ABERRATION IN PROPER MOTIONS FOR GALACTIC STARS

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ABSTRACT. Accelerations of both the solar system barycenter (SSB) and stars in the Milky Way cause a systematic observational effect on the stellar proper motions, which was first studied by J. Kovalevsky (2003). This paper intends to extend that work and aims to estimate the magnitude and significance of the aberration in proper motions of stars, especially in the region near the Galactic center (GC). We adopt two models for the Galactic rotation curve to evaluate the aberrational effect on the Galactic plane. We show that the effect of aberration in proper motions depends on the galactocentric distance of stars; it is dominated by the acceleration of stars in the central region of the Galaxy. Then we investigate the applicability of the theoretical expressions: if the orbital period of stars is only a fraction of the light time from the star to the SSB, the expression with approximation proposed by Kovalevsky is not appropriate. With a more suitable formulation, we found that the aberration has no effect on the determination of the stellar orbits on the celestial sphere. In the future this aberrational effect under consideration should be considered with high-accurate astrometry, particularly in constructing the Gaia celestial reference system realized by Galactic stars.

1. INTRODUCTION

It is well known that the velocity of the solar system barycenter (SSB) is responsible for the first order aberration in position of about 150″, however this value is a constant and not detectable. In addition, the acceleration of an observer produce aberrational effect in proper motions of celestial objects (Kovalevsky 2003; Kopeikin & Makarov 2006), which is a variational effect with respect to the aberration in position. Given astrometric measurements at micro-arcsecond level, the aberration in proper motions resulting from the acceleration of the SSB has impact on the celestial reference system realized by extragalactic radio sources (ICRS) and Earth rotation parameters, thus should be considered in the near future (Titov 2010; Liu et al. 2012).

In our Galaxy, the aberration in proper motions for stars can be written as the first time derivative of the aberration in positions:

$$\Delta\dot{\boldsymbol{p}}^{\mathrm{S}} = \frac{1}{c}\boldsymbol{p}\times[(\boldsymbol{a}^{\mathrm{B}}-\boldsymbol{a}^{\mathrm{S}})\times\boldsymbol{p}], \tag{1}$$

where $\boldsymbol{p}$ is the position vector of the star, and the superscript 'B' and 'S' represent the SSB and star, respectively. The proper motion (independent of distance of stars) resulting from $\boldsymbol{a}^{\mathrm{B}}$ is the same as the effect for extragalactic sources, which forms a dipolar field on the celestial sphere from the anti-Galactic center to the Galactic center (GC). The second part corresponding to the acceleration $\boldsymbol{a}^{\mathrm{S}}$ is more complicated and will be discussed in detail in the following.

Projecting Eq. (1) to the local tangential coordinate system $(x,\ y,\ z)$ in the Galactic coordinate system, the aberrational proper motions in longitude and latitude directions can be derived as follows:

$$\Delta\mu_\ell\cos b = \frac{1}{c}\boldsymbol{p}\times[(\boldsymbol{a}^{\mathrm{B}}-\boldsymbol{a}^{\mathrm{S}})\times\boldsymbol{p}]\cdot\boldsymbol{e}_\ell\ ;\qquad \Delta\mu_b = \frac{1}{c}\boldsymbol{p}\times[(\boldsymbol{a}^{\mathrm{B}}-\boldsymbol{a}^{\mathrm{S}})\times\boldsymbol{p}]\cdot\boldsymbol{e}_b, \tag{2}$$

where $\boldsymbol{e}_\ell$, $\boldsymbol{e}_b$ are the unit vectors in the direction of increasing longitude and latitude.

For conciseness, we define following parameters in the unit of proper motions:

$$A^{\mathrm{B}} = \frac{a^{\mathrm{B}}}{c} = \frac{V_0^2}{cR_0} \simeq 5\,\mu\mathrm{as\,yr}^{-1}\ ;\qquad A^{\mathrm{S}} = \frac{a^{\mathrm{S}}}{c} = \frac{a^{\mathrm{B}}}{c}\frac{a^{\mathrm{S}}}{a^{\mathrm{B}}} = A^{\mathrm{B}}\frac{a^{\mathrm{S}}}{a^{\mathrm{B}}} = \gamma A^{\mathrm{B}}, \tag{3}$$

where the former quantity A^{B} (constant) corresponding to the SSB was called the 'Galactic aberration constant' in Malkin (2011) and the parameter γ is the ratio of the accelerations of the star and SSB.

Using the above formulas and definitions, the proper motions in Eq. (2) can be written as:

$$\Delta\mu_\ell \cos b = A^{\mathrm{B}} \boldsymbol{p} \times [(\boldsymbol{e}_{X_{\mathrm{G}}} - \gamma\boldsymbol{\rho}) \times \boldsymbol{p}] \cdot \boldsymbol{e}_\ell \ ; \qquad \Delta\mu_b = A^{\mathrm{B}} \boldsymbol{p} \times [(\boldsymbol{e}_{X_{\mathrm{G}}} - \gamma\boldsymbol{\rho}) \times \boldsymbol{p}] \cdot \boldsymbol{e}_b, \tag{4}$$

in which $\boldsymbol{e}_{X_{\mathrm{G}}}$ and $\boldsymbol{\rho}$ are the unit vectors in the direction of $\boldsymbol{a}^{\mathrm{B}}$ and $\boldsymbol{a}^{\mathrm{S}}$, respectively; both pointing to the GC; R_0 and V_0 are the Galactocentric distance and rotation velocity of the SSB. For extragalactic radio sources whose accelerations are zero or too small to be detected (i.e. $\gamma = 0$), Eq. (4) degenerates into the form of the pure dipolar proper motion field.

2. ABERRATION IN PROPER MOTIONS BASED ON ROTATION CURVES

In order to evaluate the magnitude of aberrational proper motions, it is necessary to know the accelerations of stars, or equivalently the parameter γ. Since accelerations of stars are not available one by one, certain statistical models for the Galactic kinematics, such as rotation curves, are necessarily be used. For stars in the Galaxy, especially for those on the Galactic disk, every star revolves more or less around the GC. One simplified case is such that the rotation curve is completely flat throughout the Galaxy (see Fig. 1a). In this case, $\gamma = R_0/d$ (d is the distance from the star to the GC.), so that we have

$$\Delta\mu_\ell \cos b = -A^{\mathrm{B}} \left[1 - \left(\frac{R_0}{d}\right)^2\right] \sin\ell \ ; \qquad \Delta\mu_b = -A^{\mathrm{B}} \left[1 - \left(\frac{R_0}{d}\right)^2\right] \cos\ell \sin b. \tag{5}$$

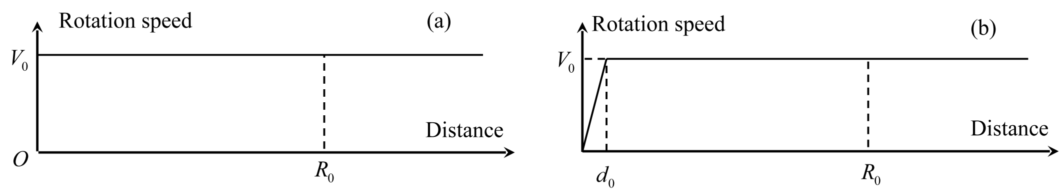

Figure 1: The simplified rotation curves of the Galaxy. The origin of both plots is the Galactic center. (a) Flat rotation curve at V_0; (b) linear rotation curve up to d_0 and then constant from d_0 at V_0.

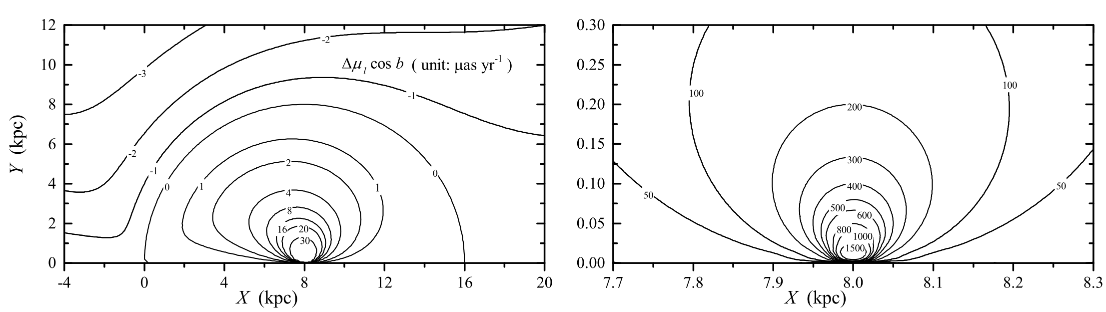

Figure 2: The amplitude of aberration in proper motions on the Galactic plane ($b = 0$, Eq. 5) corresponding to the flat rotation curve in Fig. 2(a). The Galactic center is located at (8.0, 0) and the SSB is at the (0, 0) point. The left plot covers almost the whole Galactic plane, while the right plot is the enlarged drawing for the vicinity of the Galactic center.

In the brackets of the above expressions, the second terms are inversely proportional to the squared distance of the star to the Galactic center. This means that the proper motions increase as the stars are closer to the Galactic center. Figure 2 shows the contour plot of $\Delta\mu_\ell \cos b$ for stars on the Galactic plane. The left panel is for a wider range up to 12 kpc in X and Y directions centered on the Galactic center, and the right panel is enhancement around the center up to 300 pc.

In contrast to the flat rotation curve, a more reasonable approximation of the rotation curve is the one as shown in Fig. 1b, which separate the bulge from the disk at the boundary $d = d_0$. Out of the bulge, the rotation curve is almost flat. The expression of the rigid rotation acceleration of stars within the boundary d_0 is $\gamma = R_0 d/d_0^2$. It is proportional to the galactocentric distance of the star (d), while in the previous case it is in inverse proportion to d. Then we obtain the resulting aberrational proper motions in Galactic longitude and latitude:

$$\Delta\mu_\ell \cos b = -A^{\mathrm{B}} \left[1 - \left(\frac{R_0}{d_0}\right)^2\right] \sin\ell \ ; \qquad \Delta\mu_b = -A^{\mathrm{B}} \left[1 - \left(\frac{R_0}{d_0}\right)^2\right] \cos\ell \sin b \ \ (d < d_0). \tag{6}$$

The term $1 - (R_0/d_0)^2$ in brackets is a constant of an order of 1000 (if $R_0 = 8.0$ kpc and $d_0 = 0.3$ kpc), consequently the proper motions are independent of distance of stars in the rigid rotation mode. Adopting $d_0 = 0.3\,\mathrm{kpc}$, the magnitude of proper motions are shown in Fig. 3 right panel. In this area, the largest proper motion is only about $150\,\mu\mathrm{as\,yr}^{-1}$ at $X = 8.0\,\mathrm{kpc}$ and $Y = d_0$.

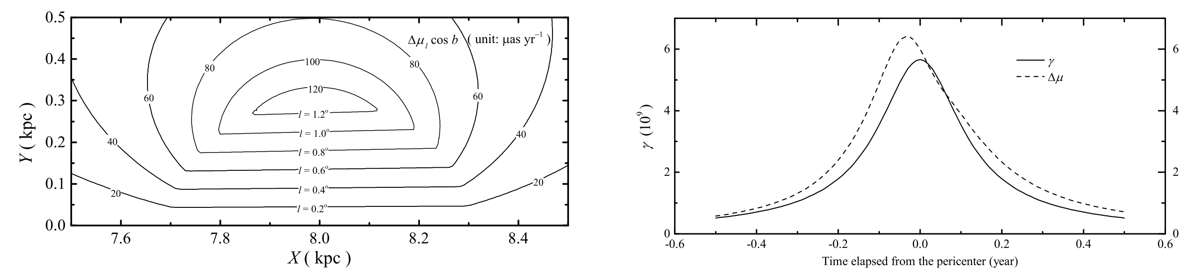

Figure 3: Left: The amplitude of aberration in proper motions on the Galactic plane ($\ell = 0$) corresponding to the rotation curve in Fig. 1b. Right: The parameter γ and the aberrational proper motions for the S0-2 star near the pericenter of the orbit. $\Delta\mu$ represents the general proper motion.

3. PROPER USE OF THE EXPRESSIONS

Conceptually, the aberration in proper motions resulting from the stellar acceleration $\boldsymbol{a}^{\mathrm{S}}$ is the variation of projected velocity on the celestial sphere during the light time from the star to the observer (SSB). Written in Eq. (1), the acceleration of the star $\boldsymbol{a}^{\mathrm{S}}$ should be a constant vector, which means that the motion of the star must be (or approximately) rectilinear during the time span of light travel. To this end, we note that Eq. (1) and follow-up expressions that describe the spurious proper motions are simplified, and should be used with caution, especially for short periodic stars.

A typical example is the S0-2 star in the central cluster of our Galaxy, the well determined orbital solutions of which has been provided by Schödel et al. (2003). We have calculated the expressions for the aberration in proper motions in the equatorial coordinate system using Eq. (1) and found

$$\Delta\mu_\alpha \cos\delta \simeq \gamma A^{\mathrm{B}} \frac{\Delta x}{d} \ ; \qquad \Delta\mu_\delta \simeq \gamma A^{\mathrm{B}} \frac{\Delta y}{d}, \tag{7}$$

where Δx and Δy is the coordinates of S0-2 in the tangential coordinate system centered on GC. Shown in Fig. 3 right panel, the parameter γ for S0-2 is on the order of $10^7 - 10^9$, and the proper motions can be up to several degrees per year. This appears unrealistic because the orbital period of S0-2 is only about 15 yr, while the light time from S0-2 to the SSB is about $T_{\mathrm{L}} \simeq 8000\,\mathrm{pc} \times 3.26\ \mathrm{yr\,pc}^{-1} = 26000\,\mathrm{yr}$, which is about 1700 times larger than P ($\tau \simeq 1/1700$). The calculation using Eq. (1) extrapolates the acceleration at the staring point to the whole time span as if the acceleration was a constant, and this inappropriate procedure causes accumulated high proper motion corrections.

For short periodic stars, after a time span covering integer multiple of orbital periods, the star moves back to the same point on the orbit which means that its velocity is the same as it is at the beginning of that time span, and the corresponding effect of aberration resulting from the acceleration of the star is zero. Only orbital period fraction (P_{f} = remainder of T_{L} divided by P) within the light time is responsible for effective aberration in proper motions, and Eq. (1) should be written as a more suitable form:

$$\Delta\dot{\boldsymbol{p}}^{\mathrm{S}} = \frac{1}{c}\boldsymbol{p} \times \left[\left(\boldsymbol{a}^{\mathrm{B}} - \frac{\boldsymbol{v}_2^{\mathrm{S}} - \boldsymbol{v}_1^{\mathrm{S}}}{T_{\mathrm{L}}}\right) \times \boldsymbol{p}\right], \tag{8}$$

where $\boldsymbol{v}_1^{\rm S}$ and $\boldsymbol{v}_2^{\rm S}$ are the velocities of the star on its orbit at the beginning (t_1) and the end (t_2) of the effective fraction of the period, respectively. To calculate the value for Eq. (8), it would require that the accuracy of the light time be measured to an accuracy at least better than $P_{\rm f}$.

Note that most of the short periodic stars are near the Galactic center, where the influence caused by the acceleration of the SSB can be ignored, we only consider the effect resulting from the motion of the star. Projecting Eq. (8) on the celestial sphere, we obtain the aberrational proper motions in the equatorial coordinate system:

$$\Delta\mu_\alpha \cos\delta = \frac{1}{\kappa}\frac{v_{2,x} - v_{1,x}}{r}, \quad \Delta\mu_\delta = \frac{1}{\kappa}\frac{v_{2,y} - v_{1,y}}{r}, \tag{9}$$

where the subscript x, y means that the velocity vectors are decomposed in the tangential coordinate system that is established by ($\boldsymbol{e}_\alpha$, $\boldsymbol{e}_\delta$, $\boldsymbol{e}_r$) triad, and $\kappa = 4.74047$ is a constant factor for unit transformation if proper motions are in unit of μas yr^{-1}, velocities in km s^{-1}, and r in pc. Because the true proper motions of the star at t_1 are such that:

$$[\Delta\mu_\alpha \cos\delta]_1^{\rm true} = \frac{1}{\kappa}\frac{v_{1,x}}{r}, \quad [\Delta\mu_\delta]_1^{\rm true} = \frac{1}{\kappa}\frac{v_{1,y}}{r}, \tag{10}$$

we have the observed proper motions at t_1 by adding the corrections in Eq. (9):

$$[\Delta\mu_\alpha \cos\delta]_1^{\rm obs} = \frac{1}{\kappa}\frac{v_{2,x}}{r} = [\Delta\mu_\alpha \cos\delta]_2^{\rm true} \ ; \quad [\Delta\mu_\delta]_1^{\rm obs} = \frac{1}{\kappa}\frac{v_{2,y}}{r} = [\Delta\mu_\delta]_2^{\rm true} . \tag{11}$$

This shows that the effect of aberration in proper motions only changes the observed phase of the star on the stellar orbit. It does not change the shape of the orbit on the celestial sphere, although we can not measure the exact value of the correction.

4. DISCUSSION AND CONCLUSION

In this paper we have improved the results of Kovalevsky (2003) to a more concise form. Two kinds of rotation curves of the Galactic disk were adopted to examine the property of the aberrational proper motions, especially in the vicinity of the Galactic center. A flat rotation curve starting from the Galactic center leads to enlargement of the proper motions to 1000 μas yr^{-1} at $d = 0.2$ kpc, while the more realistic rotation curve rising linearly from the Galactic center to the bulge-disk boundary gives limited proper motions up to about 150 μas yr^{-1}. If the period of the stellar orbit is shorter than the light time from the star to the observer, the assumption of constant acceleration in this period of time does not hold. In this circumstances, one need more basic expression as written in Eq. (8). The magnitudes of the aberration in proper motions are difficult to measure, however we have shown that there is no effect on determining the orbit of stars.

Because the amplitudes of the systematic proper motions is at some places much larger than the Gaia accuracy for the proper motion measurements, this effect should be considered to eliminate the rotation and distortion in the future Gaia celestial reference system realized by stars in optical bandpass. However, this would be possible only if the accelerations of stars are known with satisfactory precision, or we have more reliable kinematics of the Galaxy for modeling those accelerations.

5. REFERENCES

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