ROTATIONAL-OSCILLATORY MOTIONS OF THE DEFORMABLE EARTH IN THE SHORT TIME INTERVALS

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ABSTRACT. Based on the celestial mechanics' methods namely the spatial version of the problem of the Earth-Moon system in the gravitational field of the Sun a mathematical model of the rotary-oscillatory motion of the elastic Earth is developed. It is shown that the perturbing component of the gravitational-tidal forces normal to the lunar orbit's plane is responsible for some short-term perturbations in the Moon's motion. With the aid of the numerical-analytical approach a comparison between the constructed model and the high-frequency International Earth Rotation and Reference System Service (IERS) measurements is made.

1. INTRODUCTION

Mathematical models of rotary-oscillatory motion of the deformable Earth specify its rotational parameters using the observation data with a high degree of accuracy and provide their reliable prognosis. These models are an essential research tool for investigating a number of problems in astrometry, geodynamics, and navigation. The construction of theoretical models is accomplished through a compromise between the complexity of the model and the measuring accuracy. A meticulous analysis of the basis functions and their number, as well as the parameter settings, is required. A theoretical model should qualitatively and quantitatively correspond to astrometric data of IERS observations [1] and contain only a few essential unknown parameters (low-parametric model) subject to small variations due to nonstationary perturbing factors. These factors can be singled out and taken into account on short timescales.

2. MATHEMATICAL MODEL OF THE ROTARY-OSCILLATORY MOTION OF THE EARTH

We described the rotational motions of the deformable Earth and the oscillations of the Earth's pole using a simplified mechanical model for the viscoelastic rigid body of the Earth [2-4]. To take into account gravitational-tidal effects, we assumed the Earth to be axially symmetric ($(C-A)/B \approx 1/292$, $(B-A)/C \approx 2 \cdot 10^{-6}$) and two-layered, i.e., consisting of a rigid (spherical) core and a viscoelastic mantle. We could have used some more complex model. However, employing anymore complex figure for the Earth is not justified, since we cannot determine the geometrical and physical parameters of the Earth with the required accuracy and completeness via a statistical processing of indirect data from seismic measurements. We adhere to the idea that the complexity of a model must strictly correspond to the problem formulated and to the accuracy of the data used. To construct a model for the polar oscillations, we can determine a small number of some mean (integrated) characteristics of the inertia tensor. Comparison with measurements and further analysis indicate that our simplifications are justified [3, 4].

The proposed dynamical model contains relatively few parameters (it is a few-parameter model) that can be determined from observations; the model enables us to reliably interpret and the statistical characteristics of oscillations in the Earth orientation parameters (EOP), and also to forecast these [2, 3] over comparatively long time intervals (reaching several years). Using the dynamic Euler-Liouville equations with the varying inertia tensor and taking into account estimates of its terms in the harmonic composition of the variations in the tidal coefficients after averaging over the Earth's proper rotation, we obtain a set of differential equations for the EOP in the tied reference frame; i.e., for the quantities x_p,

y_p, $l.o.d.(t)$, $UT1 - TAI$

$$
\begin{aligned}
&\dot{x_p} + N y_p + \sigma_x x_p = \kappa_q r_0^2 + M_p^{S,L} + \\
&+ \varepsilon \left[2 r_0 \delta r(t) \kappa_q + r_0^2 \sum_{i=1}^{N} A_i \cos(2\pi\vartheta_i \tau + \alpha_i) + \Delta M_p^{SL}(\Omega, I) \right], \\
&\dot{y_p} - N x_p + \sigma_y y_p = -\kappa_p r_0^2 + M_q^{S,L} + \\
&+ \varepsilon \left[-2 r_0 \delta r(t) \kappa_p + r_0^2 \sum_{i=1}^{N} B_i \cos(2\pi\vartheta_i \tau + \beta_i) + \Delta M_q^{SL}(\Omega, I) \right], \\
&\left[1 + \varepsilon \sum_{i=1}^{N} C_i \cos(2\pi\vartheta_i \tau + \gamma_i) \right] \frac{d}{dt} l.o.d.(t) = -\frac{D_0}{r_0} M_r^{S,L} + \\
&+ \varepsilon \left[\sum_{i=1}^{N} \frac{C_i}{2\pi\vartheta_i} \sin(2\pi\vartheta_i \tau + \gamma_i) l.o.d.(t) - \frac{D_0}{r_0} \Delta M_r^{SL}(\Omega, I) \right], \\
&\frac{d\left[UT1 - TAI\right](t)}{dt} = -D_0^{-1} l.o.d.(t), \quad D_0 = 86400.
\end{aligned}
\tag{1}
$$

Here, the unknown coefficients must be determined from a least-squarse fit to the IERS data; ϑ_j – are the frequencies of the variations of the inertia tensor (it is assumed that the frequencies ϑ_j can be corrected during the numerical modeling) [3]; the tidal coefficients $\kappa_{p,q}$ are periodic functions with the frequencies ϑ_j; $\Delta M_{p,q,r}^{SL}(\Omega, I)$ are additional terms of the specific lunar-solar gravitational-tidal moment in the spatial Earth-Moon system subject to the solar gravitation [3]; Ω is the longitude of the ascending node of the lunar orbit; I is the ecliptic inclination of the plane of the lunar orbit.

Let us present the results of our numerical simulations of the intrayear variations in the tidal irregularity of the Earth's axial rotation without taking the additional lunar perturbations into the account. Fig. 1 presents the theoretical curve for the interpolation (from September 1, 2010 to September 1, 2011)

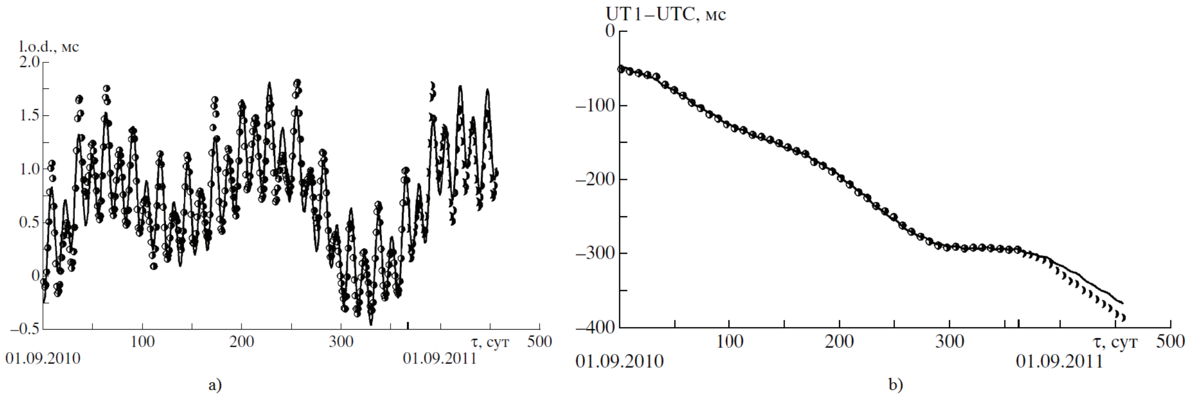

Figure 1: Interpolation (01.09.2010-01.09.2011)and forecast till 01.12.2011 in comparison between the observation data and (a) the variations of the length of the day $l.o.d.$ (b) time correlation $UT1 - UTC$.

and forecast (from September 2, 2011 to December 1, 2011) of the variations (a) in the length of the day $l.o.d.$ and (b) in $UT1 - UTC$. The solid curves show the theoretical model, while the points and half-moons show the IERS data compared to the model interpolation and forecast, respectively.

3. SPECIFIC FEATURES OF THE PROBLEM APPLIED TO SHORT-TERM FORECASTING OF THE EOP

Improving the coordinate-time support for satellite navigation requires high-precision forecasting of the Earth's rotation (the trajectory of the pole and UT1) over short time intervals. Extremely accurate forecasting for intervals lasting from 1 - 2 to 20 - 30 days could be of interest for various applications.

Constructing mechanical models capable of forecasting small-scale, high-frequency polar oscillations and irregularities in the Earth's rotation over short time intervals and explaining the observed irregularities encounter significant difficulties. Below, we consider some difficulties encountered in modeling the EOP (the polar oscillations and variations in the length of the day) using celestial mechanics; i.e., the spatial problem of the Earth-Moon system subject to the Sun's gravitation. The equations for the perturbed motion of the node of the lunar orbit Ω_M and the ecliptic inclination of the plane of the lunar orbit I take the form [3]:

$$\begin{aligned} \frac{d\Omega_M}{dt} &= -\frac{3}{4}\frac{n_S^2}{n_M}\left[1-\cos 2(l_M-\Omega_M)-\cos 2(l_S-\Omega_M)+\cos 2\lambda\right], \\ \frac{dI}{dt} &= \frac{3}{4}\frac{n_S^2}{n_M}\sin I\left[\sin 2(l_S-\Omega_M)-\sin 2(l_M-\Omega_M)+\sin 2\lambda\right]. \end{aligned} \tag{2}$$

Here, n_M and n_S are the sidereal mean motions of the Moon and the Sun; l_M, l_S are the mean longitudes of the Moon and Sun; $(l_M - \Omega_M)$ is the angle between the Moon and the ascending node of the lunar orbit, and $\lambda = (n_M - n_S)t + \lambda_0$ is the difference between the lunar and solar longitudes. The quantity λ is not a linear function of time, since the mean motion n_M is subject to at least periodic changes. The observational data can be used to determine the argument 2λ.

The right-hand sides of (2) contain both long-period and short-period terms, which contribute with fairly small amplitudes. Note that the period of the terms with the argument $2(\lambda - (l_M - \Omega_M)) = -2(l_s - \Omega_M)$ reaches 173 days (the time between two successive solar passages across the line of nodes). The terms with the arguments 2λ and $2(l_M - \Omega_M)$ have periods of half the synodic ($T_M = 29.53$ days) and zodiacal ($T_{\Omega_M} = 27.21$ days) periods, respectively. The zodiacal lunar period mainly determines the variation in the lunar latitude.

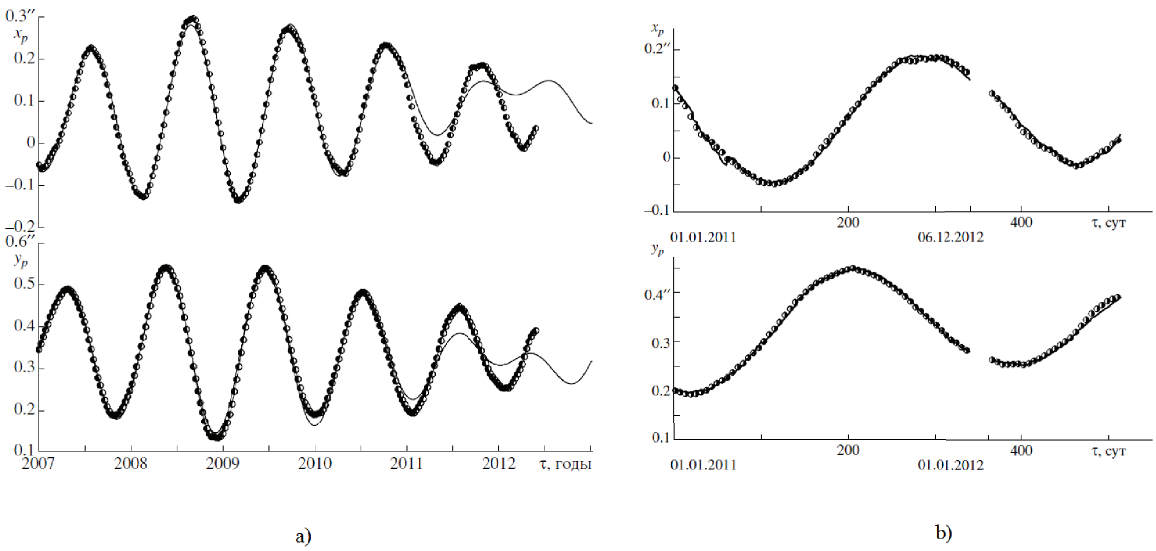

Figure 2: a) Interpolation (2007-2010) and forecast (01.01.2011-28.05.2012) of the oscillations of the Earth pole coordinates x_p, y_p without taking additional lunar perturbations into account; b) twenty-day forecasts for the coordinates of the Earth pole x_p, y_p corresponding to the time interval 01.01.2011–06.12.2011, and forecasts for the interval 01.01.2012–28.05.2012 considering the additional lunar harmonics.

Taking into account quasi-periodic lunar effects, analysis of the amplitude-frequency and amplitude-phase characteristics of the EOP reveals more complex small-scale features contained in the observations [1]. For that purpose we used refined equations for the oscillatory motions of the pole that include those terms on the righthand side of (1) containing the small parameter ε:

$$\begin{aligned} &\Delta\dot{x}_p + N\Delta y_p = \varepsilon\left[2r_0\delta r(t)\kappa_q + r_0^2\sum_{i=1}^{N} A_i\cos(2\pi\vartheta_i\tau + \alpha_i) + \Delta M_p^{SL}(\Omega, I)\right], \\ &\Delta\dot{y}_p - N\Delta x_p = \varepsilon\left[-2r_0\delta r(t)\kappa_q + r_0^2\sum_{i=1}^{N} B_i\cos(2\pi\vartheta_i\tau + \beta_i) + \Delta M_q^{SL}(\Omega, I)\right], \\ &x_p = \bar{x}_p + \Delta x_p, \qquad y_p = \bar{y}_p + \Delta y_p. \end{aligned} \tag{3}$$

Here, $\bar{x}_p$, $\bar{y}_p$ are the solutions for the equations (1) without taking into account the small parameter terms; Δx_p and Δy_p are additional terms for the coordinates of the Earth pole considering additional high-frequency lunar perturbations; A_i, B_i, α_i and β_i are unknown coefficients; and $\Delta M_{p,q}^{SL}$ are terms of higher orders of smallness in the expansion of the lunar-solar gravitational-tidal moment for the spatial problem considered.

The effect of the high-frequency model oscillations (3) is clearly seen in the beats (at the minimum amplitude of polar oscillations), when the irregular perturbations become clearer and comparatively stronger (fig. 2). The points show the IERS data while the solid curves show (a) a four-year interpolation and two-year forecast without taking additional lunar perturbations into account; (b) a twenty-day forecast for the Earth pole coordinates x_p, y_p considering high-frequency additional lunar harmonics. This approach requires a thorough analysis of the oscillations included in both the main and high-frequency models in the interpolation intervals. Our numerical simulations testify to the qualitative and quantitative improvement of the model.

4. REFERENCES

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