APPROXIMATION OF ORBITAL ELEMENTS OF THE TELLURIC PLANETS BY COMPACT ANALYTICAL SERIES

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ABSTRACT. We take the long-term numerical ephemeris of the major planets DE424 (Folkner 2011) and approximate the orbital elements of the telluric planets from that ephemeris by trigonometric series. Amplitudes of the series' terms are the second- or third-degree polynomials of time, and arguments are the fourth-degree time polynomials. The resulting series are precise and compact; in particular the maximum deviation of the planetary mean longitude calculated by the analytical series from that given by DE-424 over [-3000; 3000], the total time span covered by the numerical ephemeris, is:

- for Mercury: 0.0016 arcsec (the series includes 767 terms);
- for Venus: 0.015 arcsec (648 terms);
- for the Earth-Moon barycenter: 0.019 arcsec (535 terms);
- for Mars: 0.056 arcsec (770 terms).

1. DEVELOPMENT METHOD

In order to approximate the orbital elements of the telluric planets by analytical series we used an author's modification of the spectral analysis method (Kudryavtsev 2004, 2007). According to this modification, the expansion of an arbitrary tabulated function F of time t is directly made to trigonometric series, where both arguments and amplitudes of the series' terms are high-degree polynomials of time. Therefore, the approximating function $f(t)$ has the following form

$$f(t) = \sum_{k=0}^{N} \Big[\big(A^c_{k0} + A^c_{k1}t + \cdots + A^c_{kh}t^h\big)\cos\omega_k(t) + \big(A^s_{k0} + A^s_{k1}t + \cdots + A^s_{kh}t^h\big)\sin\omega_k(t) \Big], \tag{1}$$

where $\omega_k(t)$ are some pre-defined arguments which are assumed to be q-degree polynomials of time

$$\omega_0(t) \equiv 0, \qquad \omega_k(t) = \nu_k t + \nu_{k2}t^2 + \cdots + \nu_{kq}t^q \quad \text{if } k > 0, \tag{2}$$

$A^c_{k0}, \cdots, A^s_{kh}, \nu_k, \cdots, \nu_{kq}$ are constants, and N is the number of terms in the expansion.

In the present study the polynomial arguments are various combinations of multipliers of the planetary mean mean longitudes, where the latter are defined by Simon et al. (1994). In order to obtain such an expansion, we first find the projections of $F(t)$ on a basis generated by the functions

$$\mathbf{c}_{kl}(t) \equiv t^l \cos\omega_k(t), \quad \mathbf{s}_{kl}(t) \equiv t^l \sin\omega_k(t); \quad k = 0, 1, \cdots, N; \; l = 0, 1, \cdots, h$$

through numerical computation of the following scalar products over a time span of $[-T; T]$

$$A^c_{kl} = < F, \mathbf{c}_{kl} > \equiv \frac{1}{2T}\int_{-T}^{T} F(t)t^l \cos\omega_k(t)\chi(t)\,dt, \tag{3}$$

$$A^s_{kl} = < F, \mathbf{s}_{kl} > \equiv \frac{1}{2T}\int_{-T}^{T} F(t)t^l \sin\omega_k(t)\chi(t)\,dt \tag{4}$$

where $\chi(t) = 1 + \cos\frac{\pi}{T}t$ is the Hanning filter chosen as the weight function.

However, the basis functions $\mathbf{c}_{kl}$, $\mathbf{s}_{kl}$ are usually not orthogonal. Therefore, at the second step we perform an orthogonalization process over the expansion coefficients in order to improve the quality of representation and avoid superfluous terms; the details are available in (Kudryavtsev 2004, 2007).

2. RESULTS

Following the above described procedure we have approximated Keplerian orbital elements of Mercury and mean orbital longitudes of all telluric planets by compact trigonometric series, where amplitudes are either the second- or third-degree polynomials of time, and arguments are the forth-degree time polynomials. The long-term numerical ephemeris DE424 (Folkner 2011) is used as a source. It covers the time span of [-3000; 3000]. The expansion is done over the complete time span covered by the ephemeris; the results are presented in Tables 1–2. The analytical series are denoted as DEA-424 there. Table 2 also includes the comparative characteristics of the VSOP2013 solution (Simon et al., 2013), the latest and most accurate analytical theory of motion of the telluric planets to date.

Keplerian element	Number of terms in DEA424, N	Maximum difference DEA424-DE424
a	479	2.1×10^{-9} AU
e	623	7.5×10^{-9}
i	282	0.0004"
Ω	569	0.0013"
π	900	0.0032"
λ	767	0.0016"

Table 1: Maximum difference between Keplerian orbital elements of Mercury given by the analytical series DEA424 and numerical ephemeris DE424 over the time span of [-3000; 3000].

Planet	DEA424,	DEA424-DE424		VSOP2013,	VSOP2013-INPOP10a(ext.)[1]	
	N	[1890; 2000]	[-3000; 3000]	Num. of terms[2]	[1890; 2000][3]	[900; 3100][4]
Mercury	767	0.0008"	0.0016"	272360	0.00003"	0.01"
Venus	648	0.0032"	0.015"	289647	0.00002"	0.002"
EMB	535	0.0046"	0.019"	294426	0.00001"	0.03"
Mars	770	0.0095"	0.056"	309140	0.00074"	0.70"

Table 2: Maximum difference between the mean longitudes of the telluric planets given by the analytical series DEA424, VSOP2013 and numerical ephemerides DE424, INPOP10a(ext.) over various time spans.
Notes:
[1] Hereafter INPOP10a(ext.) denotes the original planetary ephemeris INPOP10a by Fienga et al. (2011) extended by Manche (2012) over the time span of [-4000; 8000];
[2] The data are taken from Simon et al. (2013), Table 7;
[3] The data are taken from Simon et al. (2013), Table 8;
[4] The data are taken from Simon et al. (2013), Table 11.

While the work on developing the planetary ephemeris DE424 to analytical series was in progress, a new planetary ephemeris DE430 (Folkner, 2013) and its extension DE431 were released. The planetary ephemeris DE431 is valid over [-13000; 17000], an essentially longer time span than that of any numerical ephemeris developed before. Table 3 presents our first results in expansion of the mean longitude of Mercury from this extra long-term ephemeris to analytical series over the total time span of 30,000 years covered by DE431. The analytical development of the planetary ephemeris DE431 is denoted as DEA431 there. Table 3 also contains the comparison of the development characteristics with similar data provided by the analytical theory VSOP2013.

Difference in the mean longitude of Mercury	Time span		
	[0; 4000]	[-4000; 8000]	[-13000; 17000]
DEA431-DE431	0.0016"	0.0020"	0.030"
VSOP2013-INPOP10a(ext.)[1]	0.03"	0.12"	n/a

Table 3: Maximum difference between the mean longitude of Mercury given by the analytical series DEA431, VSOP2013 and numerical ephemerides DE431, INPOP10a(ext.) over various time spans.
Note [1] The data are taken from Simon et al. (2013), Table 11.

In Table 3 the number of terms N in series (1) used by the DE431 solution for calculating the mean longitude of Mercury is 909. The maximum degree h of polynomials for amplitudes of terms in these series is four.

Acknowledgements. Research supported in part by the Russian Foundation for Basic Research. A travel grant provided to the author by the LOC of the Journées 2013 and the French Ministry of Education and Research in the framework of the programme ACCES* is sincerely appreciated.

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