

# CONSTRUCTION OF THE NEW HIGH-PRECISION EARTH ROTATION SERIES AT LONG TIME INTERVALS

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**ABSTRACT.** In the previous investigation (Pashkevich, 2013) the high-precision Rigid Earth Rotation Series (designated RERS2012) dynamically adequate to the JPL DE406/LE406 (Standish, 1998) ephemeris over 2000 and 6000 years were constructed. The main aim of present research is improvement of the Rigid Earth Rotation Series RERS2012 by using the JPL DE422/LE422 (Folkner, 2011) ephemeris, and as a result of the construction of the new high-precision Rigid Earth Rotation Series dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years. The discrepancies between the high-precision numerical solutions and the semi-analytical solutions of the rigid Earth rotation problem with respect to the fixed ecliptic of epoch J2000 are investigated by the least-squares method and by the spectral analysis methods. The problem is solved only for the relativistic (Kinematical) case in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account.

## 1. INTRODUCTION

The present research is the continuation of the investigation of the rigid Earth rotation at long time intervals (Pashkevich, 2013). The purposes of this studies are construction of the new high-precision Rigid Earth Rotation Series RERS2013 dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years and comparison of new solution RERS2013 with the previous solution RERS2012 (Pashkevich, 2013). The dynamics of the rotational motion of the rigid Earth is studied numerically by using Rodrigues-Hamilton parameters over 2000 and 6000 years. The numerical solution of the problem is obtained by solving the Lagrange differential equations of the second kind for the rigid Earth rotation with respect to the fixed ecliptic and equinox of epoch J2000 (Pashkevich, 2013). The orbital motions of the disturbing celestial bodies are defined by the DE422/LE422 ephemeris. These investigation is carried out for the relativistic (Kinematical) case, in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account. The mathematical model of the problem is described in detail in the paper (Pashkevich, 2013).

## 2. ALGORITHMS AND RESULTS

The results of the numerical solutions of the problem are compared with the semi-analytical solutions of the rigid Earth rotation RERS2012. The residuals of these comparison are studied by means the iterative algorithm:

1. Numerical solution of the rigid Earth rotation is implemented with the quadruple precision of calculations. The initial conditions are computed by the semi-analytical solution of the rigid Earth rotation (RERS2012). Discrepancies between the numerical solution and the semi-analytical solution are obtained in Euler angles over all investigation time interval with one-day spacing (presented in Figure 1). The expressions for these discrepancies are as follows

$$\left. \begin{aligned} \Delta\psi &= \sum_{k=0}^6 \psi_k t^k + \sum_j \sum_{k=0}^4 [\psi_{Sjk} \sin(\nu_{j0} + \nu_{j1}t) + \psi_{Cjk} \cos(\nu_{j0} + \nu_{j1}t)] t^k \\ \Delta\theta &= \sum_{k=0}^6 \theta_k t^k + \sum_j \sum_{k=0}^4 [\theta_{Sjk} \sin(\nu_{j0} + \nu_{j1}t) + \theta_{Cjk} \cos(\nu_{j0} + \nu_{j1}t)] t^k \\ \Delta\phi &= \sum_{k=0}^6 \phi_k t^k + \sum_j \sum_{k=0}^4 [\phi_{Sjk} \sin(\nu_{j0} + \nu_{j1}t) + \phi_{Cjk} \cos(\nu_{j0} + \nu_{j1}t)] t^k \end{aligned} \right\}, \quad (1)$$

where  $\psi$  is the longitude of the ascending node of the Earth's dynamical equator on the fixed ecliptic J2000;  $\theta$  is the angle of the inclination of the Earth's dynamical equator to the fixed ecliptic J2000;  $\phi$  is the proper rotation angle of the Earth between the ascending node of the Earth's dynamical equator and the principal axis of the minimum moment of inertia;  $\nu_{j0}, \nu_{j1}$  are the phases and the frequencies of the corresponding semi-analytical solutions, respectively;  $j = 1, \dots, 4113$ ;  $t$  is the time in the Julian days;  $\psi_k, \theta_k, \phi_k$  are the coefficients of the secular terms;  $\psi_{Sjk}, \theta_{Sjk}, \phi_{Sjk}, \psi_{Cjk}, \theta_{Cjk}, \phi_{Cjk}$  are the coefficients of the periodic and Poisson terms.

2. Investigation of the discrepancies is carried out by the least squares method and by the spectral analysis method (Pashkevich and Eroshkin, 2005). The set of the frequencies of the semi-analytical solution is used without change. Only the coefficients of the periodical terms and the coefficients of the Poisson terms are improved. The secular, periodic and Poisson terms representing the new high-precision rigid Earth rotation series RERS2013<sub>*i*</sub> (where *i* is the number of iteration) are determined:

$$\left. \begin{aligned} \psi_{\text{RERS2013}_i} &= \Delta\psi_{i-1} + \psi_{\text{RERS2013}_{i-1}} \\ \theta_{\text{RERS2013}_i} &= \Delta\theta_{i-1} + \theta_{\text{RERS2013}_{i-1}} \\ \phi_{\text{RERS2013}_i} &= \Delta\phi_{i-1} + \phi_{\text{RERS2013}_{i-1}} \end{aligned} \right\}, \quad (2)$$

where  $\psi_{\text{RERS2013}_0} = \psi_{\text{RERS2012}}$ ,  $\theta_{\text{RERS2013}_0} = \theta_{\text{RERS2012}}$  and  $\phi_{\text{RERS2013}_0} = \phi_{\text{RERS2012}}$ .

3. Numerical solution of the rigid Earth rotation is constructed anew with the new initial conditions, which are calculated by RERS2013<sub>*i*</sub>.

4. Steps 2 and 3 are repeated till the assumed convergence level has been achieved.

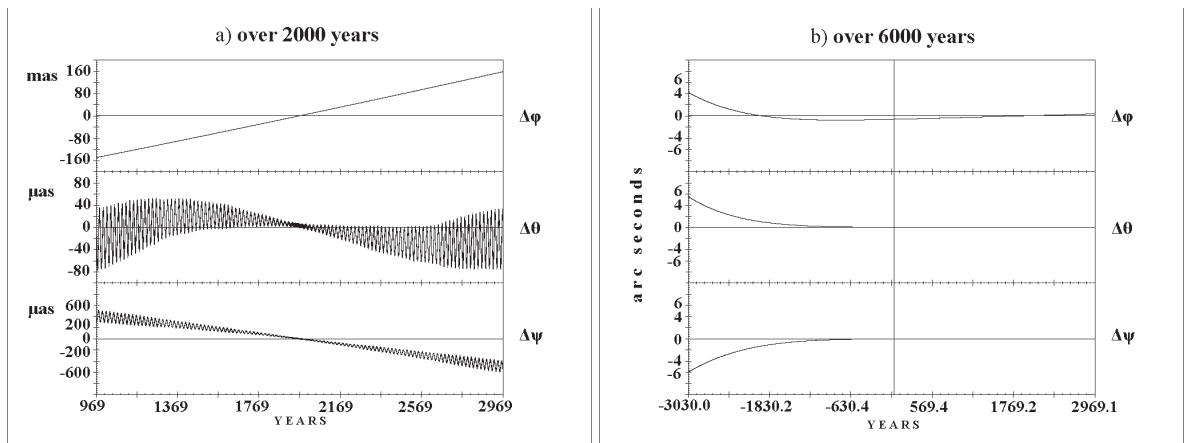


Figure 1: Discrepancies between the numerical and RERS2012 semi-analytical solutions of the Earth rotation (dynamically adequate to the DE406/LE406 ephemeris)

At first this investigation is carried out on 2000 years time interval. In Figure 1a the discrepancies are depicted between the numerical and RERS2012 over 2000 years. The secular trend does not surpass 600  $\mu\text{as}$  over 2000 years for  $\psi$  and 160 mas over 2000 years for  $\phi$ . The behavior of  $\Delta\theta$  residuals are only periodic character and do not exceed 80  $\mu\text{as}$  over 2000 years. The convergence level was achieved after

application of the third iteration of the iterative algorithm. So, the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series (RERS2013) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 2000 years. The discrepancies between the new numerical solutions and the semi-analytical solutions of RERS2013 do not surpass  $4 \mu\text{as}$  over 2000 year time interval (presented in Figure 2).

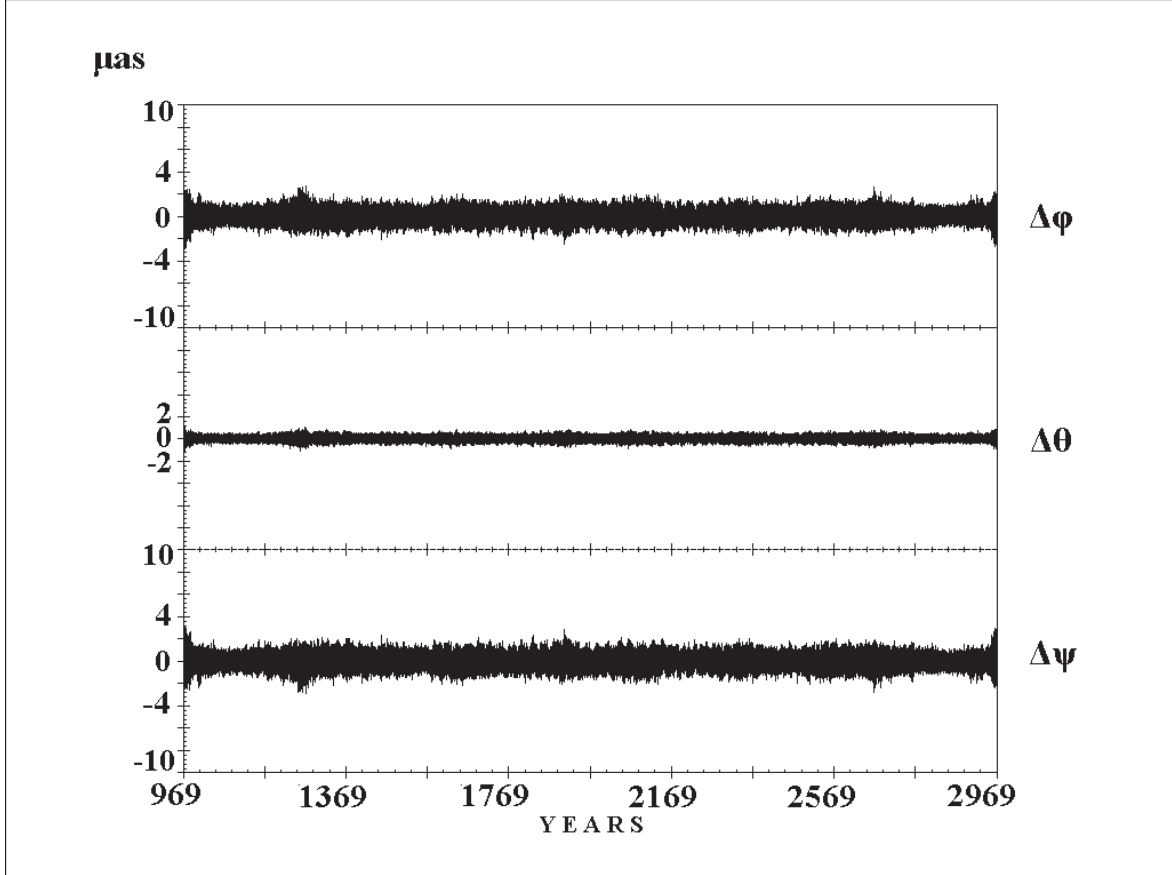


Figure 2: Discrepancies between the numerical and RERS2013 semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 2000 years after applied 3rd iterations of the iterative algorithm

This investigation is finished at 6000 years time interval. In Figure 1b the discrepancies are depicted between the numerical and RERS2012 over 6000 years. The secular trend in all Euler angles does not surpass 6 arc seconds over 6000 years. After application of the third iteration of the iterative algorithm, the convergence level was achieved and the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series (RERS2013) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 6000 years. The discrepancies between the new numerical solutions and the semi-analytical solutions of RERS2013 do not surpass  $1 \text{ mas}$  over 6000 year time interval (presented in Figure 3).

Thus, the result of the comparison on 2000 and 6000 years demonstrates a good consistency of RERS2013 series with the DE422/LE422 ephemeris.

### 3. CONCLUSION

As the results of this investigation, the new improved high-precision Rigid Earth Rotation Series RERS2013 dynamically adequate to the DE422/LE422 ephemeris over 2000 and 6000 years have been constructed. The series RERS2013 include about 4113 periodical and Poisson terms terms (without attempt to estimate new sub-diurnal and diurnal periodical and Poisson terms). The sub-diurnal and diurnal periodical and Poisson terms have not been investigated in this study. Therefore, they entered

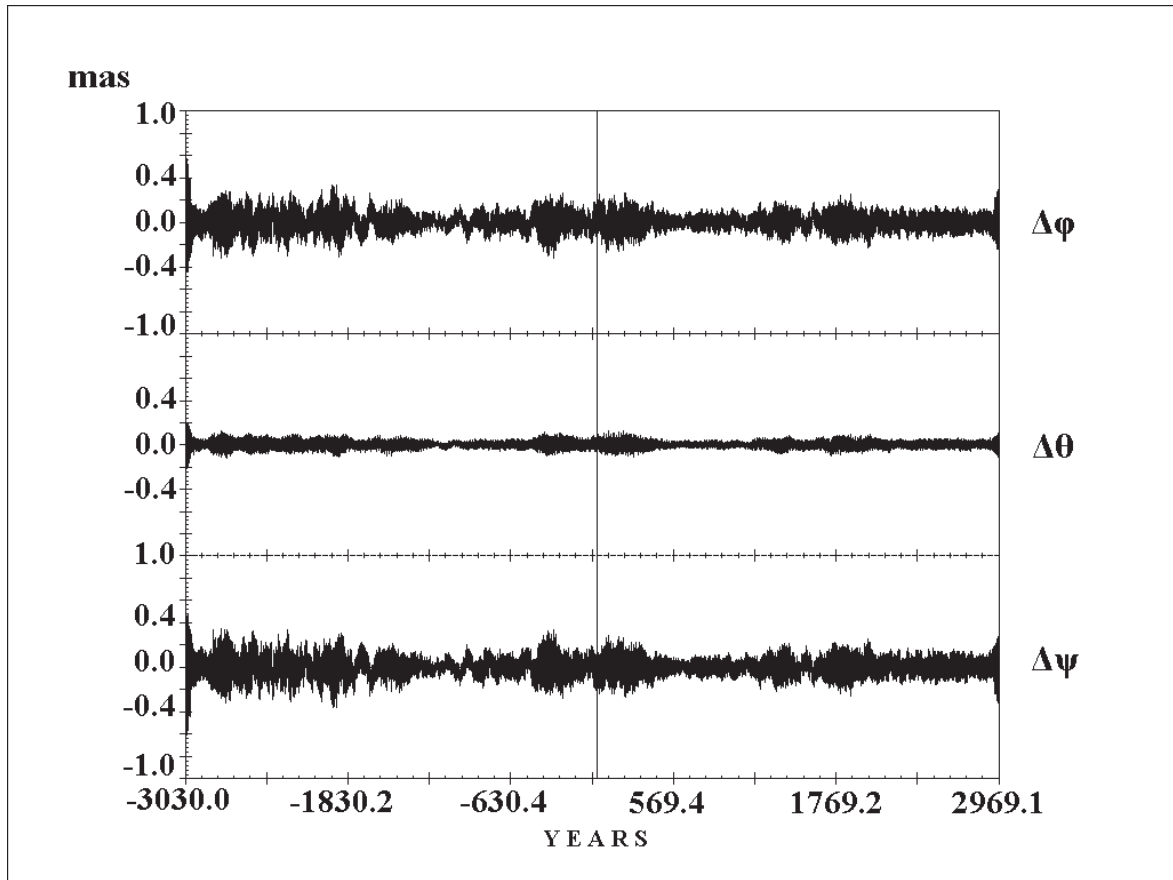


Figure 3: Discrepancies between the numerical and RERS2013 semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 6000 years after applied 3rd iterations of the iterative algorithm

into new solutions RERS2013 without change from RERS2012. The discrepancies between the numerical solution and RERS2013 do not surpass:  $4 \mu\text{as}$  over 2000 years, 1 mas over 6000 years. It means a good consistency of the RERS2013 series with the DE422/LE422 ephemeris. The RERS2013 series is more accurate than the RERS2012 series, which is dynamically adequate to the DE406/LE406 ephemeris.

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