## AMPLITUDE-FREQUENCY ANALYSIS OF THE EARTH ORIENTATION PARAMETERS AND THE VARIATION OF THE SECOND ZONAL HARMONIC OF THE GEOPOTENTIAL

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ABSTRACT. An amplitude-frequency analysis of the rotary-oscillatory Earth motion under the action of gravitational-tidal perturbing torques from the Sun and the Moon is carried out using the classical mechanics' methods. The simulation results of the oscillatory process in the motion of the Earth pole and the variations of the second zonal harmonic  $\delta c_{20}$  of the geopotential are studied. Based on the dynamic Euler-Liouville equations expressions for amplitude and phase of the Earth pole oscillations are obtained. A comparison of the spectral power densities of the time series between the Earth pole coordinates and the  $\delta c_{20}$  variations of the geopotential is carried out. A functional dependence of the aforementioned component of the geopotential from the amplitude and phase of the Earth's pole oscillatory process is shown.

## 1. INTRODUCTION

The study of the time variations of the geopotential as a result of the rotary-oscillatory processes of the Earth motion is of a significant natural-sciences and practical interest. Oscillations of the Earth's inertia tensor components depend on many factors, among them the mechanical and physical parameters of the planet, the motions of tide-forming bodies, and the observed large-scale phenomena in nature. Time-dependent variations of these and other factors (regular and irregular oscillations, stochastic fluctuations, secular variations) affect the Earth rotary-oscillatory processes and the rotational parameters of the planet. The dynamic processes of the Earth orientation parameters (EOP) in turn have an effect on its figure and lead to the fluctuations of the gravitation field. Observed variations of the EOP, the variations of the Earth's gravitational field and oscillations in the large-scale geophysical events appear to be in a considerable correlation.

## 2. DYNAMIC PROCESSES OF EOP AND SECOND ZONAL HARMONIC OF THE GEOPOTENTIAL

From the SLR (Satellite Laser Ranging) observation data [1] of the zonal coefficient  $c_{20}$  main variations are divided out being caused by the quasi-periodic annual perturbation component. It is known [2–4], that the external perturbation also leads to the six-year modulations of the oscillatory process of the Earth pole. Thus, the annual variations in the moments of inertia and the Earth rotary-oscillatory motions occur in phase. Annual oscillations in the polar motion, if considered in terms of celestial mechanics [3, 4], are accounted for by the gravitational tidal torque and its combination structure dependent on the nutation angle. The annual components of the solar gravitational tidal torque constitute a vector rotating in a related coordinate system with the mean angular velocity of the Earth orbiting the Sun.

The correspondence between these perturbations can be found from the comparison of the spectral power densities of the time series between the Earth pole coordinates observations (e.g.  $y_p$ ) [2] and the variations of the  $\delta c_{20}$  [1]. Based on the modelling of the Earth pole oscillations a division of the oscillatory process into the components is performed. Observation frequency spectrum  $\delta c_{20}$  and the  $y_p^{\nu_t}$  series

$$y_{p}^{\nu_{t}} = \hat{y}_{p} - y_{p}^{N},\tag{1}$$

which is obtained from the observations  $\hat{y}_p$  [2] by subtracting the Chandler component  $y_p^N$ , are congruent in the two main frequency regions - in the frequency interval from 0.8 to 1.2 and less than 0.3 cycles per year. It should be noted that here the Chandler component  $y_p^N$  is defined as a set of the oscillations with the main Chandler frequency N and close to it frequencies.

Assumed that  $x_p = c_x + a \cos \psi$  and  $y_p = c_y + a \sin \psi$ , the equations for amplitude and phase variables of the Earth Pole modulation motion have the form  $\dot{a} = \mu_t \cos(\psi - \nu_t)$ ,  $\dot{\psi} = N - a^{-1}\mu_t \sin(\psi - \nu_t)$ , where  $\mu_t = \mu(t)$ ,  $\nu_t = \nu(t)$  are the function of time, which are expanded into a sum of periodical components; the values  $\mu_t \cos \nu_t$ ,  $\mu_t \sin \nu_t$  are perturbations which lead to the observed oscillations of the Earth pole and which have the dimensions of specific torque. The expression for  $\delta c_{20}$  is assumed in the form:

$$\delta c_{20} = \lambda_t^c \cos \nu_t + \lambda_t^s \sin \nu_t + f(a(\nu_i, \xi_i, \alpha_i), \psi(\vartheta_j, \eta_j, \beta_j)),$$
  
$$f(a, \psi) = \varepsilon a \int \frac{\mu_t \sin(\psi - \nu_t)}{a} dt.$$
 (2)

Here f is the function of amplitude and phase of the pole oscillation; dimensional coefficient  $\varepsilon$  is refined from SLR observations and has the order  $10^{-3}$  (the amplitude a of the Earth pole oscillations is expressed in angular milliseconds).



Figure 1: a) Interpolation of the variation of the second zonal harmonic  $\delta c_{20}$  of the geopotential from 1984 till 2008 and extrapolation for six years (2009–2014). b) A comparison of the function  $f(a, \psi)$  from the amplitude and phase of the Earth pole oscillations (upper curve) that is built directly from the IERS observation data from 1980 till 2008 with the corresponding component divided out of the observations of the second zonal harmonic  $c_{20}$  of the geopotential (lower curve).

Fig. 1 a) shows the interpolation of expression (2) using the observation data of the SLR from 1984 till 2008 inclusive and extrapolation for six years with the forecast for two years. The contrast line on the figure is obtained theoretical curve, the connected by line asterisks are the measurements data. The component with oscillations corresponded to expansion of function  $f(a, \psi)$  for harmonic components is divided out during interpolation. The observed curve is shown on fig. 1 b) with comparison of function  $f(a, \psi)$  that built directly from observations of the Earth pole oscillations.

## 3. REFERENCES

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