ON THE CHANGES OF IAU 2000 NUTATION THEORY STEMMING FROM IAU 2006 PRECESSION THEORY

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ABSTRACT. The adoption of IAU 2006 precession theory (Capitaine et al. 2003) introduced some small changes in IAU 2000A nutation theory, relevant at the mircroarcsecond level. These adjustments were derived in Capitaine et al. (2005) and are currently considered in international standards like, for example, IERS Conventions (2010) or in the Explanatory Supplement to the Astronomical Almanac (2013). We reexamine the issue, working out the induced modifications due to a change in the value of the obliquity of the ecliptic and to the secular variation of the Earth dynamical flattening. In particular, within the framework of the Hamiltonian theory of the rotation of the Earth we derive analytical expressions of those changes for the motion of the figure axis. These expressions and their corresponding numerical contributions will be compared with those obtained in Capitaine et al. (2005).

1. INTRODUCTION

Precession–nutation motion is a basic ingredient to establish the transformation that relates celestial and terrestrial reference systems. It provides the evolution of a celestial pole with respect to the reference celestial system. From a dynamical perspective precession–nutation is a single entity, although it is conventionally separated into secular and long term parts, precession, and a non long term part, nutation (see, for example, IERS Conventions 2010, chapter 5 and references therein).

This motion is realized by International Astronomical Union (IAU) model for precession–nutation plus additional contributions provided by the International Earth Rotation Service (IERS) like, for example, the Free Core Nutation (FCN) caused by the interacting fluid outer core. That model reproduces the evolution of the Celestial Intermediate Pole (CIP) due to the external torques exerted by the Moon, the Sun, and the planets on the non–rigid, non–spherical Earth.

From a methodological point of view, last IAU models for precession–nutation have been constructed by considering two parts that comprise precession and nutation separately. For example, IAU 1980 nutation model was based on the works by Wahr (1981) and Kinoshita (1977), whereas its precession counterpart was given in Lieske et al. (1977). Later, by IAU Resolution B1.6, in XXIVth General Assembly (Manchester, 2000) the nutational part was replaced by the model developed in Mathews et al. (2002). The precessional component, however, was unchanged, keeping basically the theory of Lieske et al. (1977) with some corrections to precession rates.

In XXIVth General Assembly (Prague, 2006), Resolution B1 adopted the model by Capitaine et al. (2003) as the new IAU precession theory, which entered in forced on 1 January 2009. So, the current IAU model for precession–nutation is made up by two components: one for the nutation (Mathews et al. 2002) and other for the precession (Capitaine et al. 2003). To short, they are commonly referred as IAU 2000A nutation and IAU 2006 precession models.

At nowadays accuracy levels this two–component approach requires the introduction of some corrections in the nutation or the precessional part, ensuring in this way the compatibility and consistency between them, although as far as we know there is no explicit mention to this issue in IAU resolutions.

The corrections, or adjustments, of IAU 2000A nutation due to the adoption of IAU 2006 precession are mainly induced by the change in the value of the obliquity, with respect to IAU 1976 precession model (Lieske et al. 1977), and also by the introduction of a time rate of Earth J_2 parameter, not considered in previous models. The relevant values are

$$\epsilon_{A\ \mathrm{IAU\ 2006}} = 84381.40600 - 46.836769\, t + \cdots\,,\ \epsilon_{A\ \mathrm{IAU\ 1976}} = 84381.448 - 46.8150\, t + \cdots\,, \qquad (1)$$

where the obliquity is expressed in arcseconds and time is measured in Julian centuries since J2000.0; and by $\dot{J}_2 = -3 \times 10^{-9}\, cy^{-1}$ with $J_2 = 1.08263558 \times 10^{-3}$.

These modifications provide corrections to IAU 2000A nutation model (Capitaine et al. 2005), in such a way that the total nutation in longitude $\Delta\psi$ and in obliquity $\Delta\epsilon$ can be written as

$$\Delta\psi = \left(\frac{\sin \epsilon_{0\ \mathrm{IAU\ 2000}}}{\sin \epsilon_{0\ \mathrm{IAU\ 2006}}} + t\, \frac{\dot{J}_2}{J_2} \right) \Delta\psi_{\mathrm{IAU\ 2000A}},\ \ \Delta\epsilon = \left(1 + t\, \frac{\dot{J}_2}{J_2} \right) \Delta\epsilon_{\mathrm{IAU\ 2000A}}, \tag{2}$$

where ϵ_0 designates the value of the obliquity at J2000. They consist in a global rescaling of IAU 2000A nutation model, common for all the terms of the trigonometrical polynomial in which nutation is usually expanded. Besides, let us note that the nutations in obliquity are no affected by the change in ϵ_0 .

Numerically (Capitaine et al. 2005), the derived corrections greater than 1 microarcsecond (μas) are

$$d_{\epsilon_A} \Delta\psi = -8.1 \sin\Omega - 0.6 \sin\left(2F - 2D + 2\Omega\right), \tag{3}$$

for the change in the value of the obliquity at J2000.0, and

$$\begin{aligned} d_{\dot{J}_2} \Delta\epsilon &= -25.6\ t \cos\Omega - 1.6\ t \cos\left(2F - 2D + 2\Omega\right), \\ d_{\dot{J}_2} \Delta\psi &= +47.8\ t \sin\Omega + 3.7\ t \sin\left(2F - 2D + 2\Omega\right) + 0.6\ t \sin\left(2F + 2\Omega\right) - 0.6\ t \sin\left(2\Omega\right), \end{aligned} \tag{4}$$

due to the time rate of the J_2 parameter. In these expressions F denotes the mean argument of latitude of the Moon; D the mean elongation of the Moon from the Sun, and Ω the mean longitude of the Moon's mean ascending node, which in combination with the mean anomalies of the Moon, l, and the Sun, l' form the fundamental arguments of nutation. Usually (e.g., Kinoshita 1977), they are represented as

$$\Theta_i = m_{i1} l + m_{i2} l' + m_{i3} F + m_{i4} D + m_{i5} \Omega,\ \ m_{ij} \in \mathbb{Z}. \tag{5}$$

In spite of the smallness of corrections (3) and (4), they are considered in some relevant sources for Earth Rotation standards like IERS Conventions (2010), sec. 5.6.3; the Explanatory Supplement to the Astronomical Almanac (2013), p. 211; or Standards of Fundamental Astronomy (SOFA) routines (e.g., Hohenkerk 2012). When incorporated to IAU 2000A they give raise to the IAU 2006/2000A$_{\mathrm{R06}}$ precession–nutation model, although there is no official nomenclature to designate it (Urban & Kaplan 2012).

In this work we aim at providing an alternative, independent, and analytical derivation of the adjustments of nutation series induced by the obliquity value changes and the J_2 time rate. That is to say, our goal is to check the validity and scope of the adjustment nutation formulas given by Equations (2).

2. ANALYTICAL MODELING

The contributions to be discussed are very small, so, at this stage we will consider a first order theory and a rigid–like symmetrical Earth model, which incorporates the J_2 time rate.

To obtain their analytical expressions, we will make use of the Hamiltonian formalism of the rigid Earth (Kinoshita 1977). The Hamiltonian is given by the sum of the rotational kinetic energy and the first order term of the gravitational disturbing potential due to the Moon and the Sun, conveniently expressed in terms of the Andoyer canonical variables. Then, it is possible to construct an approximate analytical first order solution with the help of a canonical perturbation method. Following this procedure, with the proper modifications over Kinoshita's scheme, we have derived the nutation of the figure axis.

Our preliminary results show that at the μas level the adjustments can be modeled through the motion of the angular momentum axis (Poisson terms), whose expressions are much simpler than the figure axis ones and almost independent of the Earth model at the first order. In this way, the adjustments due to a change of the value in ϵ_A can be derived from the formulas

$$\Delta\psi = \frac{k}{\sin\epsilon_0} \sum_{i \neq 0} \frac{1}{\bar{n}_i} B_i'\left(\epsilon_0\right) \sin\Theta_i,\ \ \Delta\epsilon = -k \sum_{i \neq 0} \frac{m_{i5}}{\bar{n}_i} \frac{B_i\left(\epsilon_0\right)}{\sin\epsilon_0} \cos\Theta_i. \tag{6}$$

For the sake of conciseness we have just displayed the corrections of quasi–periodic nature, omitting mixed secular terms proportional to t that also must be considered. In Eqs. (6), k is a constant proportional to the dynamical ellipticity of the Earth H_d (Kinoshita 1977), and $\bar{n}_i$ is approximately equal to the time derivative of Θ_i (Eq. 5). The functions $B_i(\epsilon_0)$ are defined as

$$B_i(\epsilon_0) = -\frac{1}{6}A_i^{(0)}\left(3\cos^2\epsilon_0 - 1\right) + \frac{1}{2}A_i^{(1)}\sin 2\epsilon_0 - \frac{1}{4}A_i^{(2)}\sin^2\epsilon_0, \tag{7}$$

the coefficients $A_i^{(0,1,2)}$ depending on the orbital motion of the external bodies. A list of the arguments Θ_i and the numerical value of $A_i^{(0,1,2)}$ are given in (Kinoshita 1977).

With respect to the modifications coming from the J_2 time rate, let us point out that it induces a time dependence in dynamical ellipticity of the Earth that, in turn, is translated to the constant k (Eqs. 6). In this way, we can write

$$k = k_0\left(1 + t\frac{\dot{H}_d}{H_d}\right) \simeq k_0\left(1 + t\ \frac{\dot{J}_2}{J_2}\right), \tag{8}$$

appearing in the nutations the following mixed secular terms

$$\Delta\psi = \frac{k_0}{\sin\epsilon_0}\left(1 + t\ \frac{\dot{J}_2}{J_2}\right)\sum_{i\neq 0}\frac{1}{\bar{n}_i}B_i'(\epsilon_0)\sin\Theta_i\ ,\ \ \Delta\epsilon = -k_0\left(1 + t\ \frac{\dot{J}_2}{J_2}\right)\sum_{i\neq 0}\frac{m_{i5}}{\bar{n}_i}\frac{B_i(\epsilon_0)}{\sin\epsilon_0}\cos\Theta_i. \tag{9}$$

However, the precise computation of this effect by means of the perturbation method also leads to the appearance of some quasi–periodic out of phase terms. They are given by

$$\Delta\psi = \frac{k_0}{\sin\epsilon_0}\frac{\dot{J}_2}{J_2}\sum_{i\neq 0}\frac{B_i'(\epsilon_0)}{\bar{n}_i^2}\cos\Theta_i,\ \ \Delta\epsilon = k_0\frac{\dot{J}_2}{J_2}\sum_{i\neq 0}\frac{m_{i5}}{\bar{n}_i^2}\frac{B_i(\epsilon_0)}{\sin\epsilon_0}\sin\Theta_i. \tag{10}$$

To be consistent in the development of the theory, the inclusion of the J_2 time rate forces one to consider also the time rate existing in the orbital coefficients $A_i^{(0,1,2)}$. This dependence is due to the secular variation of sun eccentricity (Kinoshita 1977). From a theoretical point of view, its treatment is quite similar to that of J_2 time rate, providing also out of phase nutations and mixed secular terms with analytical expressions similar to Equations (9) and (10).

3. DISCUSSION

Next, we will evaluate numerically the corrections to IAU 2000A nutations induced by the adoption of IAU 2006 precession model through the former analytical equations, and compare them with those derived in Capitaine et al. (2005). As regard to the adjustments due to the change in the ϵ_0 value, Equations (6) lead to a global rescaling in longitude arising from the denominator of the first factor $\sin\epsilon_0$. The obliquity is not affected, since for it the denominator is equal to 1. These results are equivalently to Equations (2), with $\dot{J}_2 = 0$, taken from Capitaine et al. (2005).

However, accordingly to Equations (6), this is not the only change, since all nutational terms in longitude and obliquity also depend on ϵ_0. In the case of the longitude this dependence just comes from $B_i'(\epsilon_0)$, whereas for the obliquity it is originated from the functions $B_i(\epsilon_0)$ and $\sin\epsilon_0$. Hence, a variation in the value of ϵ_0 will affect in a different way the amplitude of each argument Θ_i of the nutation series.

This new contribution has not only theoretical interest, but also a practical one, since the derived numerical values are of the same order of magnitude as those given in Eqs. (3). Namely, with a cutoff of 0.5 μas, we have found

$$d_{\epsilon_A}\Delta\epsilon = +0.8\cos\Omega,\ \ d_{\epsilon_A}\Delta\psi = -7.5\sin\Omega + 0.5\sin(2F - 2D + 2\Omega)\,. \tag{11}$$

As we have mentioned Eqs. (6) are complemented with other providing mixed secular terms. These terms are not present in Capitaine et al. (2005), although one has an amplitude greater than 1 μas

$$d_{\epsilon_A}\Delta\psi = -8.1\ t\sin\Omega \tag{12}$$

In the case of the corrections induced by the J_2 rate, by comparing Eqs. (9) with Eqs. (6), it turns out that the modifications in the nutations are mixed secular terms proportional to $\dot{J}_2/J_2$ both in longitude and obliquity, in agreement with the results derived in Capitaine et al. (2005) given in Eqs. (2). We can conclude that, strictly speaking, those formulas are only valid for the first order terms of the nutations, since some second order terms are proportional to k^2 and the rescaling factor to be considered would be different from $\dot{J}_2/J_2$ (see Eq. 8).

We have also derived that the inclusion of the J_2 time rate is responsible for some out of phase terms, not considered previously (Eqs. 10). Numerically, the terms relevant at the μas level are

$$d_{\dot{J}_2}\Delta\epsilon = -0.8\sin\Omega,\ \ d_{\dot{J}_2}\Delta\psi = -1.4\cos\Omega. \tag{13}$$

An analogous consideration must be done for consistency with respect to the time rate of the orbital coefficients $A_i^{(0,1,2)}$, what gives the correction

$$d_{\dot{A}}\Delta\psi = -0.5\cos l'. \tag{14}$$

4. SUMMARY

We have developed an analytical model that provides the adjustments of IAU 2000A nutation model (Mathews et al. 2002) stemming from the updating to IAU 2006 precession model (Capitaine et al. 2003). Our results present some differences with respect to the computed ones in Capitaine et al. (2005) that are included in IERS Conventions (2010) and SOFA routines (e.g., Hohenkerk 2012).

In the case of the variation due to the change in the obliquity value, it seems that the global rescale (Eqs. 2, with $\dot{J}_2 = 0$) must be supplemented with additional terms of similar magnitude that affect both longitude and obliquity (Eqs. 11). There are also new secular mixed terms not considered previously (Eqs. 12). Therefore, the total *new corrections* (in μas) to be considered for this variation are given by the sum of Eqs. (11) and (12), providing

$$d_{\epsilon_A}\Delta\epsilon = +0.8\cos\Omega,\ d_{\epsilon_A}\Delta\psi = -7.5\sin\Omega + 0.5\sin(2F - 2D + 2\Omega) - 8.1\ t\sin\Omega. \tag{15}$$

The secular mixed terms emerging from the time rate of J_2 are in agreement with those determined previously (Eqs. 4). However, we have found new out of phase terms (Eq. 13) that, in combination with those coming from the time rate of $A_i^{(0,1,2)}$ (Eq. 14), give the *new contributions* (in μas)

$$d_{\dot{J}_2+\dot{A}}\Delta\epsilon = -0.8\sin\Omega,\ \ d_{\dot{J}_2+\dot{A}}\Delta\psi = -1.4\cos\Omega - 0.5\cos l'. \tag{16}$$

The source of the discrepancies between our treatment and that of Capitaine et al. (2005) should be further investigated, introducing the corresponding modifications into the corrections considered in current standards and models.

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