STUDY OF THE PROGRADE AND RETROGRADE CHANDLER EXCITATION

L.V. ZOTOV[1,2], C. BIZOUARD[2],
[1] Sternberg Astronomical Institute of Moscow State University, Laboratory of Gravimetry
Universitetski pr., 13, Moscow, 119992, Russia
e-mail: tempus@sai.msu.ru
[2] SYRTE, Observatoire de Paris, CNRS, UPMC
61, Avenue de l'Observatoire, 75014, Paris, France
e-mail: christian.bizouard@obspm.fr

ABSTRACT. Observed motion of the Earth's rotation axis consists of components at both positive and negative frequencies. New generalized equations of Bizouard, which takes into account triaxiality of the Earth and asymmetry of the ocean tide, show that retrograde and prograde excitations are coupled. In this work using designed narrow-band filter and inversion we reconstruct geodetic excitation at the prograde and retrograde Chandler frequencies. Then we compare it with geophysical excitation, filtered out from the series of the oceanic angular momentum (OAM) and atmospheric angular momentum (AAM) for 1960-2000 yrs. Their sum coincides well with geodetic excitation only in the prograde Chandler band. The retrograde excitation coincides worse, probably in result of amplification of observational noises.

1. INTRODUCTION AND METHOD

Precise observations of Polar Motion (PM) require improvement of the theory, in particular, the introduction of triaxiality into the Euler-Liouville equations and considering the consequences for the ellipticity of the main wobbles, namely annual and Chandler (Gross, 2012). Despite Chandler excitation was found to be provided by the sum of AAM and OAM (Gross, 2000), the variability of the Chandler wobble amplitude over more then one century of PM observations still remains elusive. The Earth's PM is commonly modelled by the linear Liouville equation (Munk, MacDonald, 1960), (Lambeck, 1980)

$$\frac{i}{\sigma_c}\frac{dm(t)}{dt} + m(t) = \Psi(t), \tag{1}$$

where the complex Chandler angular frequency $\sigma_c = 2\pi f_c(1 + i/2Q)$ depends on real Chandler frequency $f_c = 0.8435$ yr^{-1} and quality factor $Q = 100$ (used below). In the dynamical system (1) the complex PM trajectory $m = m_1 + im_2$ is a filtered response to the input excitation $\Psi = \Psi_1 + i\Psi_2$. In (Bizouard, Zotov, 2013) new generalized version of Euler-Liouville equation was derived. The main equation has the form

$$(1-U)m + (1+eU)\frac{i}{\sigma_e}\frac{dm(t)}{dt} - Vm^* + eV\frac{i}{\sigma_e}\frac{dm^*(t)}{dt} = \Psi^{Pure}(t), \tag{2}$$

where parameter U depends on rheology, V characterises the asymmetric response, brought by triaxiality of the Earth and ocean pole tide, σ_e is the Euler frequency, asterisk $*$ means complex conjugation. The geophysical excitation free from rotational excitation stands in the left-hand side of (2), it is related to the effective excitation as $\Psi^{Pure}(t) = (1-U)\Psi(t)$.

Introducing the inverse symmetric transfer function

$$L^{-1}_{sym} = 1 + \frac{(1+eU)}{1-U}\frac{i}{\sigma_e}(i\omega) \approx 1 + \frac{i}{\sigma_c}(i\omega), \tag{3}$$

which coincides with the inverted transfer function of the classical equation (1), and asymmetric inverse transfer function

$$L^{-1}_{asym} = \frac{eV\frac{i}{\sigma_e}(i\omega) - V}{1-U}, \tag{4}$$

the equation (2) can be rewritten in the frequency domain as

$$L_{sym}^{-1}(\omega)\hat{m}(\omega) + L_{asym}^{-1}\hat{m}^*(-\omega) = \hat{\Psi}_{sym}(\omega) + \hat{\Psi}_{asym}(\omega) = \hat{\Psi}(\omega), \tag{5}$$

where $\hat{:}$ is Fourier transform and the rule $\widehat{m^*}(\omega) = \hat{m}^*(-\omega)$ was applied. According to this rule, asymmetric operator (4) acts on the conjugated PM spectrum with inverted frequency $\hat{m}^*(-\omega)$.

In linear equation (1) the input at a particular frequency produces an output at the same frequency. In equation (2) the presence of both direct and conjugated variable m makes input at one frequency producing an output at both prograde and retrograde frequencies. Taking $\hat{m}$ at a particular frequency, we can reconstruct symmetric and asymmetric excitations for it (both of these excitations have prograde and retrograde components), using operators (3), (4), whose amplitude responses as function of ω are shown in Fig. 1, left.

In this work we shall use the new equation (5) to study Chandler wobble. In this framework the output at Chandler frequency is produced by input at prograde and retrograde frequencies. We will isolate the prograde and retrograde Chandler frequencies with the narrow-band Gauss filter modified according to the corrective filtering scheme proposed in (Zotov, Bizouard, 2012). The transfer function of the Gauss filter centered at prograde/retrograde Chandler frequency $\pm f_c$ is

$$L_h(f) = \exp\left(-\frac{(f \mp f_c)^2}{2f_0^2}\right). \tag{6}$$

The plots of these filters are also given in Fig. 1, left. The filter parameter (defining its width) was selected to be $f_0 = 0.04$ yr^{-1}. For the selected f_0 and f_c the filter (6) is narrow-band, not changing the phase of the signal. A time-window of more than 20 years extent corresponds to it. As the filtered signal undergo edge effects, it is not reliable for the first and last 10 years of the considered time interval. The trustful region is depicted by the red rectangle on the plots.

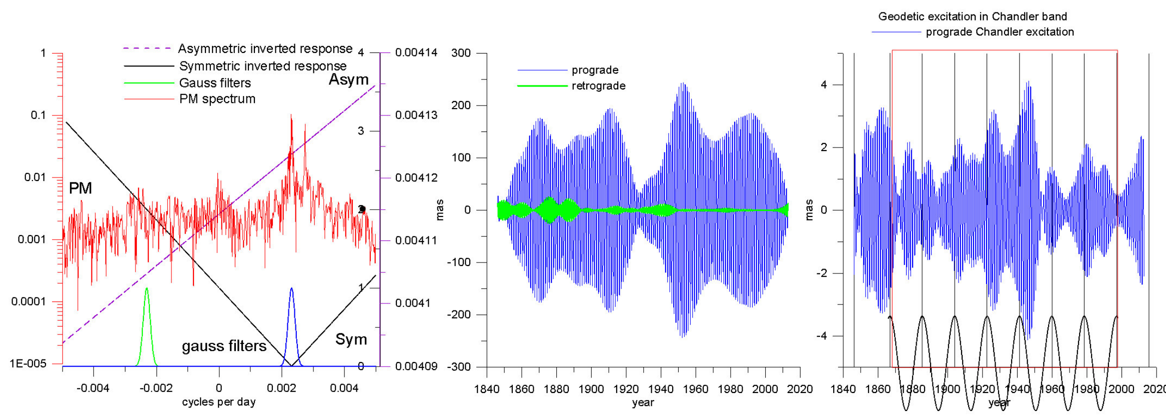

Figure 1: Amplitude responses of inverse operators $|L_{sym}^{-1}(\omega)|, |L_{asym}^{-1}(\omega)|$, prograde and retrograde Gauss filters, and PM spectrum (left). Filtered prograde and retrograde Chandler wobble (center). Prograde Chandler excitation obtained through classical inversion with $L_{sym}^{-1}(\omega)$ (right). Lunar 18.6-yr tide is shown along abscissa.

2. ANALYSIS AND COMPARISON OF RESULTS

Firstly, PM was filtered with Gaussian filter (6) in prograde and retrograde Chandler band (Fig. 1, center). Then the symmetric and asymmetric parts of geodetic excitation in prograde and retrograde Chandler band were obtained through multiplication by the symmetric (3) and asymmetric (4) inverse operators in frequency domain. The classical prograde Chandler excitation is shown in Fig. 1, right. As it was noted in (Zotov, Bizouard 2012), it has an amplitude modulation, often synchronous with the Lunar 18.6-yr tide.

The prograde and retrograde geodetic Chandler excitations are shown in Fig. 2, its classical (symmetric) part Ψ_{sym} is presented to the left, asymmetric part Ψ_{asym} is to the right. Asymmetric part of both prograde and retrograde component has an order of magnitude of 1 mas or less, and is thus smaller than the symmetric contribution. Nevertheless, asymmetric part's contribution is significant at the contemporary level of observational precision ($\sim$ 0.05 mas). Retrograde asymmetric part with an amplitude up to 1 mas repeats the shape of the prograde Chandler wobble (Fig. 1, center), because it was obtained by multiplying this wobble by the linear function $L^{-1}_{asym}(\omega)$ (Fig. 1, left). It dominates in the total asymmetric excitation (sum of asymmetric prograde and retrograde parts) and is by far the most important innovation brought by the new equation (2).

On the contrary, the classical (symmetric) prograde Chandler excitation, as seen from Fig. 2, left, looks small over the background of retrograde part. The letter is especially large before 1900. It could be caused by the observational noise amplification at the retrograde Chandler frequency by the inverse operator $L^{-1}_{sym}(\omega)$, Fig. 1 left, where its amplitude response is quite large (if to compare with prograde). Then symmetric and asymmetric parts of geodetic excitation were added together and compared to the

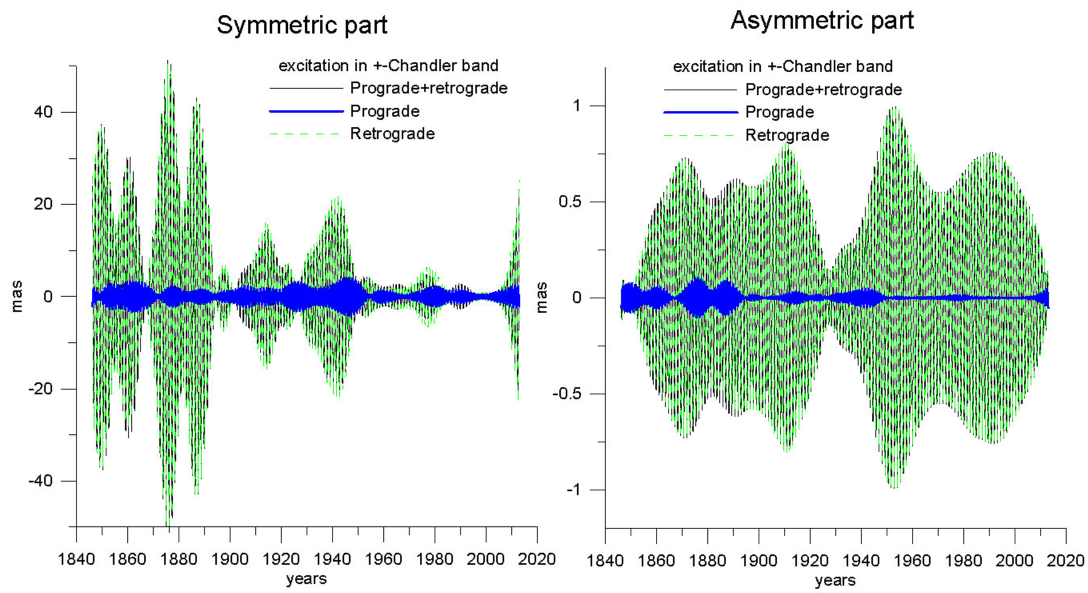

Figure 2: Symmetric Ψ_{sym} (left) and asymmetric Ψ_{asym} (right) components of the geodetic excitation at the prograde and retrograde Chandler frequencies.

geophysical excitation. The atmospheric contribution was obtained by filtering of NCEP/NCAR AAM combination of pressure (IB) and wind terms by the Gaussian filter (6) with parameters chosen above. Oceanic part of excitation was obtained in the same way from ECCO OAM time series, sum of bottom pressure and current terms. In Fig. 3 we plot geodetic and geophysical excitations in prograde (top) and retrograde (bottom) Chandler bands for AAM (left), OAM (center) and their sum (right). Despite the initial time series span is 1949-2010, we compare the results of filtering only for 1960-2000 (depicted with red rectangle) because of the edge effect. The agreement is good in the prograde band, while in the retrograde band OAM+AAM sum does not explain the geodetic excitation.

Table 1 presents the correlation coefficients between geodetic excitation and OAM, AAM, OAM+AAM geophysical excitations in prograde and retrograde Chandler bands. The misfit and worse correlation at the retrograde frequency could have several explanations. Firstly, as a result of observational noise amplifications during inversion. Secondly, by existence of some other factors, which excite the retrograde wobble. Finally, some defect could remain in the transfer function of dynamical equation (2), causing overestimation of the inverse amplitude response at this frequency. In any case, new equation (2) introduces an asymmetric part much smaller than the symmetric one, bringing the results presented in Fig. 3 and Table 1 in close agreement to what would have been obtained with classical modelling of Eq. (1).

3. CONCLUSION

We derived the geodetic excitation in prograde and retrograde Chandler band in the framework of generalised Euler-Liouville equation (2), accounting for asymmetry brought by ocean pole tide and triaxiality, and compared it to the geophysical excitation. Excitation at the retrograde Chandler frequency

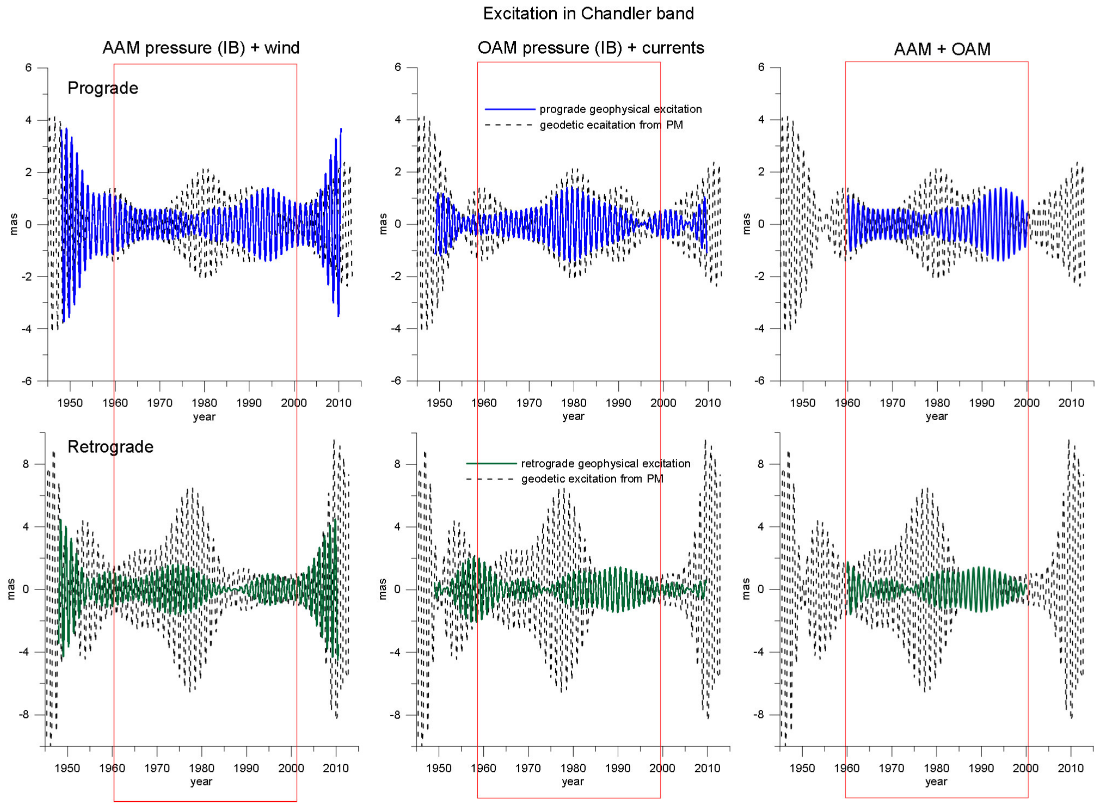

Figure 3: Comparison of geodetic excitation in prograde (top) and retrograde (bottom) Chandler frequency bands (sum of symmetric and asymmetric parts) with the geophysical excitation, related to AAM (left), OAM (center), AAM+OAM (right).

	AAM X	AAM Y	OAM X	OAM Y	AAM+OAM X	AAM+OAM Y
Prograde Chandler	0.598	0.596	0.896	0.897	0.920±0.010	0.920±0.011
Retrograde Chandler	0.428	0.430	0.123	0.126	0.438±0.056	0.439±0.056

Table 1: Correlation coefficients between geodetic and geophysical (AAM, OAM, their sum) excitations.

is found to be larger than at the prograde one. New formalism introduces the asymmetric input at the level of 1 mas, what is important at the contemporary level of observational precision, but does not sufficiently improve the geophysical budget of PM excitation. In particular, misfit between geodetic and geophysical excitation at the retrograde Chandler frequency remains questionable.

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