# CORRECTIONS TO TIDAL VARIATIONS OF THE GEOPOTENTIAL DUE TO FREQUENCY DEPENDENCE OF LOVE NUMBERS

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## EXTENDED ABSTRACT[1]

[1] The research details are available in Kudryavtsev (2013)

The main effect of the solid Earth tides on the Earth gravitational potential (defined as "Step 1") can be described through variations $\Delta\bar{C}_{nm}^{ST}$, $\Delta\bar{S}_{nm}^{ST}$ in the instant values of the normalized standard geopotential coefficients of degree $n$ and order $m$ (Eanes et al. 1983)

$$\Delta\bar{C}_{nm}^{ST} - i\Delta\bar{S}_{nm}^{ST} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{\mu_j}{\mu_E}\left(\frac{R_E}{r_j}\right)^{n+1}\bar{P}_{nm}\left(\sin\phi_j\right)e^{-im\lambda_j}, \quad (1)$$

where $i \equiv \sqrt{-1}$; $k_{nm}$ are frequency-independent complex Love numbers; $R_E$, $\mu_E$ are, respectively, the Earth's equatorial radius and gravitational parameter; $\mu_j$, $r_j$, $\phi_j$ and $\lambda_j$ are, respectively, the gravitational parameter, geocentric distance, geocentric latitude and East longitude (from Greenwich) of the Moon ($j = 2$) and Sun ($j = 3$) at epoch $t$; $\bar{P}_{nm}$ is the normalized associated Legendre functions.

Anelasticity of the Earth's mantle leads to frequency dependence of Love numbers. Therefore, as "Step 2", the IERS Conventions (2010) (Petit & Luzum, 2010) recommend to calculate some additional tidal corrections to the gravitational coefficients due to deviations $\delta k_{2mf}^R$, $\delta k_{2mf}^I$ of the degree 2 complex Love numbers at frequency $f$ from their nominal values. The presently recommended by Petit & Luzum (2010) formulae use the tide height values from the Earth tide-generating potential (TGP) expansion by Cartwright and Tayler (1971), Cartwright and Edden (1973). However, tide heights are not included to the HW95 format (Hartmann and Wenzel, 1995), which is considered as a common standard for TGP catalogues now. The known factors for conversion of amplitudes of tidal terms from Hartmann and Wenzel's conventions to Cartwright-Tayler-Edden's ones (Petit & Luzum, 2010) do not take into account the phase of tidal waves. The latter is zero for all tidal terms in the TGP catalog by Cartwright and Tayler (1971), Cartwright and Edden (1973), but it is not the case in the modern TGP catalogs. In particular, the phase of some waves used in computing the tidal corrections at "Step 2" is not zero there.

The Earth TGP development in the HW95 format represents the TGP value at a surface point $P$ as

$$V(t) = \sum_{n=2}^{n_{max}}\left(\frac{r}{R_E}\right)^n\sum_{m=0}^{n}\bar{P}_{nm}\left(\sin\varphi'\right)\sum_{f(n,m)}\left[\bar{C}_{nmf}^*(t)\cos\omega_{nmf}^*(t) + \bar{S}_{nmf}^*(t)\sin\omega_{nmf}^*(t)\right], \quad (2)$$

where

$$\bar{C}_{nmf}^*(t) = \bar{C}_{0nmf}^* + \bar{C}_{1nmf}^* t, \qquad \bar{S}_{nmf}^*(t) = \bar{S}_{0nmf}^* + \bar{S}_{1nmf}^* t, \quad (3)$$

$\omega_{nmf}^*$ are arguments based on the development frequencies $f$, and $\bar{C}_{0nmf}^*$, ..., $\bar{S}_{1nmf}^*$ are constants.

Then the in-phase $A_{2mf}^{(ip)}$ and out-of-phase $A_{2mf}^{(op)}$ amplitudes ($m = 0, 1, 2$) are (Kudryavtsev 2013):

$$A_{20f}^{(ip)} = \frac{R_E}{\mu_E}\left(\delta k_{20f}^R\bar{C}_{20f}^* + \delta k_{20f}^I\bar{S}_{20f}^*\right), \qquad A_{20f}^{(op)} = \frac{R_E}{\mu_E}\left(-\delta k_{20f}^R\bar{S}_{20f}^* + \delta k_{20f}^I\bar{C}_{20f}^*\right), \quad (4)$$

$$A_{21f}^{(ip)} = \frac{R_E}{\mu_E}\left(\delta k_{21f}^R\bar{S}_{21f}^* - \delta k_{21f}^I\bar{C}_{21f}^*\right), \qquad A_{21f}^{(op)} = \frac{R_E}{\mu_E}\left(\delta k_{21f}^R\bar{C}_{21f}^* + \delta k_{21f}^I\bar{S}_{21f}^*\right), \quad (5)$$

$$A_{22f}^{(ip)} = \frac{R_E}{\mu_E}\delta k_{22f}^R\bar{C}_{22f}^*, \qquad A_{22f}^{(op)} = -\frac{R_E}{\mu_E}\delta k_{22f}^R\bar{S}_{22f}^*. \quad (6)$$

Eqs. (3)–(6) give updated amplitudes of the tidal corrections to the geopotential coefficients of the second degree due to frequency dependence of Love numbers. They employ the Earth TGP development represented in the HW95 format and take into account the phases of tidal waves.

By using Eqs. (3)–(6) and the Earth TGP development by Hartmann and Wenzel (1995) we re-calculated amplitudes of the "Step 2" corrections and compared them with those recommended by the IERS Conventions (2010), Tables 6.5a,b,c. Tables 1–2 show the updated in-phase amplitudes $A^{(ip)}_{21f}$ and $A^{(ip)}_{22f}$ for which the differences with the corresponding amplitudes given by the IERS Conventions (2010) have only been found.

| $f$, deg/hr | Doodson No. | $A^{(ip)}_{21f}$ IERS Conventions (2010) | $A^{(ip)}_{21f}$ This study |
|---|---|---|---|
| 15.07749 | 166,455 | 0.1 | 0.2 |
| 15.08214 | 166,554 | -20.6 | -20.5 |

Table 1: Updated values for in-phase amplitudes $A^{(ip)}_{21f}$, units: $10^{-12}$

| $f$, deg/hr | Doodson No. | $A^{(ip)}_{22f}$ IERS Conventions (2010) | $A^{(ip)}_{22f}$ This study |
|---|---|---|---|
| 28.43973 | 245,655 | -0.3 | 0.2 |
| 28.98410 | 255,555 | -1.2 | 0.8 |

Table 2: Updated values for in-phase amplitudes $A^{(ip)}_{22f}$, units: $10^{-12}$

Our further analysis proves that such relatively large corrections to $A^{(ip)}_{22f}$ values are not only due to use of either modern TGP catalogs or more accurate Eqs. (3)–(6), but also can be obtained with use of older TGP catalogs, e.g. that by Cartwright and Tayler (1971), Cartwright and Edden (1973), and present formulae from Petit & Luzum (2010). Therefore, the noticed differences in the presently recommended by the IERS values for $A^{(ip)}_{22f}$ in-phase amplitudes are likely due to some errors in the IERS Conventions (2010), Tables 6.5c, which should be corrected.

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