ON THE SPATIAL DISTRIBUTION OF MAIN BELT ASTEROIDS

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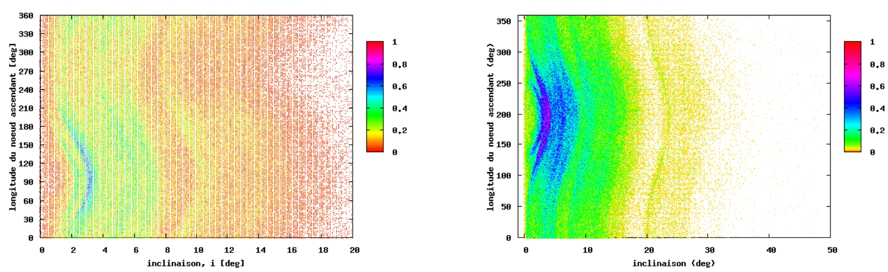
ABSTRACT. We investigate here the distribution of main belt asteroids in the (i: inclination, Ω: longitude of the ascending node) space with respect to the ecliptic-equinox J2000. We identify and confirm a sinusoidal behaviour of this distribution, which disappears when the inclination is given with respect to Jupiter's orbital plane, or with respect to the invariable plane (IP). This behaviour is explained by planetary secular effects, mainly due to Jupiter. Furthermore, we identify three different orbital behaviours that explain the density distribution in this space.

1. INTRODUCTION

We present here the analysis of a not so common distribution, the Main Belt Asteroid (MBA) population in the osculatory element domain defined by the inclination, and the longitude of the ascending node (i,Ω). We show (Fig.1(a)) that a particular sinusoidal aspect of the distribution is observed and that it is due to secular effects of the planets, mainly Jupiter.

2. MODEL AND ARGUMENTS

One should ask two questions: (i) is this distribution an artefact due to an observational bias? (ii) does the area of maximal density correspond to a particular dynamical grouping?



(a) (24) Themis, $i = 0°, 75$ (b) (24) Themis, $i = 0°, 75$

Figure 1: Density distribution of the asteroids (333.841 numbered MBAs) in term of orbital plans with respect to the ecliptic-equinox J2000 (Fig.1(a)), with respect to the Invariable Plane (IP) (Fig.1(b)).

The answer to both question is *"No"*. In fact, the "waves" (Fig.1(a)) are also observed in the distribution defined by mean i vs mean Ω. Though, when the elements are given with respect to the solar system's (IP) (Souami & Souchay, 2012) (Fig.1(b)) or with respect to Jupiter's or Saturn's orbital planes, this sinusoidal behaviour tends to disappear and the distribution is flattened.

Some people have looked into the problem, for example Michkovitch (1947) suggested the secular perturbations to be at the origin of a similar distribution for the longitude of the perihelium. Scheirich (2005) observed the distribution but did not provide an explanation. Here, using a simplified secular model following the approach of Murray & Dermott (1999; Chap. 7), we provide an answer to the problem. We consider the truncated Hamiltonian to the second order in both eccentricities and inclinations. For a massless asteroid, of semi-major axis a, and mean motion n; the perturbing function is written:

$$\mathcal{R} = na\Big[\frac{1}{2}A(h^2+k^2) + \frac{1}{2}B(p^2+q^2) + \sum_{j=1}^{2} A_j(hh_j+kk_j) + \sum_{j=1}^{2} B_j(pp_j+qq_j)\Big], \qquad (1)$$

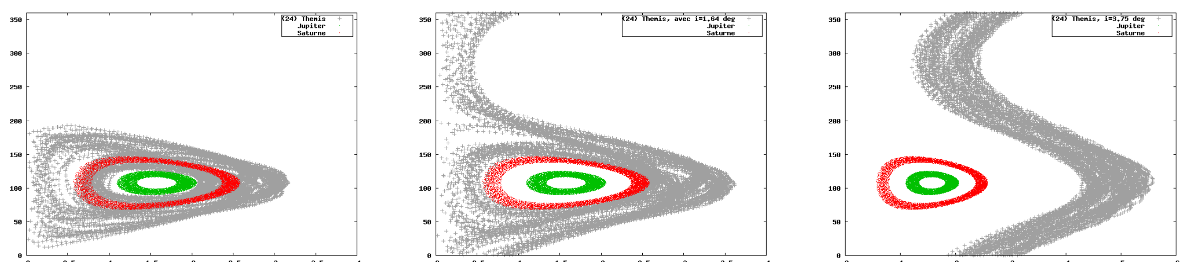
where p and q are the components of the inclination vector; h and k those of the eccentricity vector. The matrices A et B, which are function of the Laplace coefficients are given by their classical expressions (Murray & Dermott 1999). The indices $i = 1$, and $i = 2$ are used for Jupiter and Saturn, respectively.

The perturbing function given by Eq (1) is uncoupled in eccentricity and inclination, thus we can restrict ourselves to the secular terms in inclination. The solution is given by

$$p(t) = I_{free} \sin(Bt + \gamma) + p_0(t) \quad , \quad q(t) = I_{free} \cos(Bt + \gamma) + q_0(t) \tag{2}$$

It is written as the sum of the forced solution $(p_0(t), q_0(t))$ and the free one (which is periodic and has an amplitude I_{free}). The free motion depends upon the initial conditions of the inclination vector (of amplitude I_{free}, and phase γ). This is clearly observed for the asteroid (24) Themis (Fig.2(a)), and two fictitious Themis (Figs:2(b),2(c)) for which we vary the value (I_{free}).
$p_0(t) = -\sum_{i=1}^{2} \frac{\mu_i}{B-f_i} \sin(f_i t + \gamma_i)$, $q_0(t) = -\sum_{i=1}^{2} \frac{\mu_i}{B-f_i} \cos(f_i t + \gamma_i)$, where $\mu_i = \sum_{j=1}^{2} B_j I_{ji}$.
The forced motion is associated with a forced inclination I_{forced}, and node Ω_{forced}

$$I_{\text{forced}} = \sqrt{p_0^2 + q_0^2} \quad , \quad \tan \Omega_{\text{forced}} = \frac{p_0}{q_0}, \tag{3}$$



(a) (24) Themis, $i = 0°, 75$ (b) $i = 1°, 64$ (c) $i = 3°, 75$

Figure 2: The secular motion in (i, Ω) plane over 10 000 years, for the asteroid (24) Themis (in grey): $a = 3,14$ au, $e = 0,13$ et $i = 0°, 75$ (Fig. 2(a)). The other figures 2(b), 2(c) are for a fictitious Themis fictifs of inclination $1°, 64$ et $3°, 75$, respectively; green for Jupiter et red Saturn.

We have identified three different secular dynamics of the asteroids, depending on the initial conditions (I_{free}): *i*)- libration for small I_{free} values (Fig. 2(a)), *ii*)- homogenous or regular circulation (Fig 2(b)), *iii*)- heterogenous circulation for relatively high values of I_{free} (Fig. 2(b)).

3. CONCLUSIONS

We have explained with a simple secular model the observed distribution of minor planets in the (i, Ω) plane, with respect to the ecliptic-equinox J2000.0; proving by a simple analytical model that the distribution is mainly due to secular effects of Jupiter. The results are confirmed numerically considering a more complete model (see Souami 2012).

We were able to distinguish three different dynamical behaviours that depend on the initial inclination. The superposition of the three dynamics explains the observed distribution, though one question remains open: what are the initial conditions that would lead to the observed distribution?

4. REFERENCES

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