NEXT STEP IN EARTH INTERIOR MODELING FOR NUTATION.

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ABSTRACT. Accurate reference systems are important for many geophysical applications and satellite observations. It is therefore necessary to know the Earth rotation and orientation with high precision. Interactions between the solid Earth and its fluid layers (liquid core, atmosphere, ocean) induce variations in the Earth's speed of rotation. In addition, because the Earth is not a perfect sphere, but rather an ellipsoid flattened at its poles, the combined gravitational forces acting upon it produce changes in the orientation of its spin axis. Precession describes the long-term trend in the orientation of the Earth, while nutation refers to shorter-term periodic variations. The nutations of the Earth are the prime focus of the present paper. Models are used to predict the real-time Earth rotation and orientation, based on past observations and theoretical considerations of geophysical processes. In particular, the coupling mechanisms at the internal boundaries have been shown to be important for rotation. We here address the coupling mechanisms at the core boundaries such as the topographic, electromagnetic and viscous couplings, and discuss improvements in their computation and observation. The study uses and compares numerical and semi-analytical approaches, with the objective of both improving the nutation model and the rotation, and better understanding the interior of the Earth.

1. RECENT ADVANCES IN OBSERVATION

Nutation observations are performed using Very Long Baseline Interferometry (VLBI). The performance of the VLBI antenna networks used for these observations has increased during the recent years. There are more stations used and more stable sources observed in each session, which has improved the definition and stability of the reference frame and therewith the observation of precession and nutations relating the celestial to the terrestrial reference frame. Moreover the time elapsed since the beginning of good observations has increased, allowing a higher precision determination of the long period nutations such as the 18.6 year nutation, than at the time of the previously adopted nutation model.

The nutation observations are compared with the theory as adopted by the IAU and IUGG in 2000 and 2003. The residuals are mainly due to the Free Core Nutation (FCN), a free mode excited by the atmosphere. The FCN amplitude cannot be precisely determined due to the poor knowledge of its excitation. We here subtract the effect of the FCN free mode contribution (as determined by the IERS) on the nutation in the time domain, and also the effects of the atmosphere and ocean on nutation, which has an important contribution on the prograde annual nutation. What remains is the nutation for the non-rigid Earth without ocean and atmosphere in the time domain, from which observed nutation amplitudes can be deduced with a precision at the ten microarcsecond level. These nutation amplitudes can be compared to theoretical ones computed for an ellipsoidal Earth, with a solid inner core, a liquid outer core, and an ellipsoidal inelastic mantle. Due to resonances in the response of the Earth with the FCN and FICN (Free Inner Core Nutation), one can deduce the "observed" coupling constants at the CMB (core-mantle boundary) and at the inner core boundary. This determination necessitates the knowledge of the forcing acting on the Earth. It is computed from a rigid Earth nutation theory accounting not only for the luni-solar direct effect on the Earth but as well for the direct and indirect of the planets.

2. COUPLING MECHANISMS AT CORE-MANTLE BOUNDARY

There are several coupling mechanisms that have to be considered to explain the observed coupling constant at the CMB: (1) the classical ellipsoidal pressure-gravitational torque, already considered in the MHB2000, the adopted nutation model, (2) the electromagnetic torque, also considered in the adopted model, (3) the viscous torque, and (4) the topographic torque. In the adopted model, only the electromagnetic coupling is considered at the CMB. A revisite of this computation, together with the accounting of the viscous coupling does not lead to matching between theory and observation (coupling constants at the CMB from VLBI data). One explanation can be found by consideration of a thermal conductivity of liquid iron under the conditions in Earth's core is several times higher than previous estimates (Pozzo et al., 2012; Buffett, 2010, 2012).

Alternatively, inclusion of the topographic coupling may reduce the need of a large electromagnetic field. We know from seismology that there is a core-mantle boundary topography at the km level. The liquid pressure at the CMB on this topography induces a pressure torque able to transfer angular momentum from the core to the mantle. This phenomena is well known for the explanation of the decadal variations of Earth rotation (Hide 1977). At the nutation diurnal timescale, it is difficult and challenging to compute, but the topographic torque cannot be ruled out to explain the coupling constants determined from nutation observations. Wu and Wahr (1997) have used seismic value for the topography at the CMB and have computed the effect on nutations. They have shown that the effects on the retrograde annual nutation can be at the milliarcsecond level and that for some topography wavelength there are amplifications of the contributions. We shall examine the approach and equations and further study them. In particular we show that the amplifications can exist due to resonances with inertial waves in the rotating fluid core.

Aiming at obtaining the torque and the associated effects on nutation, we use the following strategy: (1) we establish the motion equations and boundary conditions in the fluid; (2) we compute analytically/numerically the solutions; (3) we obtain the dynamic pressure as a function of the physical parameters; and (4) we determine the topographic torque. Our results can then be compared with those of Wu and Wahr (1997) who used a numerical technique only.

The basic dynamical fluid motion equation is the linearized Navier-Stokes equation. If one considers that the equilibrium corresponds to the hydrostatic case, it can be expressed as

$$\frac{\partial \vec{V}}{\partial t} + 2\,\vec{\Omega}\times\vec{V} + \frac{1}{\rho_f}\nabla p - \nabla\phi_m + \Omega\,\frac{\partial \vec{m}}{\partial t}\times\vec{r} \;=\; 0 \tag{1}$$

where $\vec{\Omega}$ is the uniform equilibrium angular rotation of amplitude Ω, $\vec{m}$ is the scaled additional mantle angular velocity, $\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$, $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is the position of the fluid particle in the reference frame, $\vec{V}$ is the velocity of the fluid particle in the reference frame, ρ_f is the fluid density and p is the incremental effective pressure $p = P - P_0 - \rho_f\phi_1 - \rho_f\phi_e$ computed from the pressure P, the mass redistribution potential ϕ_1, and the external potential ϕ_e. Note that the angular velocity vector of the reference frame attached to the mantle $\vec{\omega} = \vec{\Omega} + \Omega\vec{m}$.

The boundary condition at the core-mantle boundary expresses that core material does not penetrate the mantle: $\vec{n}\cdot\vec{V} = 0$ ($\vec{n}$ is the normal to the surface). It depends on the boundary topography. We write the boundary surface (hydrostatic + non-hydrostatic parts) as

$$r = r_0\left[1 + \sum_{n=1}\sum_{m=-n}^{n}\varepsilon_n^m Y_n^m(\theta,\lambda)\right] \tag{2}$$

where r_0 is the surface mean radius, $Y_n^m(\theta,\lambda)$ are the spherical harmonics of the colatitude θ and the longitude λ, and ε_n^m are small dimensionless numbers related to the existence of the topography. The largest contribution is ε_2^0 due to the flattening (hydrostatic + non-hydrostatic parts) of the CMB. It must be noted that the ϵ_2^0 in a topography development in spherical harmonics usually contains the hydrostatic part and the non-hydrostatic contribution to the topography; these must be separated. Here it is separated into a hydrostatic part $\epsilon_2^{0\;hydr}$ and an additional one noted ϵ_2^0 for simplicity of writing.

We assume that the fluid is incompressible: $\nabla\cdot\vec{V} = 0$. We now decompose the velocity: $\vec{V} = \vec{v} + \vec{u} = \vec{v} + \Omega\,L\,\vec{q}$, where L is the maximum radius of the core and $\vec{q}$ is a non-dimensional velocity. One imposes that $\vec{u} << \vec{v}$. The philosophy for solving the equations is to separate the velocity into a global part ($\vec{v}$) and an additional part ($\vec{u}$ or $\vec{q}$ if normalized) and to separate the equation into two equations of which the solutions are $\vec{v}$ and $\vec{q}$ and can be computed analytically. The equation and condition for $\vec{v}$ are:

$$\begin{cases} \frac{\partial \vec{v}}{\partial t} + 2\,\vec{\Omega}\times\vec{v} + \;+\Omega\,\frac{\partial \vec{m}}{\partial t}\times\vec{r} - \nabla\phi_m \;=\; 0 \\ \nabla\cdot\vec{v} = 0 \end{cases} \tag{3}$$

The equation and condition for $\vec{q}$ are:

$$\left\{ \begin{array}{l} i\,\sigma_m\,\vec{q} + 2\,\vec{\hat{z}} \times \vec{q} + \nabla\Phi \;=\; 0 \\ \nabla\cdot\vec{q} \;=\; 0 \\ \vec{n}\cdot\vec{q} + \Omega^{-1}\,L^{-1}\vec{n}\cdot\vec{v} \;=\; 0 \end{array} \right. \tag{4}$$

where $\Phi = \frac{\phi}{\Omega^2\,L^2}$ and $\phi = \frac{p}{\rho_f}$, Φ being called the non-dimensional dynamic pressure, and where $\vec{\hat{z}}$ is the normalized vector in the direction of $\vec{\Omega}$. The time dependence of the variables is considered as $e^{i\sigma t}$ where σ is the nutation frequency in the reference frame attached to the mantle. When used in non-dimensional equations as above, the frequency to be used is σ_m instead of σ, where $\sigma = \Omega\sigma_m$.

After some algebra of the first equation of (4), one can obtain the following expression for $\vec{q}$ as a function of $\nabla\Phi$:

$$\vec{q} = \;\; \frac{-i\,\sigma_m}{4-\sigma_m^2}\,\left[\nabla\Phi \;-\; \frac{2}{i\,\sigma_m}\,\vec{\hat{z}}\times\nabla\Phi \;-\; \frac{4}{\sigma_m^2}\,(\vec{\hat{z}}\cdot\nabla\Phi)\,\vec{\hat{z}}\right] \tag{5}$$

Using the above equation for $\vec{q}$ and the incompressibility condition for this fluid velocity (second equation of (4)), one obtains the following equation for Φ:

$$\nabla^2\Phi \;-\; \frac{4}{\sigma_m^2}\,\frac{\partial^2\Phi}{\partial Z^2} \;=\; 0$$

where Z is a particular coordinate (related to the cylindrical coordinates involving the colatitude θ and used by Greenspan, 1969), which is equal to $\sqrt{\frac{\sigma_m}{2}}\cos\theta$. The factor $(1-\frac{4}{\sigma_m^2})$ being negative, this mixed differential equation is an hyperbolic differential equation and has the typical form of a wave propagation equations. It expresses that small perturbations of an equilibrium configuration can propagate in the fluid in the form of waves which are the so-called inertial waves because they are controlled by the Coriolis force as a restoring force.

The solution of this equation for Φ must be proportional to the associated Legendre functions of the first kind; it has the following form:

$$\Phi \;=\; \sum_{l=1} a_l^k\,P_{lk}(\frac{\sigma_m}{2})\,Y_l^k(\theta,\lambda). \tag{6}$$

where $P_{lk}(\frac{\sigma_m}{2})$ are the fully normalized associated Legendre polynomials, and $Y_l^k(\theta,\lambda)$ the fully normalized spherical harmonics as introduced before. The a_l^k are coefficients that will be determined in the next step using the boundary conditions (third equation of (4)).

Using the boundary condition for $\vec{q}$ (third equation of (4)) and the expression of $\vec{q}$ as a function of Φ (Eq. (5)), substituting the above solution for Φ (Eq. (6)), after a lot of algebra, one obtains for the first order in the small quantities such as ϵ_n^m:

$$\begin{aligned}
& \sin^2\vartheta\sum_{l,k} Y_l^k\left[kP_{lk}(\frac{\sigma_m}{2}) - \left(1-\frac{\sigma_m^2}{4}\right)P'_{lk}(\frac{\sigma_m}{2})\right]\,a_l^k \\
+\quad & \sin^2\vartheta\left[2\sqrt{\frac{2\pi}{15}}\left(1+3\sum_{n=0}\epsilon_n^m Y_n^m\right)\left(\frac{(\sigma_m^2+\sigma_m-2)}{2\sigma_m}Y_2^1 m_f^- + \frac{(-\sigma_m^2+\sigma_m+2)}{2\sigma_m}Y_2^{-1}m_f^+\right)\right. \\
& \qquad \left. +\sqrt{\frac{2\pi}{3}}\frac{(4-\sigma_m^2)}{2\sigma_m}\;\;\Psi\;\;\left(-Y_1^1 m_f^- \;+\; Y_1^{-1}m_f^+\right)\right] \\
+\quad & \cos^2\vartheta\sqrt{\frac{2\pi}{3}}\;\;\Psi\;\;\left(-\frac{(\sigma_m+2)}{2}Y_1^1 m_f^- \;+\; \frac{(\sigma_m-2)}{2}Y_1^{-1}m_f^+\right) \\
+\quad & \sqrt{\frac{2\pi}{15}}\;\sum_{n=1} m\epsilon_n^m Y_n^m\;\;\left(\frac{(\sigma_m+2)}{2}Y_2^1 m_f^- \;+\; \frac{(\sigma_m-2)}{2}Y_2^{-1}m_f^+\right) \\
=\quad & 0
\end{aligned} \tag{7}$$

where $Y_l^k \equiv Y_l^k(\vartheta,\lambda)$, $m_f^+ = m_1^f + im_2^f$, $m_f^- = m_1^f - im_2^f$, $P'_{lk}(x) = \frac{dP_{lk}(x)}{dx}$ and Ψ is given by:

$$\Psi = \sum_{n=1}\epsilon_n^m\left[\frac{n\sqrt{n-m+1}\sqrt{n+m+1}}{\sqrt{2n+1}\sqrt{2n+3}}Y_{n+1}^m \;-\; \frac{(n+1)\sqrt{n-m}\sqrt{n+m}}{\sqrt{2n+1}\sqrt{2n-1}}Y_{n-1}^m\right]. \tag{8}$$

Equation (7) allows us to solve for the a_l^k as a function of the ϵ_n^m and σ_m. Because we have only kept first order in ϵ_n^m, the a_l^k coefficients are linear functions of ϵ_n^m. It must be noted that this equation can be considered component per component by projection on each $Y_{l'}^{m'}$ and that we can solve as well for each ϵ_n^m separately and then sum over all the contributions.

The boundary conditions at the CMB are imposed on the total velocity and yield thus a relation between $\vec{v}$ (and thus components of the relative global fluid rotation m_1^f and m_2^f), $\vec{q}$ (and thus the a_l^k coefficients), and the topography coefficients ϵ_n^m. This allows to solve for the a_l^k in terms of the relative global relative fluid rotation.

The total pressure torque on the whole topography can then be decomposed into two parts: $\Gamma^0 + \Gamma_{topo}^{\phi}$, where (1) Γ^0 is the constant classical part of the torque for an ellipsoidal topography at equilibrium, and (2) Γ_{topo}^{ϕ} is due to the inertial rotation pressure computed from the above solution. Only the second part of the torque is of importance when computing the effects of a perturbing potential related additional rotations of the core and the mantle on a topography different with respect to the ellipsoidal hydrostatic shape.

3. RESULTS

Substituting the solution for Φ, provided in Eq. (6) as a function of the coefficients a_l^k, in the expression for $\vec{q}$ provided by Eq. (5), and computing the contribution to the torque, one gets the $\vec{q}$-contribution to topographic torque $\vec{\Gamma}_{topo}^{\phi}$ as a function of a_l^k (or equivalently ϵ_n^m by means of Eq. (7)).

4. CONCLUSIONS

From our computation we see that some topography coefficients provide larger contributions to nutation than others. We have not yet solved some differences with respect to Wu and Wahr (1997), even when using the same CMB topography. But the main conclusion remains: it is possible to have topography coefficients that enhance the coupling at the core-mantle boundary.

With this computation, we have shown analytically that the degrees and orders of the nutation with significant amplifications depend on the degrees and orders of the excitation and of the topography expressed in spherical harmonics.

We must note however that the degrees and orders that come out of our computations/conclusions may change when the effect of an inner core is taken into account.

REFERENCES

Buffett B. and Christensen U., 2007, "Magnetic and viscous coupling at the core-mantle boundary: Inferences from observations of the Earths nutations."', Geophys. J. Int., 171(1), pp. 145-152, DOI: 10.1111/j.1365-246X.2007.03543.x.

Buffett B., 2010, "Chemical stratification at the top of Earth's core: Constraints from observations of nutations.", Earth and Planetary Science Letters, 296(3-4), pp. 367-372, DOI: 10.1016/j.epsl.2010.05.020.

Buffett B., 2012, "Geomagnetism under scrutiny.", Nature, 485(7398), pp. 319-320, DOI: 10.1038/485319a.

Pozzo, M., Davies, C., Gubbins, D. and Alfè, D., 2012, "Transport properties for liquid silicon-oxygen-iron mixtures at Earth's core conditions.", Physical Review B, 87(1), Id. 014110, DOI: 10.1103/PhysRevB.87.014110, and Nature 485, pp. 355358, DOI: 10.1038/nature11031.

Wu Xiaoping and Wahr J.M., 1997, "Effects of non-hydrostatic core-mantle boundary topography and core dynamics on Earth rotation", Geophys. J. Int., 128, pp. 18-42.