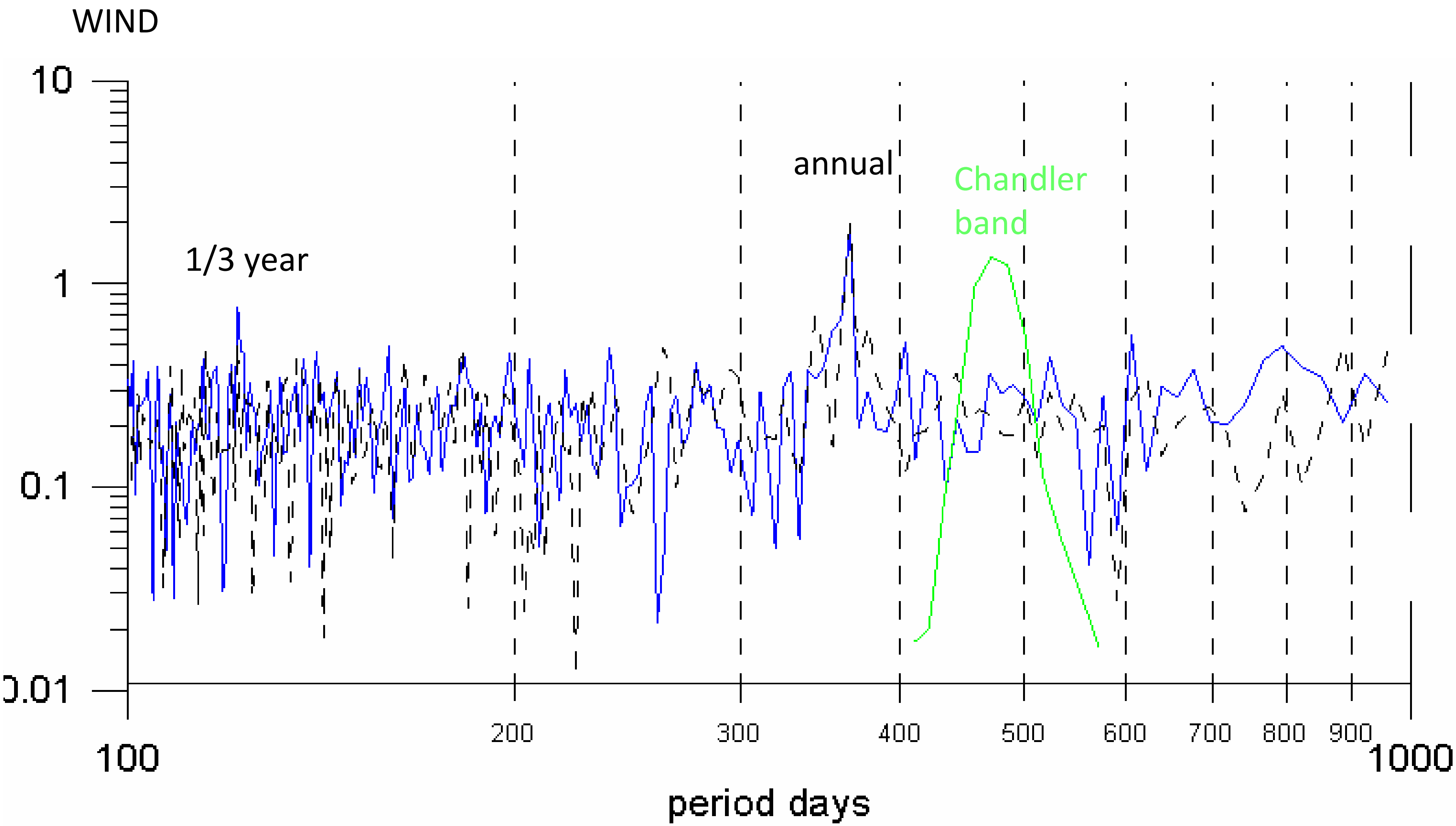
$$\chi^{\mathbf{W}} = \chi_1^W + i\chi_2^W = \frac{-1.5913R^3}{\Omega(C-A)g} \int\int\int (u \sin\phi + iv) \cos\phi e^{i\lambda} \mathrm{dpd}\lambda\mathrm{d}\phi$$

Inverted barometer hypothesis applied

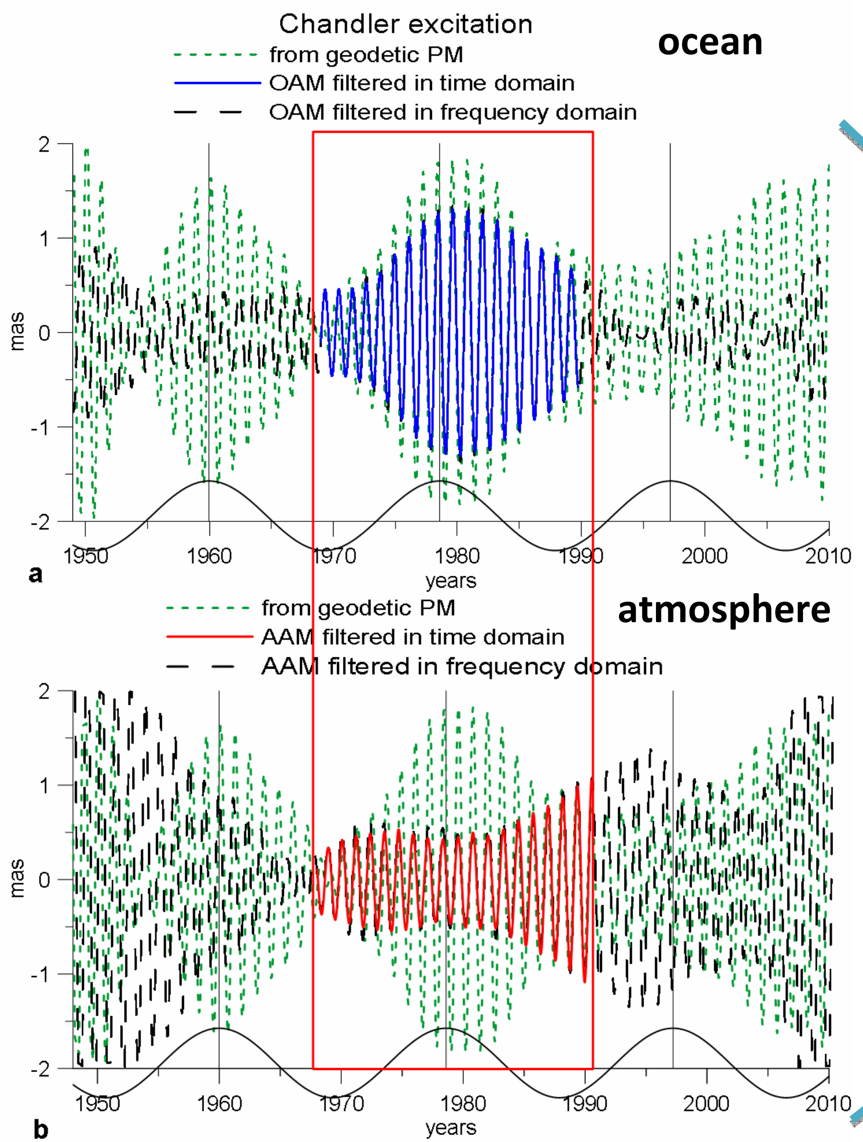
Y. H. Zhou, D. A. Salstein. and J. L. Chen, Revised atmospheric excitation function series related to Earth's variable rotation under consideration of surface topography, JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 111, D12108, doi:10.1029/2005JD006608, 2006

Oceanic Angular Momentum data with 1-day resolution in time since 1949 yr for currents and ocean bottom pressure terms from ECCO model and observations (since 1993)

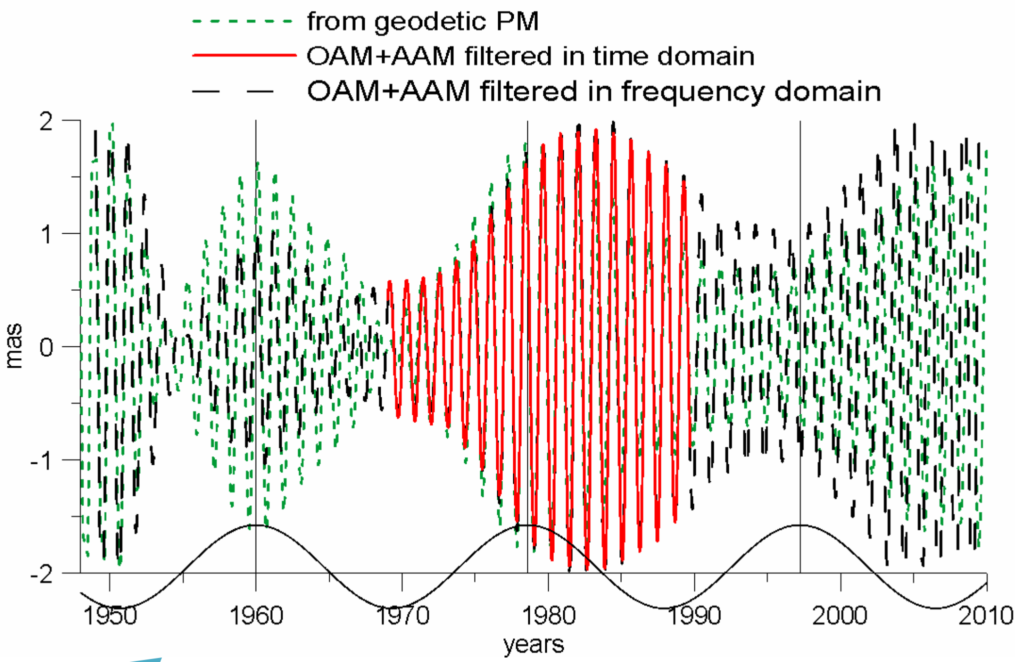
AAM spectrum, wind



Hydro-atmospheric excitation



Oceanic angular momentum from ECCO model + Atmospheric angular momentum from NCEP/NCAR



L. Zotov, C. Bizouard, On modulations of the Chandler wobble excitation, Journal of Geodynamics, special issue "Earth Rotation" 2012

Generalized Euler-Liouville equation

accounting for both triaxiality and anisotropic ocean pole tide

$$(1-U)\,m + \frac{i}{\sigma_e}\,(1+eU)\,\dot{m} - Vm^* + \frac{i}{\sigma_e}\,eV\,\dot{m}^* = \Psi^{(pure)}$$

$$\Psi_G^{sym}(t) = m + \frac{i}{\sigma_e(1-U)}\,(1+eU)\,\dot{m} \approx m + \frac{i}{\tilde{\sigma}_c}\,\dot{m}\,,$$

$$\Psi_G^{asym}(t) = \frac{-Vm^* + \frac{i}{\sigma_e}\,eV\dot{m}^*}{1-U}\,.$$

Transition to the frequency domain

Classical (symmetric) part

$$\widehat{m^*} = \hat{m}^*(-\omega)$$

Asymmetric part

$$\left(\frac{i}{\sigma_c}i\omega + 1\right)\hat{m} = \hat{\Psi}$$

$\hat{.}$ - Fourier transform

$.^*$ - conjugation

$$\frac{\frac{1}{\sigma_e}eV\omega - V}{U-1}\widehat{m^*} = \hat{\Psi}$$

$$\hat{m} = \mathrm{L}_{\mathrm{sym}}(\omega)\hat{\Psi}$$

$$\widehat{m^*} = \mathrm{L}_{\mathrm{asym}}(\omega)\hat{\Psi}$$

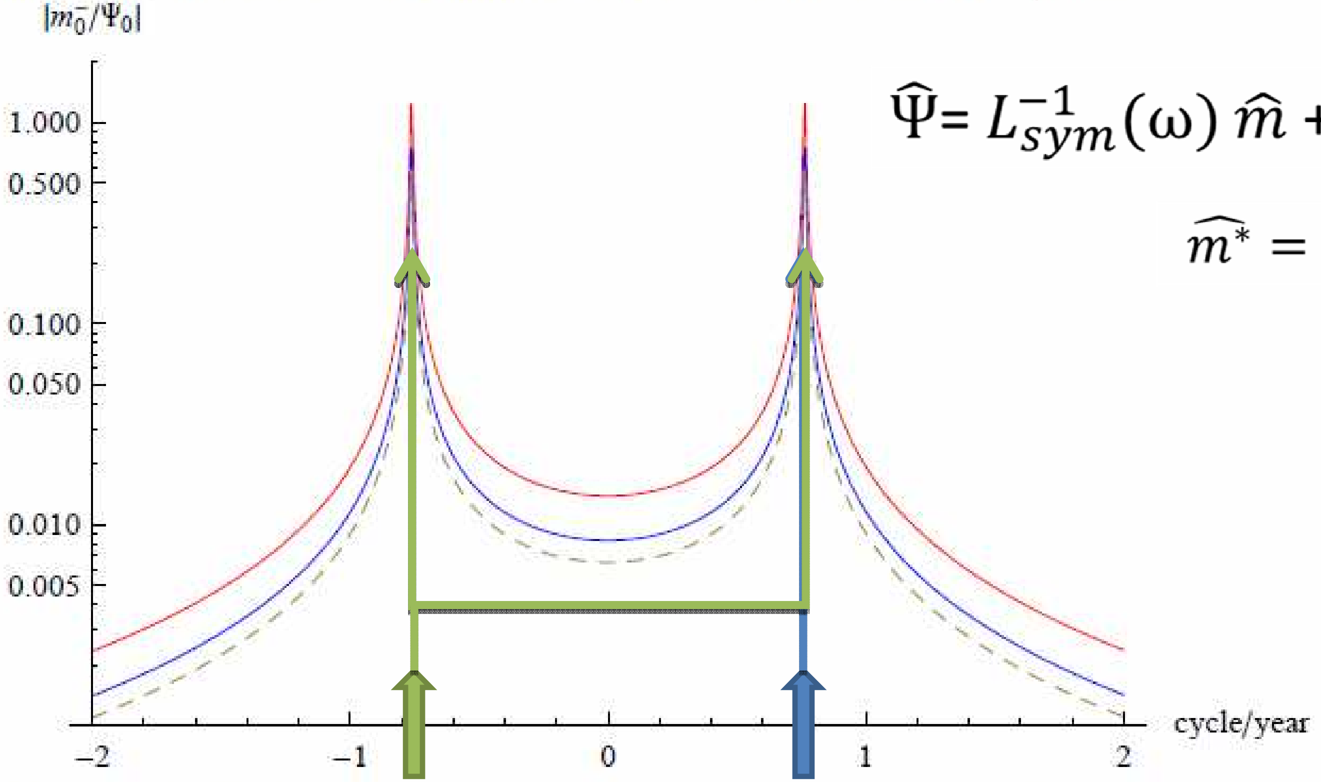
Bizouard C., Zotov L., Asymmetric effects on polar motion, Celestial Mechanics, May 2013

Intrinsic polarisation of the polar motion

Circular excitation $\Psi = \Psi_0 e^{i\sigma t}$ produces $m_\sigma(t) = m_0^+ e^{i\sigma t} + m_0^- e^{-i\sigma t}$
$\rightarrow$ common circular polar motion of same frequency (m^+)
$\rightarrow$ **circular polar motion of opposite frequency $-\sigma$ (m^-)**

$$\text{with } m_0^+ = -\Psi_0 \frac{\sigma_e}{\sigma - \tilde{\sigma}_c} \qquad m_0^- = \Psi_0^* \frac{\sigma_e V}{2\sigma_c} \left(-\frac{\sigma_e + e\sigma_c}{\sigma - \tilde{\sigma}_c} + \frac{\sigma_e - e\sigma_c}{\sigma - \tilde{\sigma}_c^-} \right)$$

Main asymmetric effect results from the rotationnal ocean response

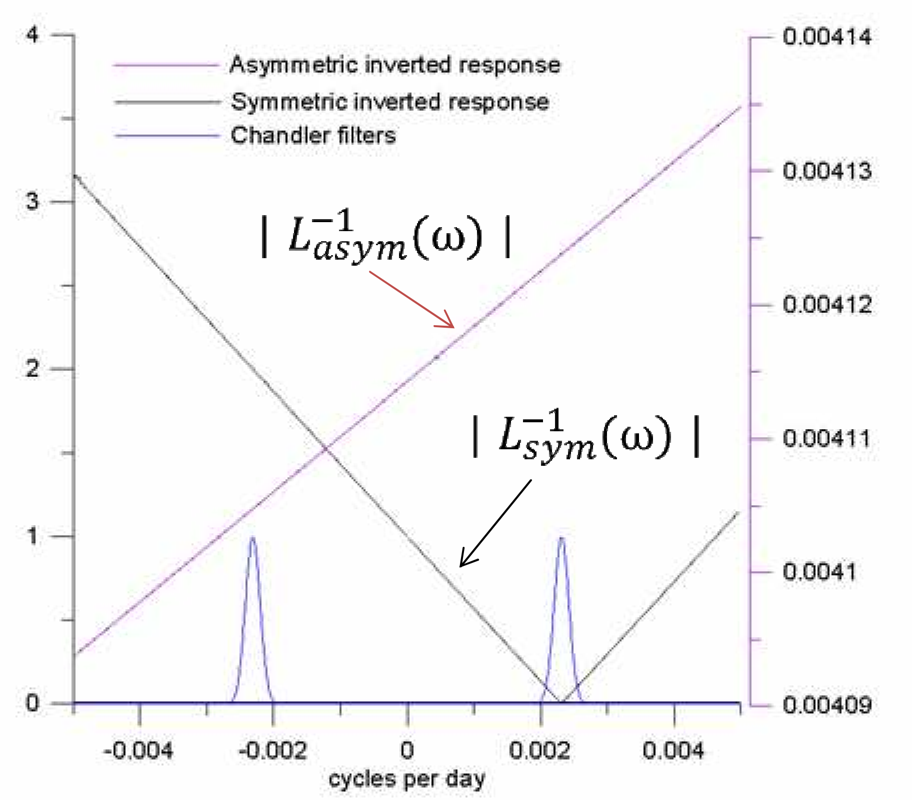


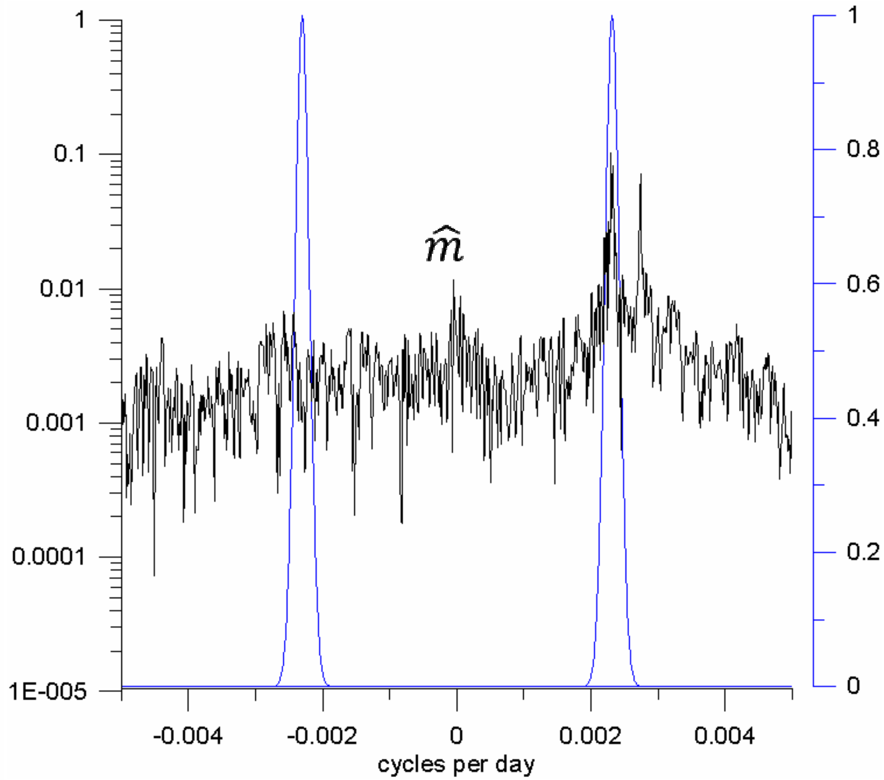
$$\hat{\Psi} = L_{sym}^{-1}(\omega)\, \hat{m} + L_{asym}^{-1}(\omega)\, \widehat{m^*}$$

$$\widehat{m^*} = \hat{m}^*(-\omega)$$

blue : triaxiality alone red : asymmetric pole tide alone : dashed : combined effect

Prograde and retrograde filtering with operator inversion





$$L_{sym}^{-1}(\omega) = \left(\frac{i}{\sigma_c} i\omega + 1\right)$$

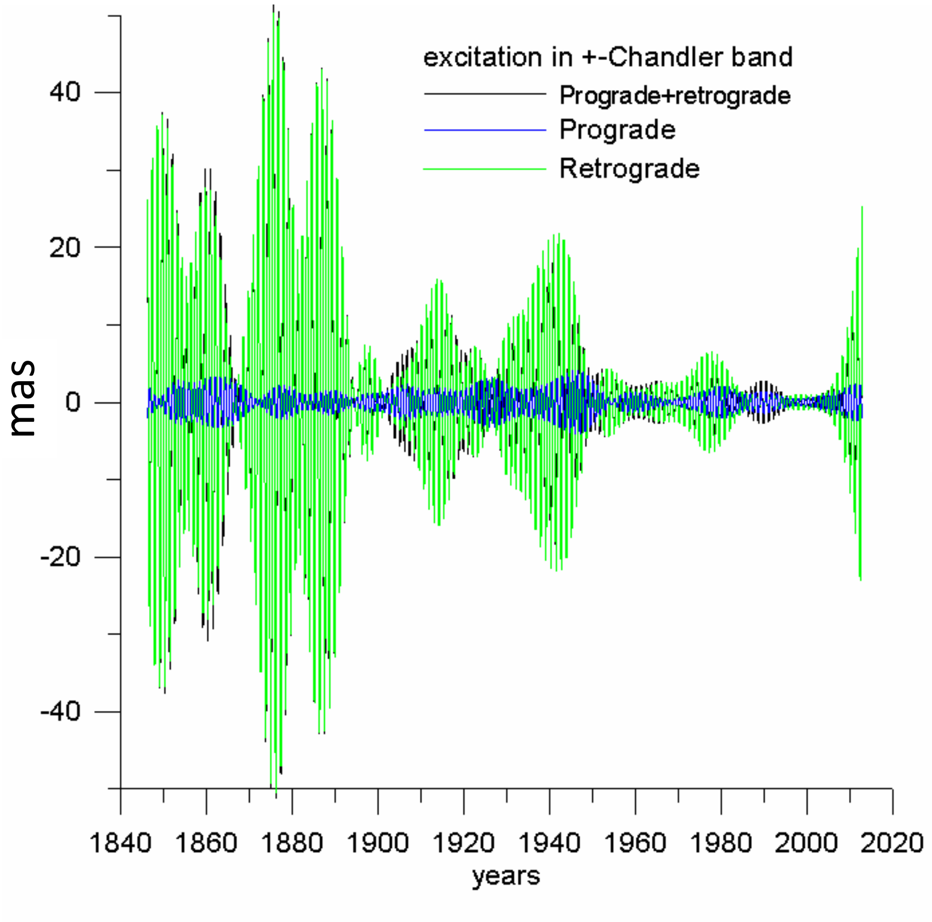
$$\widehat{m^*} = \hat{m}^*(-\omega)$$

$$L_{asym}^{-1}(\omega) = \frac{\frac{1}{\sigma_e} eV\omega - V}{U-1}$$

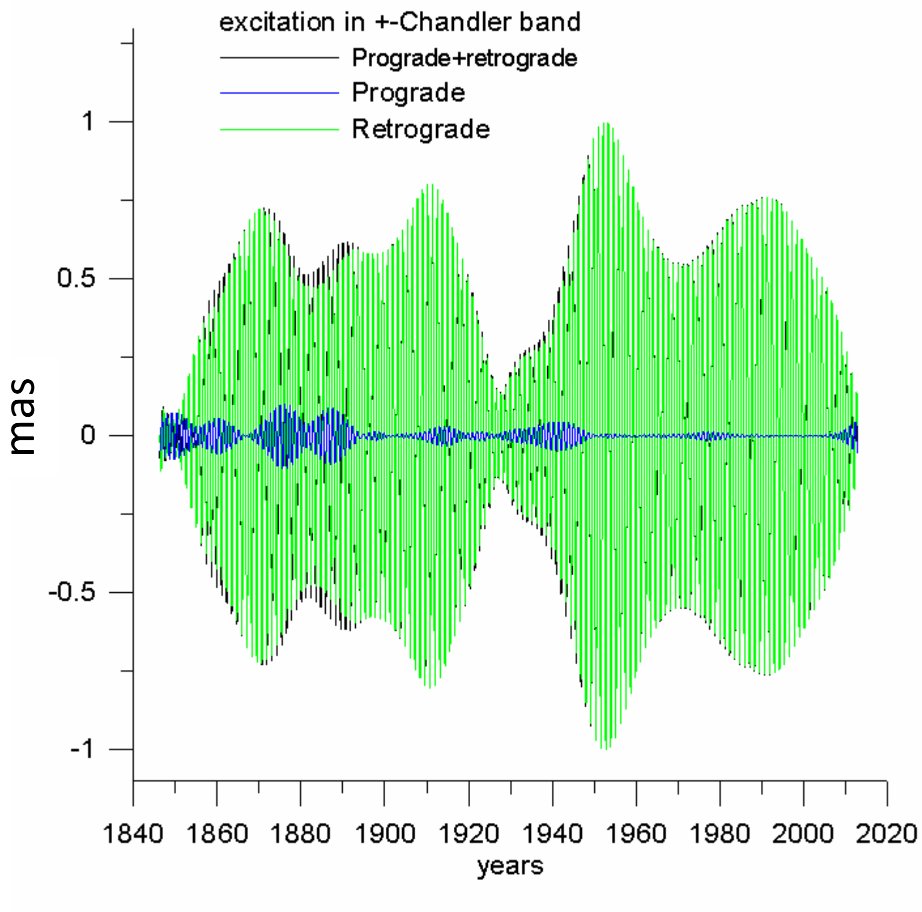
$$\hat{\Psi} = L_{sym}^{-1}(\omega)\, \hat{m} + L_{asym}^{-1}(\omega)\, \widehat{m^*}$$

Classical and asymmetric parts of Chandler excitation m_{chand}, X-coordinate

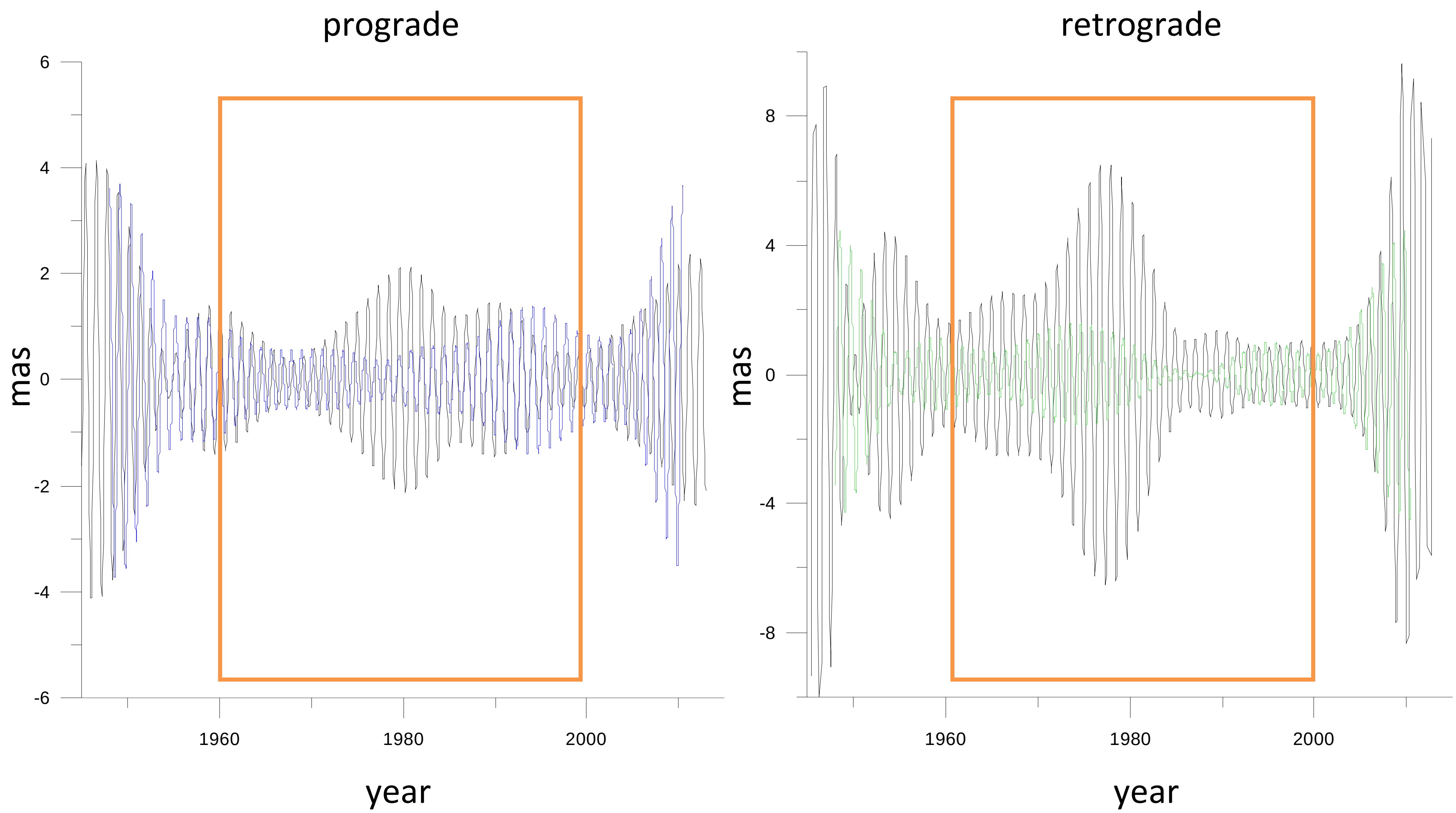
Classical (symmetric) part



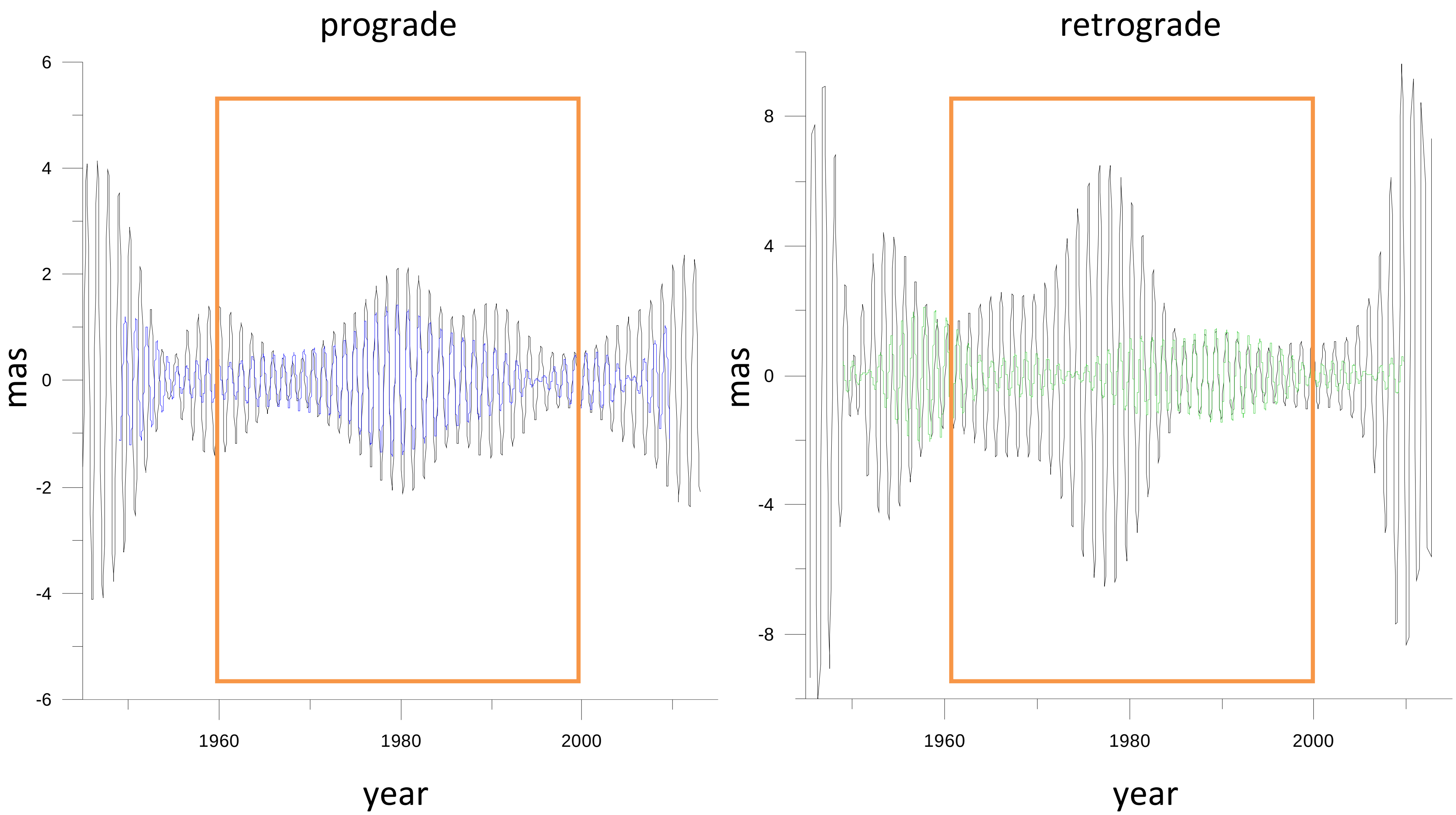
Asymmetric part



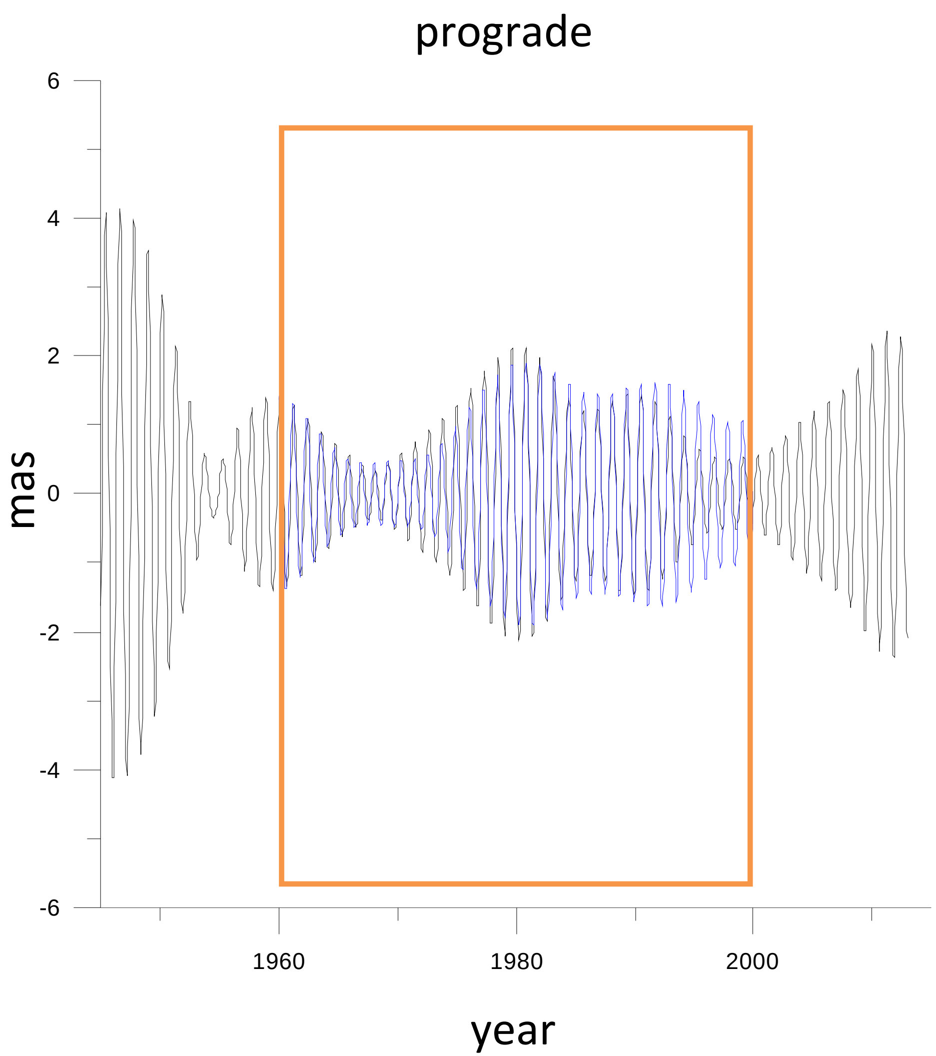
AAM



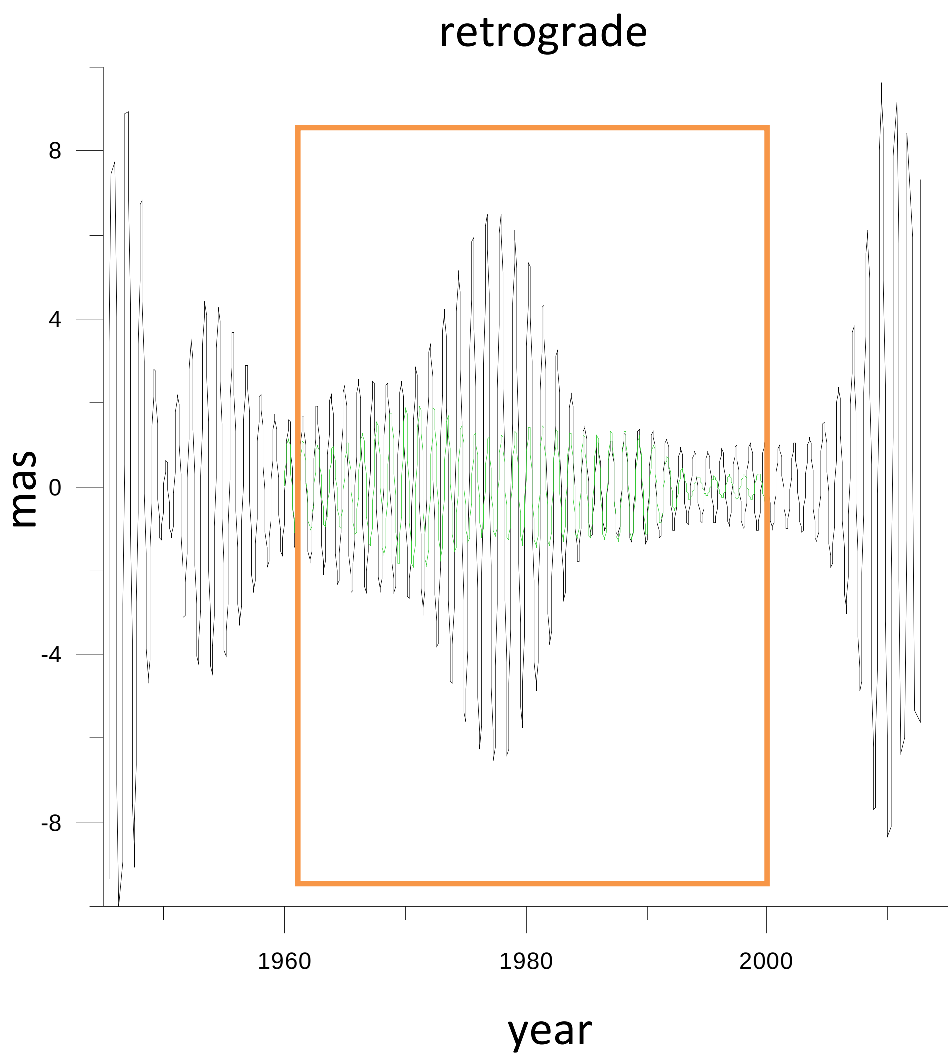
OAM



AAM+OAM



prograde
mas
6
4
2
0
-2
-4
-6
1960
1980
2000
year



retrograde
mas
8
4
0
-4
-8
1960
1980
2000
year

Correlations table

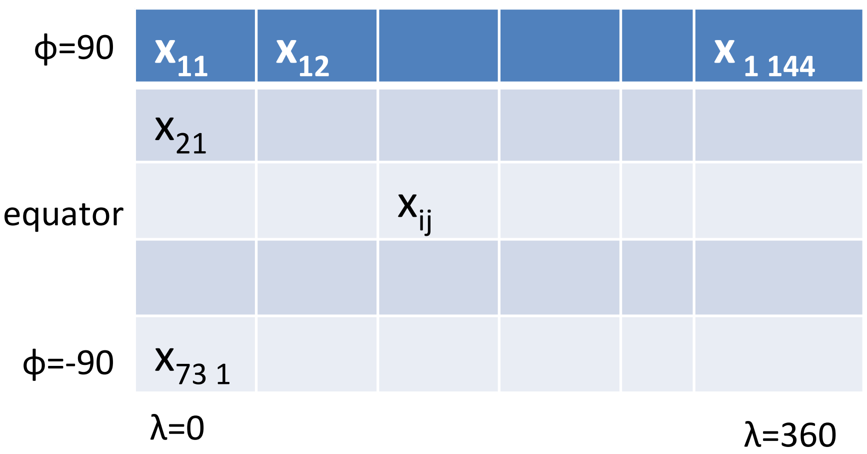
	AAM X	AAM Y	OAM X	OAM Y	AAM+OAM X	AAM+OAM Y
Prograde Chandler excitation	0.598	0.596	0.896	0.897	0.92	0.92
Retrograde Chandler excitation	0.428	0.430	0.123	0.126	0.438	0.439

Gridded AAM processing

Where on the map the sources of the Chandler excitation are located ?

EAAM grids with 2,5° resolution in space produced by ShAO

$$x + iy = |m|e^{i\varphi}$$



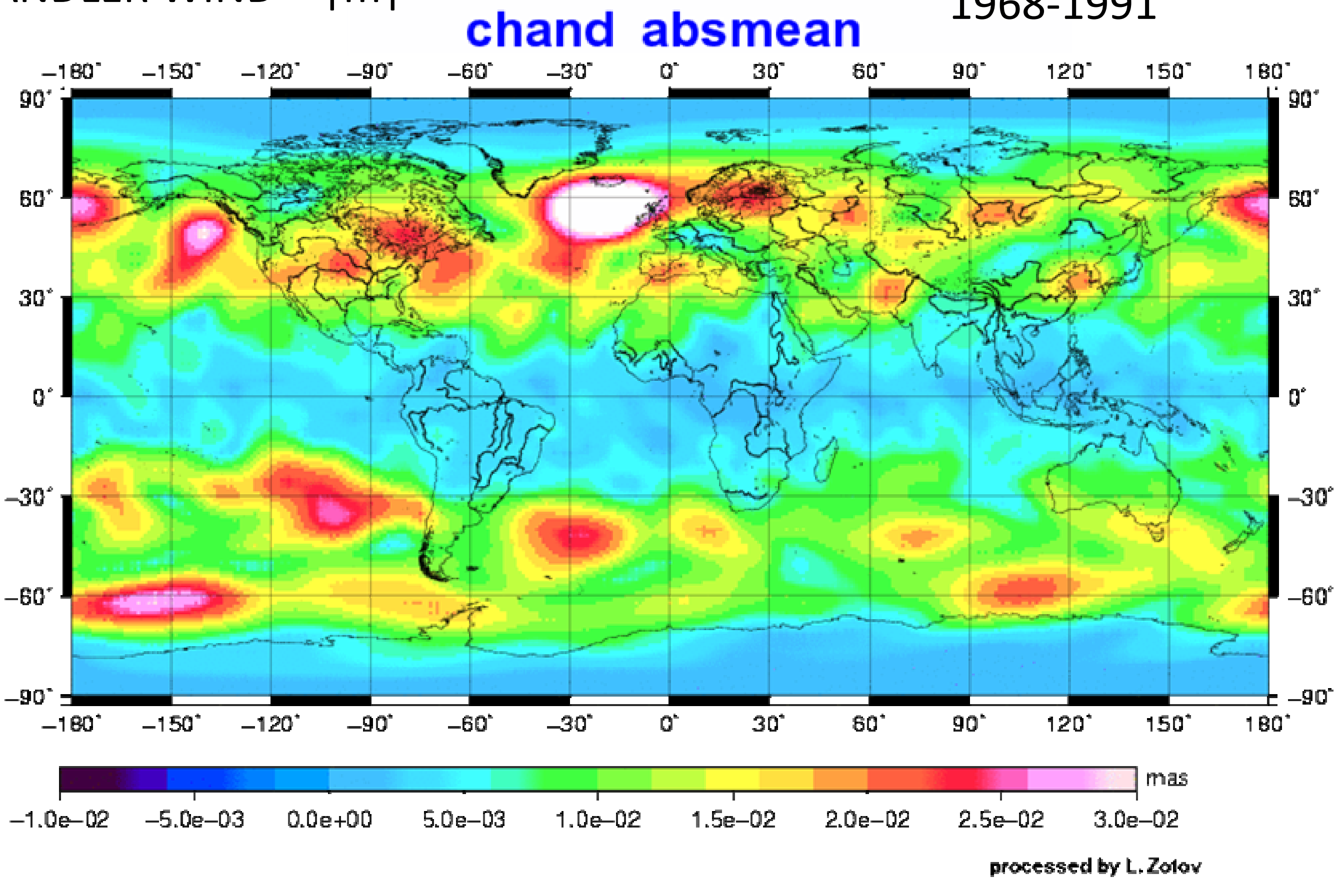
Filtering of the time series of every pixel and excitation reconstruction in it

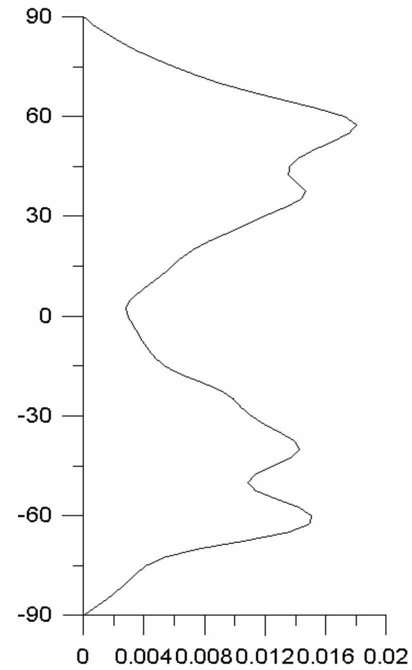
Chandler wind excitation mean

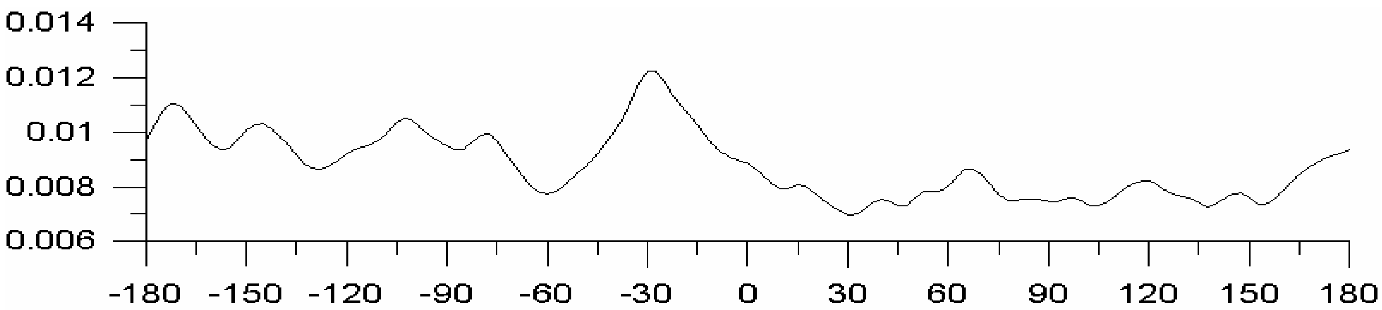
CHANDLER WIND <|m|>

1968-1991

$$|m| = \langle |m| \rangle + \Delta |m|$$

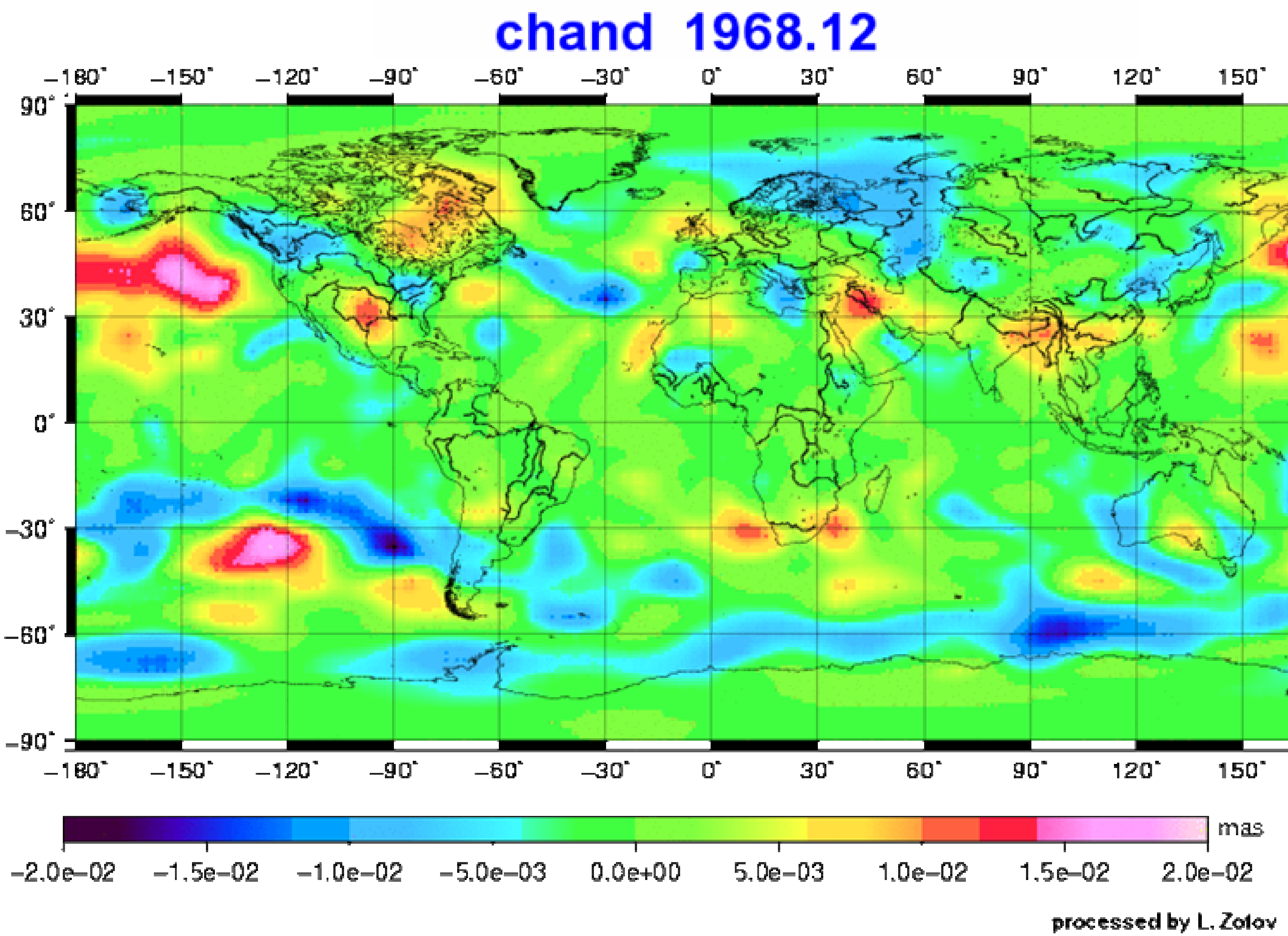






Chandler wind excitation changes

CHANDLER WIND $\Delta|m|$ $|m| = \langle |m| \rangle + \Delta|m|$



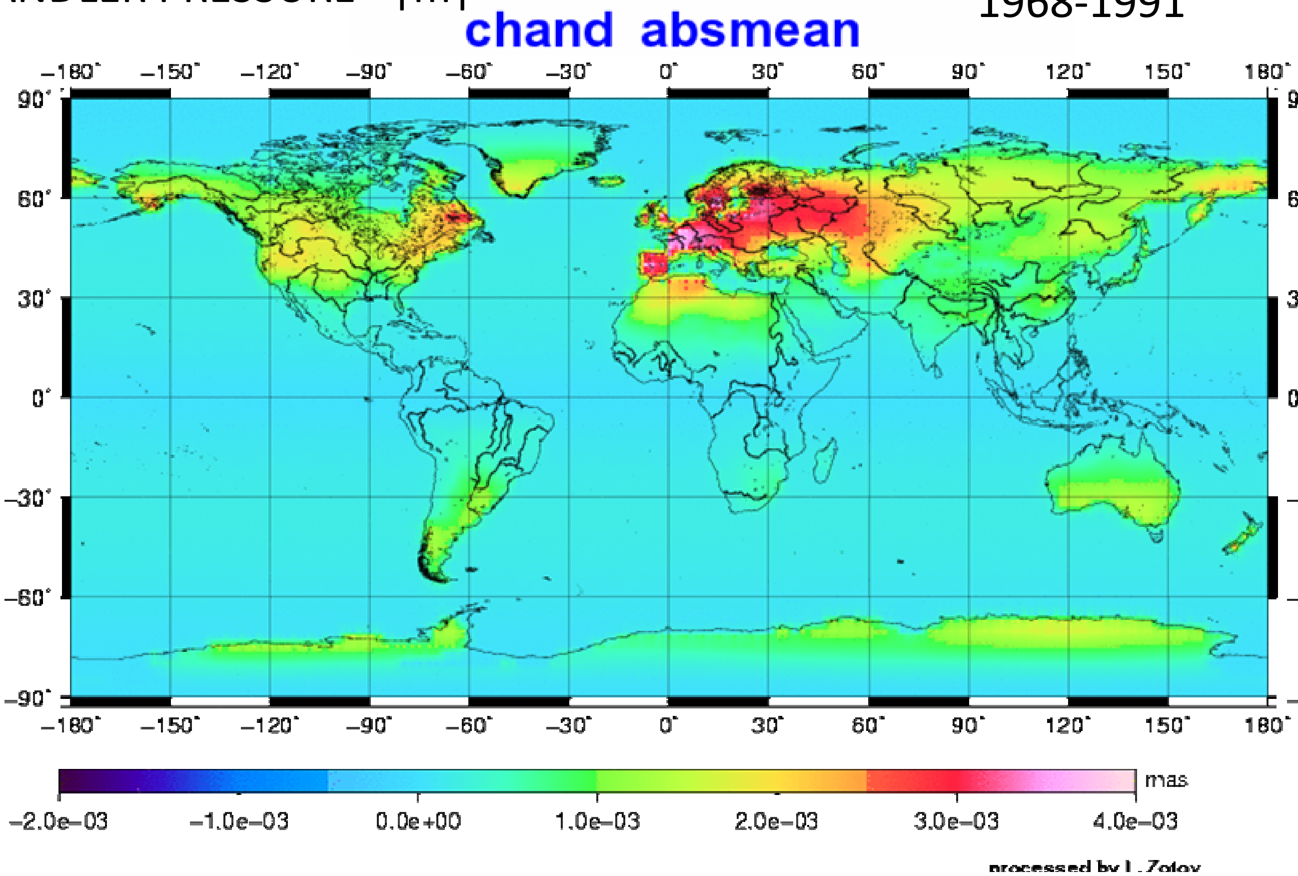
1968-1991

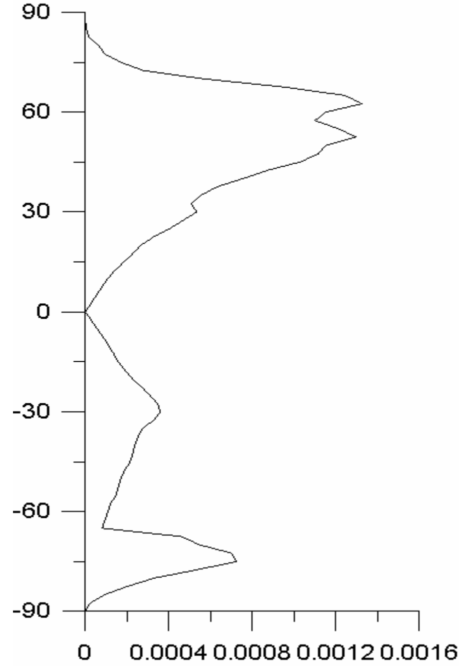
Chandler pressure excitation mean

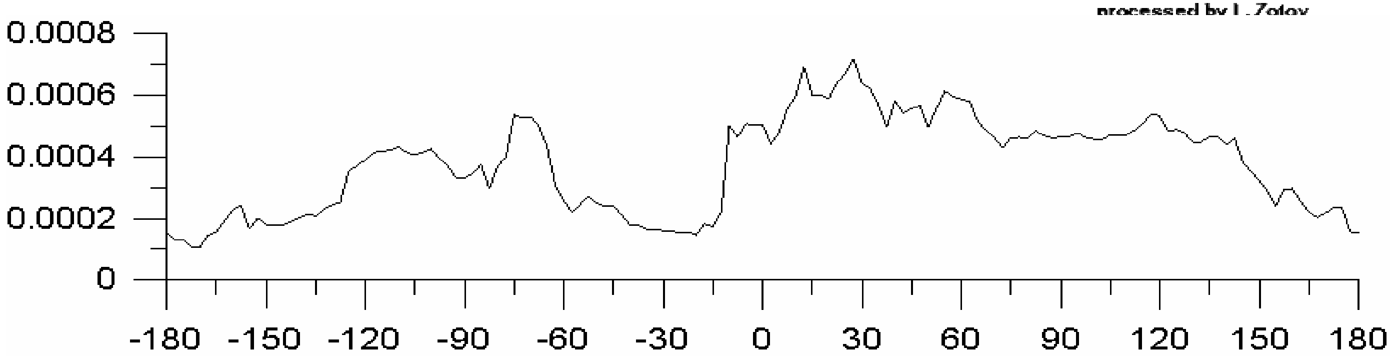
CHANDLER PRESSURE <|m|>

1968-1991

$|m| = \langle |m| \rangle + \Delta|m|$

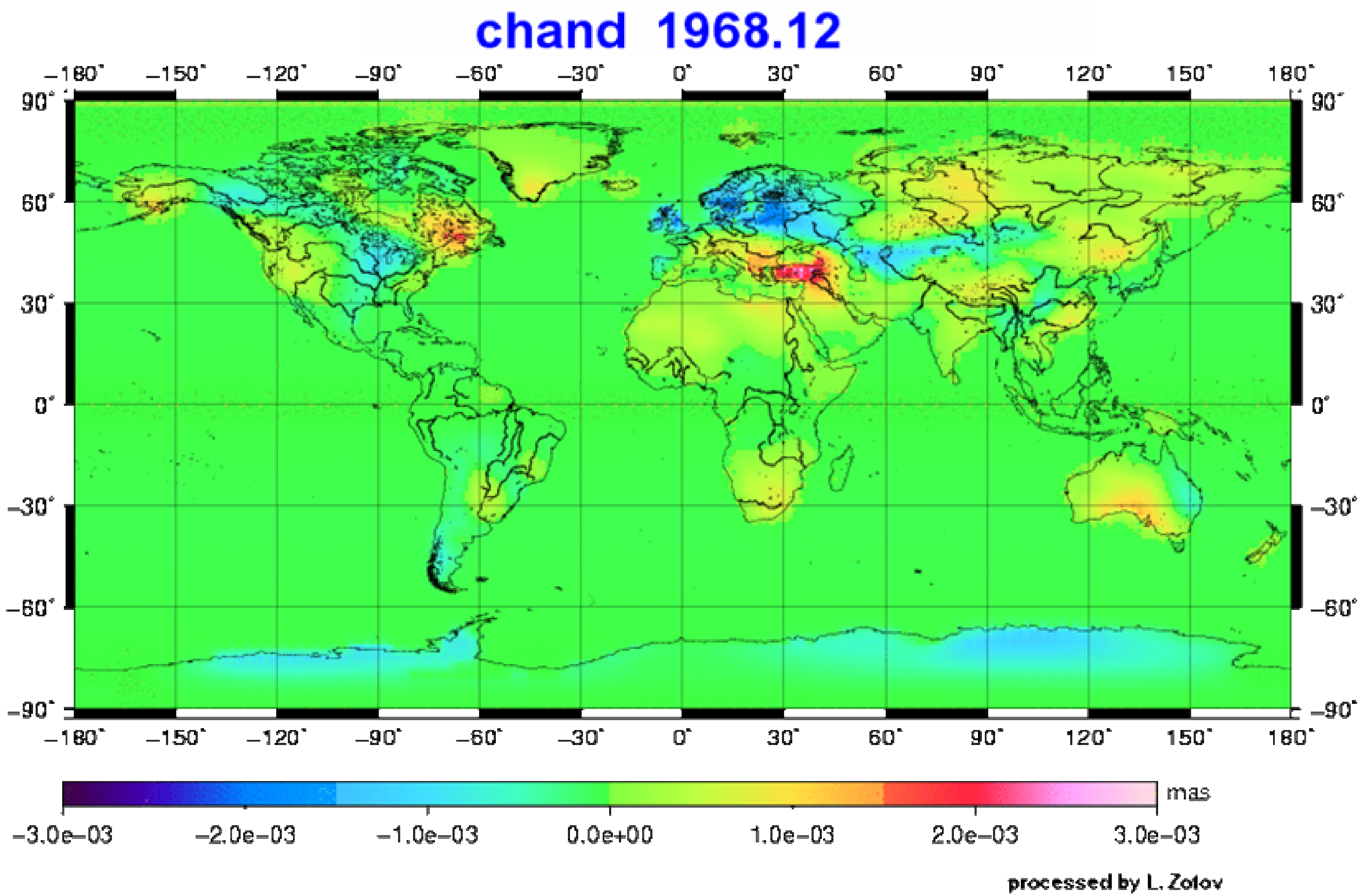






Chandler pressure excitation changes

CHANDLER PRESSURE Δ|m| $|m| = <|m|> + \Delta|m|$



1968-1991

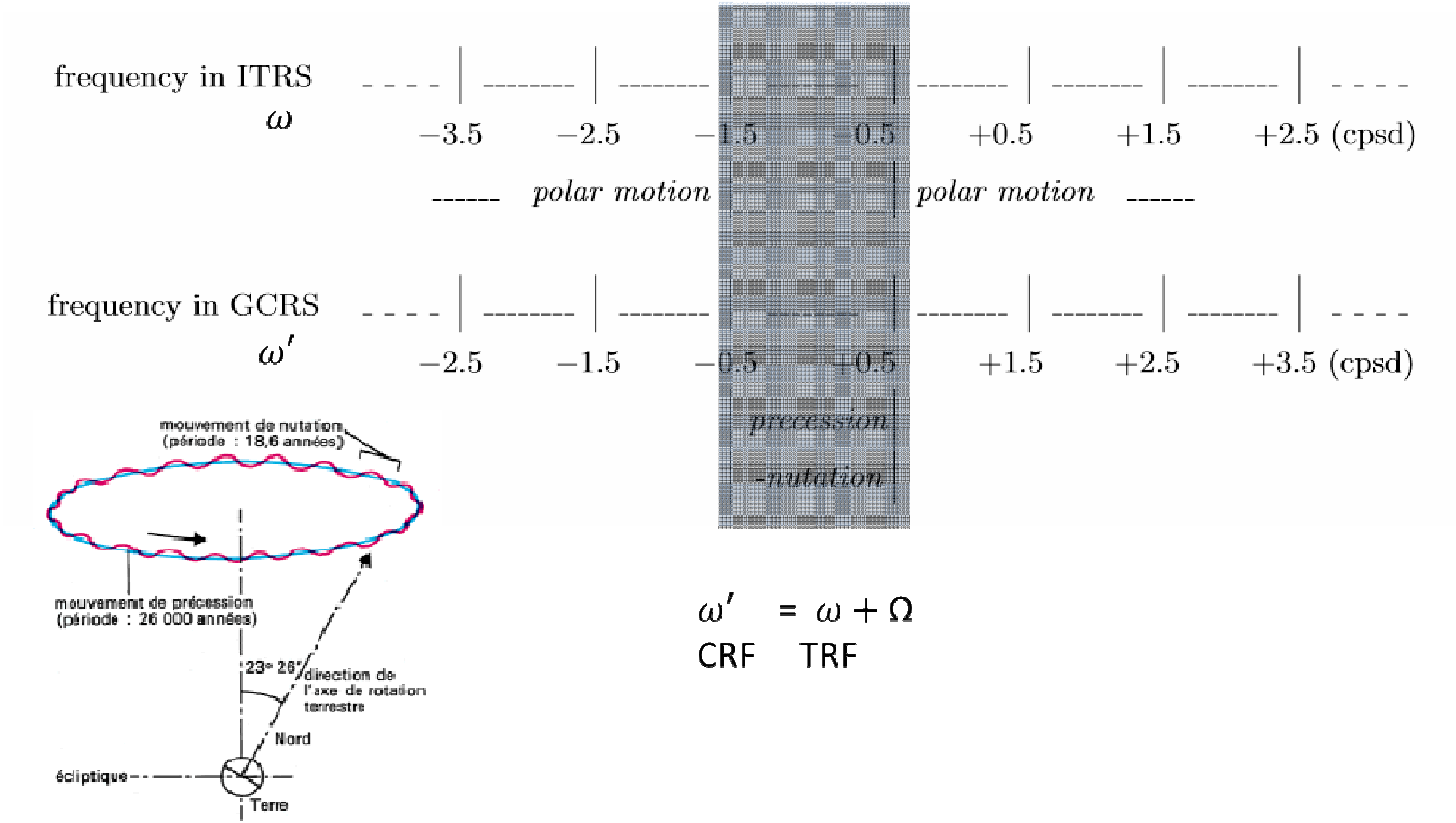
Conclusions

- Methods of excitation study around the resonant frequencies through Panteleev corrective filtering are developed
- 18.6-yr modulation, found in the reconstructed Chandler excitation, is synchronous with the Moon orbital nodes precession cycle and temperature variations on Earth
- Oceanic and atmospheric excitation together coincide well with reconstructed excitation on 1970-1990 interval
- New Generalized Euler-Liouville equation with asymmetric effects was used to study prograde and retrograde excitation of the Chandler wobble

Acknowledgements: We kindly thank Dr. Yonghong Zhou for providing AAM and OAM data grids. This work is supported by the RFBI grant N 12-02-31184


Merci
beaucoup
pour
votre
attention

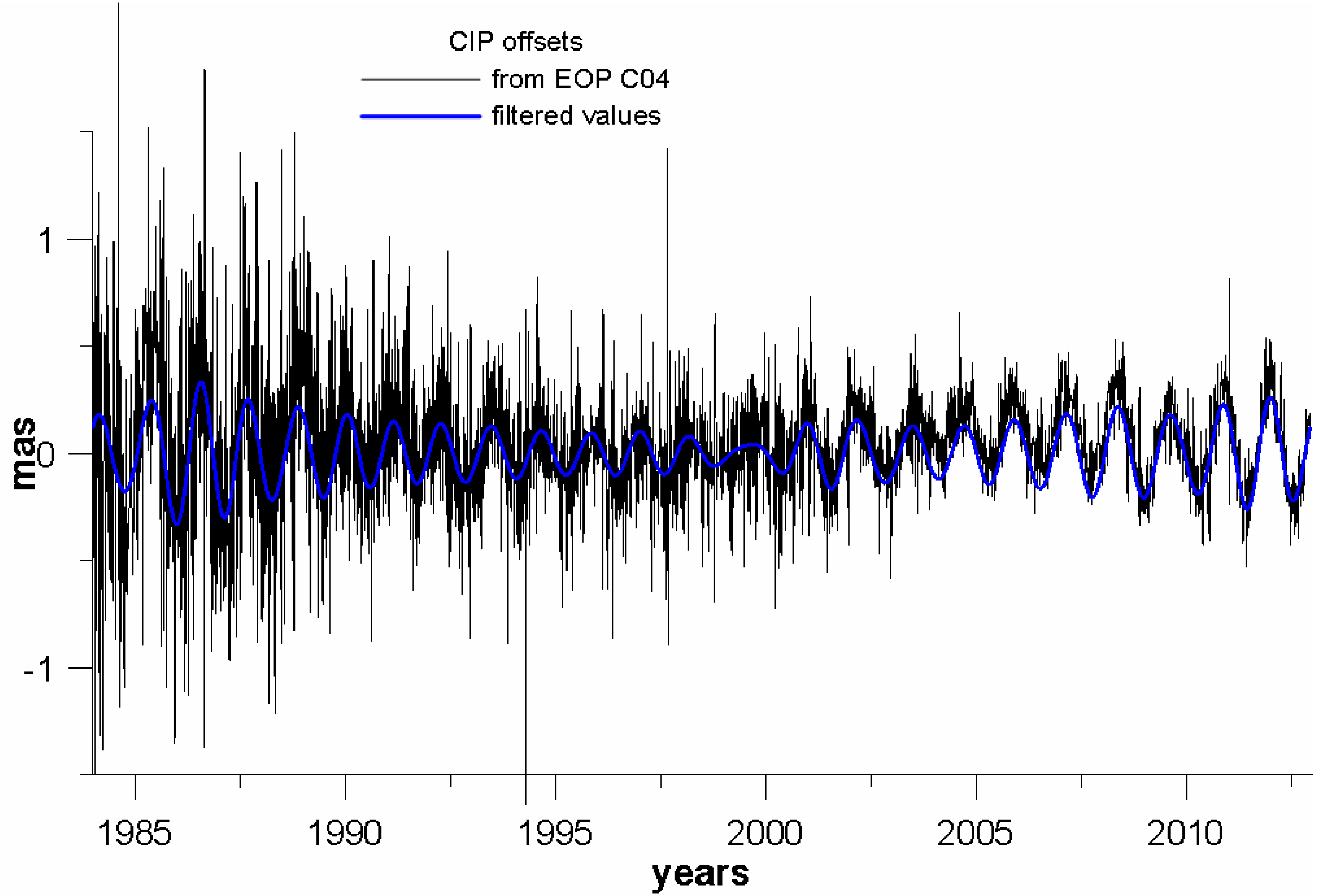
Celestial pole offsets are the discrepancies between VLBI observations and IAU 2000 precession-nutation model



$\omega' = \omega + \Omega$

CRF TRF

FCN signal in celestial pole offsets



Free Core Nutation resonance

Liouville equation" derived by Brzeziński (1994) on the basis of the dynamical theory of Sasao and Wahr (1981):

$$(D - i\sigma_c{}')\left(D - i\sigma_f{}'\right)m = i\sigma_c\left[\left(D - i\sigma_f{}'\right)(\chi'^p + \chi'^w) + (D - i\sigma_c{}')\left(a_p\chi'^p + a_w\chi'^w\right)\right]$$

$$\sigma_c = \sigma_c{}' - \Omega = \frac{\Omega}{T_c}\left(1 + \frac{i}{2Q_c}\right)$$

$$\sigma_f = \sigma_f{}' - \Omega = -\frac{\Omega}{T_f}\left(1 - \frac{i}{2Q_f}\right)$$

in TRF CRF

$$\sigma_c' = \Omega[1.002\,304 + i \cdot 0.000\,006],$$
$$\sigma_f' = \Omega[-0.002\,318 + i \cdot 0.000\,025].$$

T_f=1-1/431, Q_f=20000
T_c=433, Q_c=175

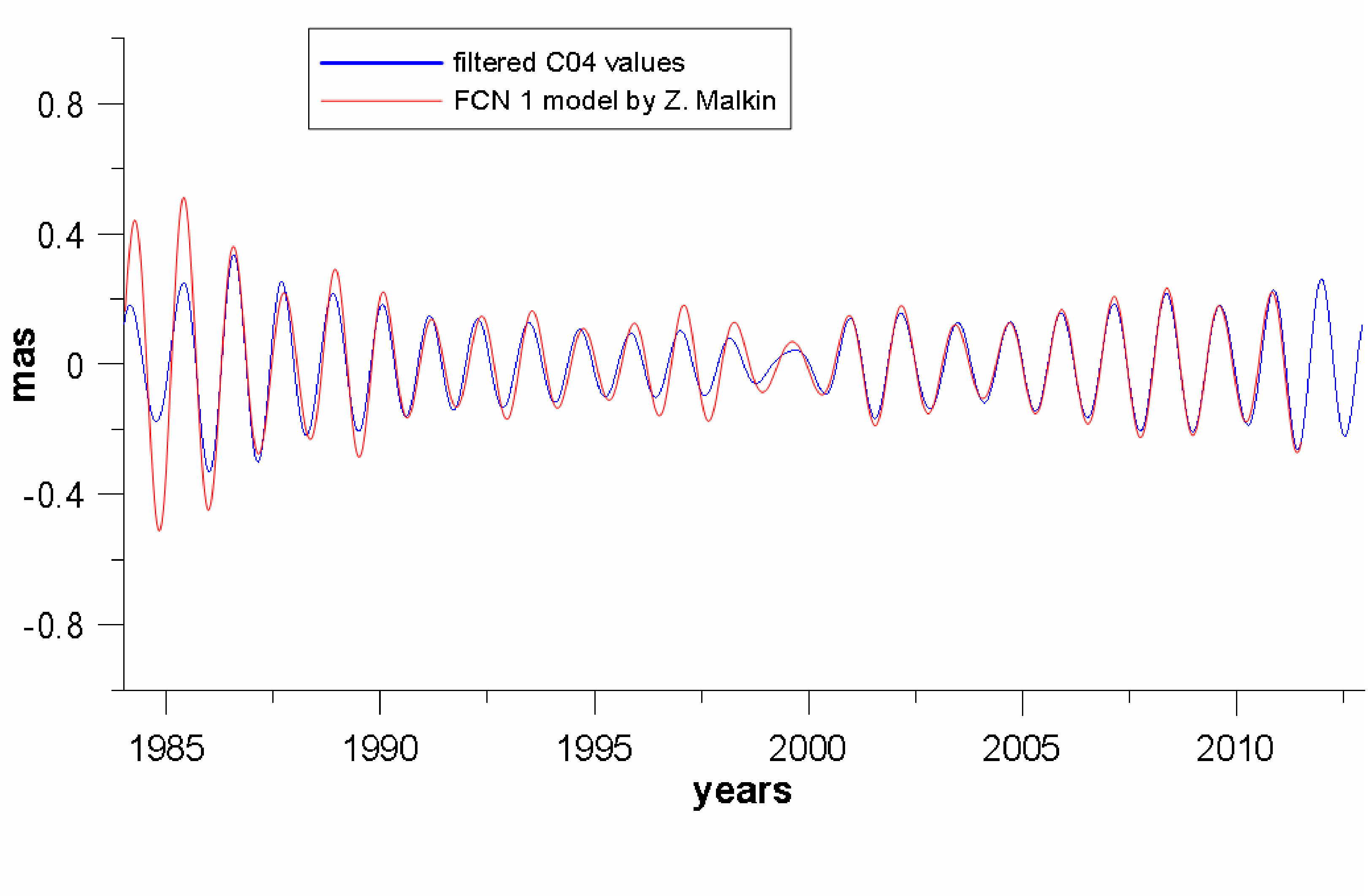
at FCN frequency $(D - i\sigma_c{}')$ can be replaced by -iΩ

Supposing $\chi'^w = 0$, we obtain in frequency domain

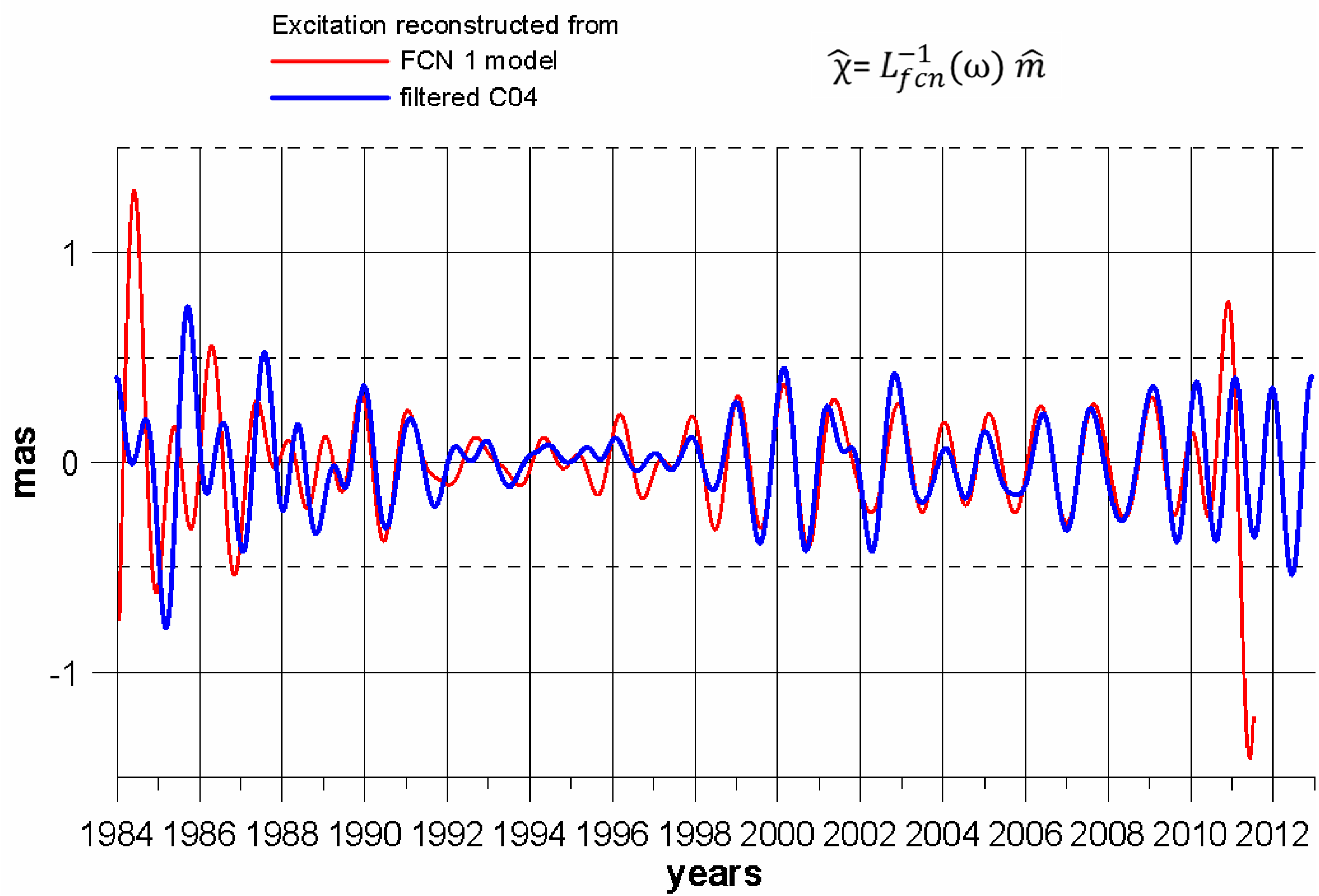
$$\hat{m} = L_{fcn}(\omega)\hat{\chi} \qquad L_{fcn}(\omega) = \sigma_c\left[\frac{1}{\sigma_c' - \omega} + \frac{a_p}{\sigma_f' - \omega}\right]$$

a_p=0.095

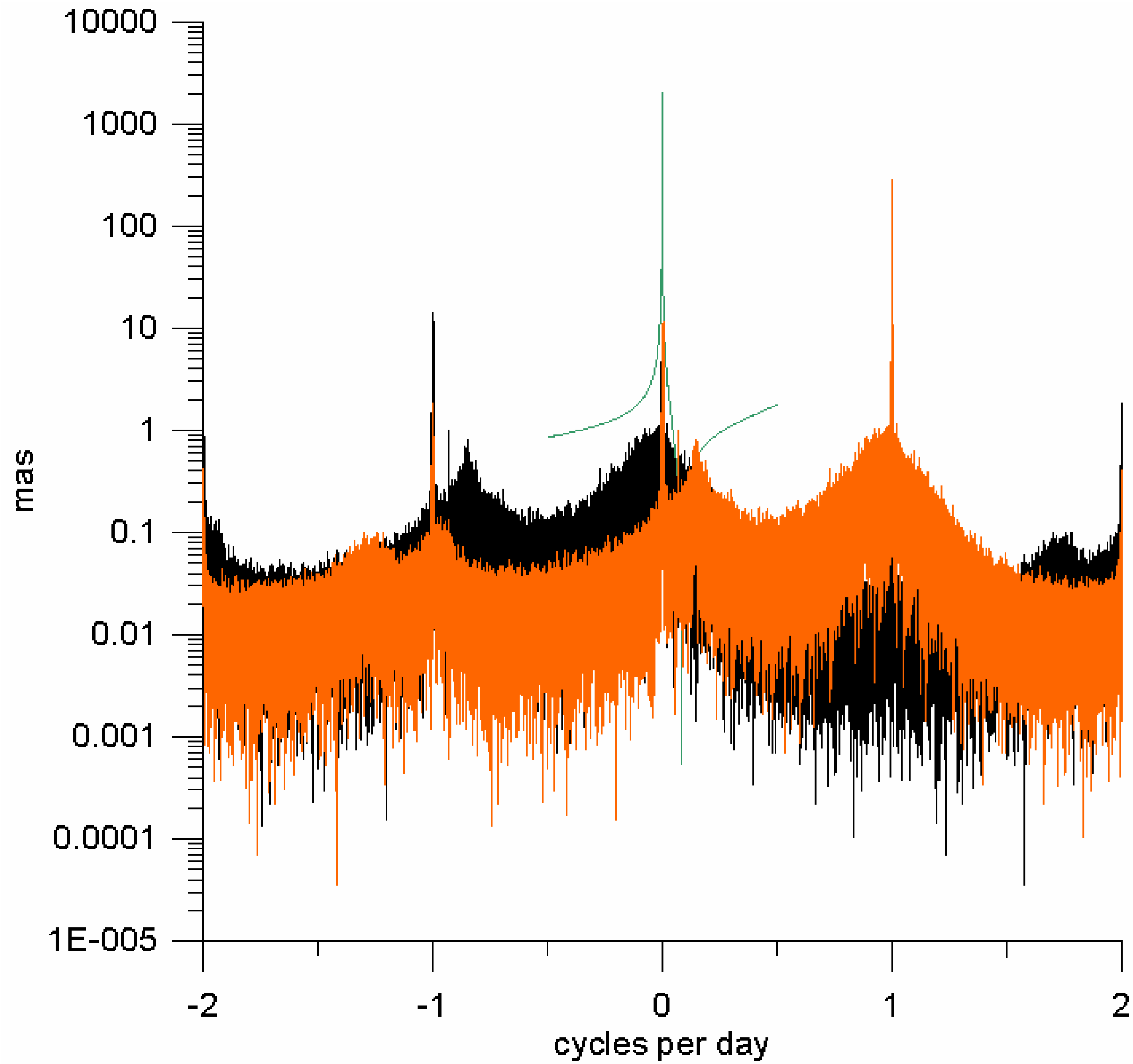
FCN model and filtered C04 data



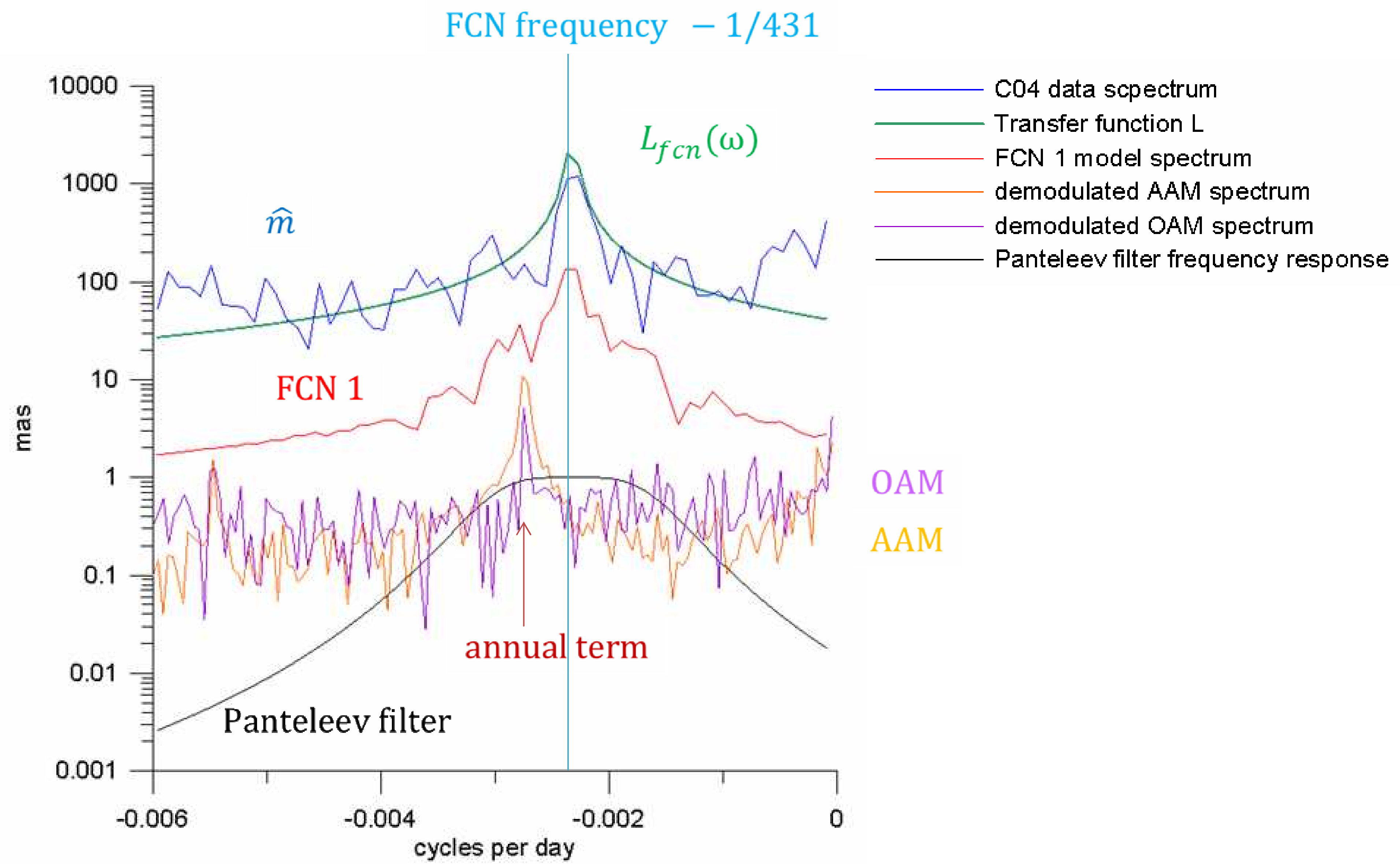
FCN excitation reconstructed



AAM transformation to CRF



Detailed spectra around FCN resonance



AAM and OAM input at FCN frequency