ROTATIONAL AND LIBRATIONAL MOTION OF SOLAR SYSTEM BODIES

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ABSTRACT. Planetary exploration has revealed the great richness and diversity of the terrestrial planets and natural satellites. The mean rotational motion of planets was determined from ground-based telescopes since few centuries and nowadays the measurement resolution afforded by space telescopes and spacecraft allows the detection of small variations in the rotational motion that bear the signature of internal properties. The investigation of the interiors of planets and satellites is instrumental to understanding planetary processes operating on a global scale. Here, we will present the knowledge of the rotation and librations (for bodies in spin-orbit resonance) of some Solar system bodies that has been measured recently.

1. INTRODUCTION

The mean rotation of planets and natural satellites have been detected few centuries ago and today the improvement and accumulation of data allow astronomers to distinguish variations in the mean rotational motion. Such variations have been identified for the Earth since 2$^{\mathrm{nd}}$ century B.C. when Hipparcus measured the precession and then in 1747 when Bradley determined the first nutation (see e.g. Souchay & Capitaine 2013). In 1839 Bessel revealed the lunar physical libration by using a heliometer (see e.g. Koziel 1985). The oscillations around uniform motion are called nutations for non-resonant rotator like the Earth and librations for bodies in spin-orbit resonance. These variations intrigued dynamisicists and geophysicists because they bear the signature of internal properties. The determination of the present core state reveals crucial information on our understanding of the thermal evolution and geophysical history of the bodies. In addition, the knowledge of the rotational state of bodies is useful for cartography and for the planning of space missions.

In this short review, we will focus on the recent determination of the librations of the Moon, Phobos, and on the first librational determination for Epimetheus, Enceladus, and Mimas. We will also mention the recent results on Mercury and Venus.

2. THE ROTATIONAL MOTION OF THE MOON

It is well known that our natural satellite is in spin-orbit synchronous resonance implying that the Moon shows on average the same face towards the Earth. The Moon's rotation is measured with a remarkable accuracy of few milli-arcseconds thanks to the Lunar Laser Ranging (LLR) experiment that has been active since 1969 (e.g. Dickey *et al.* 1994). This experiment consists in the measurement of the round-trip travel time of a short laser pulse between an Earth observatory and one of the five corner cube retroreflector arrays settled on the Moon. US astronauts established three corner cube arrays during Apollo-era missions (Apollo 11, 14, and 15) and two were added by the soviet robotic missions Lunakhod (Lunakhod 1 and 2). Echoes of the Lunakhod 1 have been obtained in April 2010 (Murphy *et al.* 2011) thanks to a new determination of the array position by Lunar Reconnaissance Orbiter. Earth observatories such as the Apollo station (Murphy 2009) and OCA station (Samain *et al.* 1998) regularly shoot the Moon in order to obtain echoes and carry on the collection of data. The LLR data processing is a very sophisticated and challenging task, even after more than 40 years of routine observational operation.

Due to the high accuracy of the LLR observations and the large amount of data, the lunar rotation is integrated numerically, for example in the DE ephemeris (Williams *et al.* 2001) and INPOP ephemeris (e.g. Fienga *et al.* 2009). These models are joint numerical integration of the orbits of the Moon, the

Earth, the planets and asteroids, and of the lunar rotation (Williams *et al.* 2001; Fienga *et al.* 2009). The dynamical partial derivatives of the orbits and lunar Euler angles with respect to solution parameters such as moment of inertia, gravity field, tides, dissipation, interaction with a fluid core and initial conditions are computed and the adjustment provides the determination of these geophysical parameters (for a complete description see Standish and Williams 2012).

The rotational motion of the Moon is mainly sensitive to the gravitational torque of the Earth acting on its dynamical figure. The rotational motion may be described through the Euler equation written in the Moon's reference frame

$$\frac{d[I]\vec{\omega}}{dt} + \vec{\omega} \wedge \vec{[I]}\vec{\omega} = \vec{\Gamma} \tag{1}$$

where $[I]$ is the inertia tensor of the Moon and $\vec{\omega}$ is the angular velocity vector. The external gravitational torque of the Earth that acts on the dynamical figure of the Moon is expressed as

$$\vec{\Gamma} = \frac{3GM}{r^3}\vec{u} \wedge I\vec{u} \tag{2}$$

where $\vec{u}$ is the unit vector toward the Earth, M the Earth's mass. In DE and INPOP models, there are also gravitational torques due to other bodies in Solar system (Sun, Venus, and Jupiter) and the gravitational field of the Moon is developed until degree 6 and the Earth at degree 2. The external gravitational torque (Eq. 2) depends on the relative distance between the Moon and the Earth. The response of the Moon to this external torque is called librational motion and corresponds to oscillatory motion of the Moon around a uniform rotational motion. The librational motion can be decomposed in a Fourier series where the forced frequencies arise from periods present in the lunar orbit from (1) Earth-Moon-Sun effects, with periods related to the Delaunay arguments of lunar theory, and (2) planetary effects. A second set of frequencies, called free frequencies are also present in the librational motion of the Moon and they correspond to the dynamical normal modes of the Moon in spin-orbit resonance when its spin is displaced from its dynamical equilibrium position (e.g. Williams *et al.* 2001; Rambaux and Williams 2011). The amplitudes of the periodic forced libration terms depend on both the strength of the torque applied to the aspherical Moon and how close the forcing period is to the resonant periods of the rotation, the free periods. The strength of a torque term can be variable in time, as for example due to the eccentricity of the Earth-Moon orbit around the Sun, which varies with time and alters the amplitude of the librations. In addition, if the tides are introduced, the tensor of inertia $[I]$ is variable with time and mixes some terms in the librational motion (see Williams *et al.* 2001).

To take into account the presence of the core an additional equation of motion of the form

$$\frac{d\vec{H}^c}{dt} + \vec{\omega} \wedge \vec{H}^c = \vec{\Gamma}_c \tag{3}$$

is introduced. The $\vec{\Gamma}_c$ represents the interaction between the core and the mantle. In Williams *et al.* (2001), the core is assumed to be spherical and the coupling is dissipative. That formulation has been supplemented in the DE model (e.g. Williams *et al.* 2013) to take into account the core oblateness.

A new vision of the interior of the Moon has emerged from the combination of the GRAIL data (Zuber *et al.* 2013), seismic lunar models (Weber *et al.* 2011; Garcia *et al.* 2011) and the LLR model (Williams *et al.* 2013). Notably, from the mean density, mean moment of inertia of the solid Moon, and the tidal potential love number, the lunar structure appears divided into a thin crust, a thick mantle layer, and a fluid outer core that probably contains an inner core. In addition, the LLR data is sensitive to oblateness of the core and to the dissipation through the detection of a shift of 0.26" in the pole direction and the determination of the amplitude of four librational terms. The dissipation factor can be described through a power law in frequency but the sign of the power law (negative) is still challenging for the geophysical models that predict a positive sign. In addition, since the free librations damp with time, the observational detection of free librations requires recent excitation or continuing stimulating mechanisms. The settlement of new laser retroreflectors or active transponders on the surface of Moon may improve significantly the understanding of the interior structure of the Moon (Dehant *et al.* 2012).

3. ENCELADUS AND OTHER SATELLITES

The detection of librations is not unique to the Moon. Today, we know four additional satellites that exhibit variations in their rotational motion: Phobos, Epimetheus, Enceladus, and Mimas. Their main librational amplitude and periods are listed in Table 1. The reported detections are on the librations in longitude that correspond to the oscillations of the body principal axis projected onto the equatorial plane of the satellite. The amplitudes of these librations are generally larger than the amplitude of the libration in latitude, North-South oscillations. The librations in longitude can be expressed as (e.g. Rambaux *et al.* 2010)

$$\gamma = A_0 \sin(\omega_0 t + \phi_0) + \sum_i \frac{\omega_0^2 H_i}{\omega_0^2 - \omega_i^2} \sin(\omega_i t + \alpha_i) \tag{4}$$

where $\omega_0 = n\sqrt{3\sigma}$ is the resonant frequency of this spin-orbit problem and A_0, ϕ_0 are two constants of integration. The H_i, ω_i, and α_i are respectively the magnitude, the frequency and the phase of the orbital perturbations. For example, the libration in longitude is caused by the variations in the orbital velocity related to the non-ezro eccentricity. In this case, the magnitude of the perturbation is $H_i = 2e$ with e the eccentricity and the frequency is the orbital frequency. Then the amplitude of physical libration γ is equal to $6e\sigma/(3\sigma - 1)$. If the perturbed orbital motion implies a long period term, then the resulting amplitude of libration is almost equal to the magnitudes of the orbital perturbation because $\omega_i \ll \omega_0$. Comstock and Bills (2003) gave a large overview of the amplitude of the librations in longitude due to the non-zero eccentricity for rigid bodies. The presence of a fluid ocean modifies the resulting amplitude libration (see e.g. Van Hoolst *et al.* 2013) but the amplitude of librations for such bodies have not yet been observed.

The main libration of Phobos was determined 20 years ago by Duxbury and Callahan (1989). They used a digital control point network based on Viking images. The amplitude of librations have been confirmed and improved by MEX data (Willner *et al.* 2010) as well as from the orbital perturbation (Jacobson 2010). These new data and adjustment stimulated the elaboration of new models of Phobos librations in order to find evidence of possible origin of Phobos (Rambaux *et al.* 2012; Le Maistre *et al.* 2013).

The NASA-ESA space mission Cassini has orbited in the Saturnian system since 2004. The numerous flybys of Cassini over Saturnian satellites led to the discovery of the librational motion of three satellites: Epimetheus, Enceladus, and Mimas. The rotational motion of Epimetheus has been obtained as a by-product of the construction of the shape model (Tiscareno *et al.* 2009). This method works well for this satellite because it has a large librational amplitude, roughly 5.9° (see Table 1). Unfortunately, the same method applied to the companion Janus does not provide a clear detection (0.33° ± 0.66°), Tiscareno *et al.* 2009). Following the detection of Epimetheus' librations, the rotational motions of Enceladus and Mimas were determined. In this case, the control point network method has been used by Giese *et al.* (2011) and Tajeddinne *et al.* (2013), respectively.

The horsechoe shaped orbital librations of Epimetheus and Janus leads to further investigation of their rotational responses by numerical (Noyelles 2010) and analytical studies (Robutel *et al.* 2010). A generalization of the rotational motion for co-orbital satellites in spin-orbit resonance has been formulated by Robutel *et al.* (2011) and applied to Telesto, Calypso, Helene, and Polydeuces in perspective of future detection. Thomas *et al.* (2013) investigated the librations of Helene based on the same approach that Tiscareno *et al.* (2009) used but did not detect librational motion. For Enceladus, the librational detection used the model developed by Rambaux *et al.* (2010) in order to discriminate between long and short period. The librations of Enceladus are dominated by the long-period librations resulting from its interaction with Dione. For Mimas, the comparison of the observed amplitude with the libration computed for a hydrostatic model developed by Noyelles *et al.* (2011) shows a large departure for the libration at the orbital period (0.944 days). The amplitude of the observed libration at the orbital period is twice the prediction whereas the five other detected librations have amplitudes in very good agreement. Such a large departure may be interpreted as the signature of an elongated core that can give information on the origin of Mimas (Tajeddinne *et al.* 2013).

Stiles *et al.* (2008, 2010) analyze the rotation of Titan from radar observations by assuming a non-constant angular velocity. Then Merigolla and Iess (2011) investigated the possibility to detect librations in the rotational motion of Titan. It appears that the amplitude of the librations is below the detection accuracy as confirmed by theoretical models (Van Hoolst *et al.* 2013).

4. MERCURY AND VENUS

The best illustration of the relationship between rotation and interior is certainly obtained for Mercury. Margot *et al.* (2007, 2012) performed Earth-based radar interferometry observations of Mercury. They

Satellite	Amplitude	Frequencies	References
The Moon(a)	90.706”	365.260	Rambaux and Williams (2011)
	16.799”	27.555	”
	16.762”	1095.220	”
Phobos	1.2° ± 0.15°	0.3190 days	Willner *et al.* (2010)
Epimetheus	5.9° ± 1.2°	0.692 days	Tiscareno *et al.* (2009)
Enceladus	0.056°	1.372 days	Giese *et al.* (2011)
Mimas(b)	43.78° ± 0.07°	25772.62 days	Tajeddine *et al.* (2013)
	42.1’ ± 1.8’	8590.87 days	”
	48.3’ ± 1.3’	0.944 days	”

Table 1: Determination of the librations in longitude for natural satellites. (a) For the Moon, the table lists only the three largest librations; for the rest of spectrum see Rambaux and Williams (2011). (b) For Mimas, Tajeddine *et al.* (2013) detected three more librations.

obtain the rotational motion of Mercury with a great accuracy and discovered the amplitude of forced libration at the orbital period equal to (38.5 ± 1.6 arcseconds) and an obliquity value of Mercury of (2.04 ± 0.08) arcminutes (Margot *et al.* 2012). The forced amplitude can be modeled as (e.g. Margot *et al.* 2007):

$$\gamma = \frac{3}{2}\frac{B-A}{C_m} f(e) \sin nt \qquad (5)$$

where A and B are the equatorial moments of inertia of Mercury, C_m is the polar moment of inertia of the mantle, and $f(e)$ is a function depending on the eccentricity. Indeed C_m represents the inertia of the body, i.e. its resistance to the motion. If the core is solid, it is all the body that responds to the gravitational torque of the Sun and the denominator C_m must be replaced by C, if the core is fluid, only the mantle will respond by assuming that the interior is decoupled from the mantle because the inertial coupling is small (Rambaux *et al.* 2007). As the moment of inertia of the mantle C_m is smaller than the total moment of inertia C, the amplitude of libration is increased for Mercury with a fluid core.

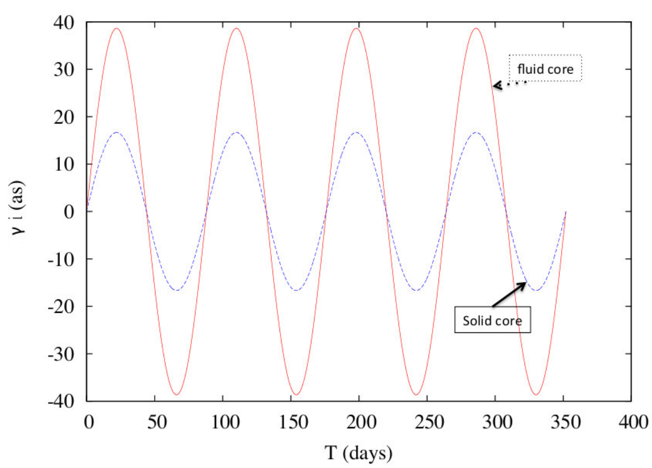

Figure 1: Amplitude of librations for Mercury for two different interiors structures (liquid and solid).

The residuals of the forced librations between the observations and the model show an additional libration of which the origin is still a mystery. Some hypotheses have been suggested and investigated: existence of a free libration, forced librations due to Jupiter, interior coupling (see the discussion and references in Margot *et al.* 2012). But no firm explanation on this additional libration has been provided. The identification of this forced libration would associate the interior properties more accurately with improving significantly the interior models (e.g. Rivoldini *et al.* 2009; Hauck *et al.* 2013).

The rotational motion of Venus is the least known among the four terrestrial planets. This is essentially due to the thick atmosphere that conceals the surface. The Magellan mission used radar images (Davies *et al.* 1992) and VEX followed thermal emissivity anomalies (Mueller *et al.* 2012) to track landmark features at the surface of the planet. The anomalous features appear to be slightly shifted when using the Davies et al. rotational model developed in 1992. Possible origins of this variation in the spin period is the exchange of angular momentum between the atmosphere and the solid body, the solar gravitational torque acting on the triaxial figure, or the presence of a fluid core (Cottereau *et al.* 2011). However, the peak-to-peak length-of-day variations induced by these mechanisms seem to be too small to explain the observational data. Understanding the mechanisms responsible for these two observations will bring crucial information on the nature of interior or atmospheric couplings in Venus.

5. CONCLUSION

The determination of the rotational motion has been improved during the last ten years by Earth-based observations and/or in-situ space missions visiting these bodies. The increase of the accuracy and accumulation of data allow a better vision of the interior of the Moon while the first measurement of the librational or rotational variations for Epimetheus, Enceladus, Mimas, Mercury, and Venus reveals extremely rich bodies. In parallel to these new measurements, the development of the modeling (analytical and numerical) including more and more physics allows a full exploration and exploitation of these observations that lifts the veil on the interior of Solar system bodies.

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