THE TIME TRANSFER FUNCTION AS A TOOL TO COMPUTE RANGE, DOPPLER AND ASTROMETRIC OBSERVABLES

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ABSTRACT. In this communication, we will show how the Time Transfer Function (TTF) can be used in the relativistic modeling of range, Doppler and astrometric observables. We will present a method to compute these observables up to second post-Minkowskian order directly from the space-time metric $g_{\mu\nu}$ without explicitly solving the null geodesic. The resulting expressions involve integrals of some functions defined by the metric tensor taken along a straight line between the emitter and the receiver of the electromagnetic signal. Some examples are given within the context of future space missions.

1. MODEL

Let us consider two observers $\mathcal{O}_\mathcal{A}$ and $\mathcal{O}_\mathcal{B}$ moving along their respective worldlines. $\mathcal{O}_\mathcal{A}$ sends an electromagnetic signal received by $\mathcal{O}_\mathcal{B}$. The signal is emitted at the coordinates $(t_A, \mathbf{x_A})$ with a frequency ν_A. It is received by $\mathcal{O}_\mathcal{B}$ at the coordinates $(t_B, \mathbf{x_B})$, with a frequency ν_B. We denote by $n^{(i)}$ the incident direction of the received signal with respect to a tetrad $\lambda^\mu_{(\alpha)}$ comoving with $\mathcal{O}_\mathcal{B}$.

2. COMPUTATION OF THE OBSERVABLES FROM THE TTF

The coordinate travel time of a light ray connecting a emission and a reception point-events is given by the Time Transfer Function $\mathcal{T}_r$ (Teyssandier and Le Poncin-Lafitte, 2008a):

$$t_B - t_A = \mathcal{T}_r(\mathbf{x}_A(t_A), t_B, \mathbf{x}_B). \tag{1}$$

It has been shown that the expression for the frequency shift can be written as (Teyssandier et al. 2008b and Hees et al. 2012)

$$\frac{\nu_B}{\nu_A} = \frac{\left[g_{00} + 2g_{0i}\ \beta^i + g_{ij}\ \beta^i\beta^j\right]_A^{1/2}}{\left[g_{00} + 2g_{0i}\ \beta^i + g_{ij}\ \beta^i\beta^j\right]_B^{1/2}} \times \frac{1 - c\ \beta^i_B\ \frac{\partial \mathcal{T}_r}{\partial x^i_B} - \frac{\partial \mathcal{T}_r}{\partial t_B}}{1 + c\ \beta^i_A\ \frac{\partial \mathcal{T}_r}{\partial x^i_A}}\ , \tag{2}$$

where $\beta^i_{A/B} = \dfrac{1}{c}\dfrac{dx^i_{A/B}}{dt}$ is the coordinate velocity of $\mathcal{O}_{\mathcal{A}/\mathcal{B}}$.

The direction of the incident light ray observed by $\mathcal{O}_\mathcal{B}$ is given by the components of the spatial part of the wave vectors in the tetrad basis (Brumberg, 1991)

$$n^{(i)} = -\frac{\lambda^0_{(i)} + \lambda^j_{(i)}\hat{k}_j}{\lambda^0_{(0)} + \lambda^j_{(0)}\hat{k}_j},$$

where $\hat{k}_j \equiv k_j/k_0$ with k_μ being the covariant coordinates of the wave vector at reception (expressed in the global coordinate system). The last relation can be expressed in term of the TTF (Hees et al, 2013)

$$n^{(i)} = -\frac{\lambda^0_{(i)}\left(1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right) - c\ \lambda^j_{(i)}\frac{\partial \mathcal{T}_r}{\partial x^j_B}}{\lambda^0_{(0)}\left(1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right) - c\ \lambda^j_{(0)}\frac{\partial \mathcal{T}_r}{\partial x^j_A}}, \tag{3}$$

where the components of the tetrad $\lambda^{\mu}_{(\alpha)}$ are evaluated at reception coordinates $(t_B, \mathbf{x}_B)$.

The expression of the TTF as a post-Minkowskian series is given in Teyssandier and Le Poncin-Lafitte (2008)

$$\mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B) = \frac{R_{AB}}{c} + \frac{1}{c}\sum_n \Delta_r^{(n)}(\mathbf{x}_A, t_B, \mathbf{x}_B),$$

where the superscript (n) stands for the nth PM order (quantity of order $\mathcal{O}(G^n)$ with G the Newton gravitational constant) and $R_{AB} = |\mathbf{x}_B - \mathbf{x}_A|$.

In Hees et al. (2013), we showed how to compute the TTF and its derivatives up to the second PM approximation as *integrals of functions depending on the metric taken along the Minkowskian path* (a straight line joining the emitter and the receiver). The corresponding expressions are then used in (1), (2) and (3). This formulation is very general and well adapted to the case of numerical evaluation. In particular, it can be applied to any space-time in GR or in alternative metric theories of gravity.

3. APPLICATION TO A GAME-LIKE SCENARIO

We apply our results to simulate the angular deflection of a light ray coming from a static light source and observed by a satellite in a 1 AU orbit around the Sun during a Solar conjunction. This configuration corresponds to a GAME-like observation (Vecchiato et al. 2009) whose expected accuracy is at the μas level. In Figure 1, we present some high-order PM corrections to the direction of light. The 2PM contribution to the light deflection has been separated in two parts: the so-called enhanced term, proportional to the factor $(1+\gamma)^2$ and the contribution proportional to $\kappa = 2(1+\gamma) - \beta + 3/4\epsilon$ (with γ, β and ϵ the PPN parameters). The 3PM term has been computed analytically by extending the results of Linet and Teyssandier (2013). As can be seen from Figure 1, the complete 2PM contribution needs to be modeled for GAME-like missions and even some third order terms show amplitudes well above the desired accuracy (at least near the Sun limb).

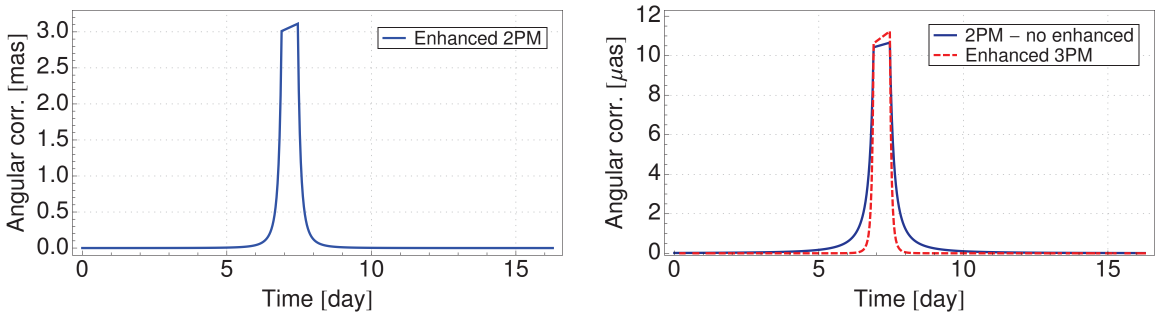

Figure 1: Contribution of the 2PM and 3PM corrections to the angular deflection of a light ray during a solar conjunction.

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