ESTIMATION OF THE CHANDLER WOBBLE PARAMETERS BY THE USE OF THE KALMAN DECONVOLUTION FILTER

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ABSTRACT. We estimate the Chandler wobble (CW) parameters, the period T and the quality factor Q, based on the stochastic models of polar motion and geophysical excitation data. We apply the Kalman deconvolution filter developed by Brzeziński (1992). This filter can be used to analyze either the polar motion data alone, or simultaneously the polar motion and the excitation data, in order to estimate the unknown residual excitation. By imposing the minimum variance constraint upon the estimated unknown excitation we can find the best value of the resonant parameters T and Q. The CW parameters estimated from different sets of polar motion and geophysical excitation data are compared to each other as well as to the earlier results derived by the alternative algorithms.

1. INTRODUCTION

The parameters of the Chandler wobble, the frequency F (or, equivalently, the period T=1/F) and the quality factor Q, are important for studying global dynamics of the Earth because they define the equation describing geophysical excitation of polar motion (PM) and are closely related to various geophysical parameters. The first attempts to estimate the CW parameters started shortly after the establishment of the International Latitude Service in 1899. Several important results are shown in Table 1; for more complete review of the investigations concerning the Chandler wobble see (Plag et al., 2005).

An important contribution to modeling the observed Chandler wobble had been done by Jeffreys (1940, 1968). He assumed that the excitation function of the free wobble can be adequately modeled as a white noise process. After using the linear equation of polar motion he could arrive at the stochastic model of the Chandler wobble, which is the randomly excited harmonic oscillator with damping. Jeffreys (1940, 1968) used his model and the maximum likelihood method (MLM) scheme to analyze the ILS observations of polar motion. The corresponding estimates of T and Q are given in Table 1. The model of Jeffreys had been further developed by Ooe (1978), Wilson and Vicente (1980, 1990); see Table 1 for their results. The most frequently used estimate of T and Q is that by Wilson and Vicente (1990) based on the ARIMA model and the MLM algorithm: $T = 433.0 \pm 1.1$ days and $Q = 179$ $(74 - 789)$.

A new approach to the estimation of the CW parameters was proposed by Kuehne and Wilson (1996), and Furuya and Chao (1996). They adopted the optimization condition stating that the values of T and

Table 1: Selected estimates of the CW parameters with 1σ uncertainties, the period T_c in days and the quality factor Q_c.

Source	Method‡	F_c	Q_c		data (length in yr)
Jeffreys (1940)	AR	446.7 ± 6.8	46	(37–60)	ILS (42)
Jeffreys (1968)	AR	433.2 ± 3.4	61	(37–193)	ILS (68)
Ooe (1978)	MEM–AR	434.8 ± 2.0	96	(50–300)	ILS (76)
Wilson & Vicente (1980)	MLM–ARIMA	433.3 ± 3.6†	175	(48–1000)†	ILS (78)
Wilson & Vicente (1990)	MLM–ARIMA	433.0 ± 1.1	179	(74–789)	ILS+BIH (86)
Kuehne & Wilson (1996)	LSQ–PM/AAM	439.5 ± 1.2	72	(30–500)	Space93+AAM (9)
Furuya & Chao (1996)	OPT–PM/AAM	433.7 ± 1.8	49	(35–100)	Space94+AAM (11)

† These are not 1-σ but 90% uncertainties
‡ AR – autoregressive model, ARIMA – autoregressive integrated moving-average model, MEM – maximum entropy method, MLM – maximum likelihood method, LSQ – least squares method, OPT – optimization method, PM/AAM - simultaneous analysis of PM and AAM data

Q are such that the corresponding polar motion transfer function yields the best agreement between the observed polar motion and atmospheric angular momentum (AAM) series. This condition was then applied to the simultaneous analysis of the PM and AAM data, and the CW parameters were estimated by the least-squares (LSQ) or the optimization (OPT) procedure.

Here we follow the algorithm described by Brzeziński (2005) based on the Kalman deconvolution filter developed by Brzeziński (1992). This filter can be used to analyze either the polar motion data alone, or simultaneously the polar motion and the excitation data, in order to estimate the unknown residual excitation. By imposing the minimum variance constraint upon the estimated unknown excitation we can find the best value of the resonant parameters T and Q. We will apply this algorithm to different sets of polar motion and geophysical (atmospheric, oceanic and hydrological) excitation data. The estimated CW parameters will be compared to each other as well as to the earlier results given in Table 1.

2. DESCRIPTION OF THE MODEL

Geophysical excitation of the Chandler wobble is governed by the following first-order differential equation (e.g. Brzeziński, 1992)

$$\dot{p} - i\,\sigma_c p = -i\,\sigma_c \chi, \tag{1}$$

in which $i = \sqrt{-1}$ denotes the imaginary unit, $p = x_p - i\,y_p$ describes the change of the terrestrial direction of the Celestial Intermediate Pole, that is polar motion; and $\chi = \chi_1 + i\,\chi_2$ is the forcing (excitation) function of the geophysical fluid. The complex angular frequency of the Chandler resonance is $\sigma_c = 2\pi F_c\,(1 + i/2Q_c)$. The underlying terrestrial reference system is geocentric with its z-axis pointing towards the North pole, the x-axis towards the Greenwich meridian and the y-axis towards 90^o East longitude.

We assume for the rest of the paper that the excitation χ is a stochastic process with power spectral density (PSD) near F_c being approximately constant and equal to S_c.

As a next step we assume that the excitation of polar motion is a sum of the observed excitation χ_o (AAM, oceanic AM – OAM, hydrological AM – HAM) and the residual unknown excitation denoted χ_u

$$\chi = \chi_o + \chi_u. \tag{2}$$

These excitation functions are generated by the stochastic processes described by the following stochastic differential equations

$$\dot{\chi}_o = k_o \chi_o + w_o; \qquad \dot{\chi}_u = k_u \chi_u + w_u, \tag{3}$$

in which the coefficients k_o, k_u fulfill the stationarity condition $\Re k_o, \Re k_u \leq 0$ (with $\Re$ denoting the real part of a complex number) and $\{w_o\}, \{w_u\}$ are zero-mean, white Gaussian noises. In the following analysis we assumed $k_o = k_u = 0$, that is the random walk model for both χ_o and χ_u.

Having defined the state vector $[p\ \chi_o\ \chi_u]^T$ we can implement Kalman smoother for the linear system defined by equations (1), (2), (3), as described in details by Brzeziński (1992). By using simultaneous observations of p and χ_o as an input for the Kalman recursion we can smooth p and χ_o and estimate the unknown excitation χ_u. Several options of this algorithm are possible when applying to real data. First, we can use only the polar motion data. In this case the filtering becomes a pure deconvolution procedure. Second, it is possible to use either one or several observed excitation series (AAM and OAM, or AAM, OAM and HAM). In case of two or three excitation series we can treat them either as separate state variables or add them together prior to the analysis (we adopted this option here).

A straightforward approach to the CW parameters estimation is to find F_c, Q_c which minimize the mean squared value of the estimated unknown excitation χ_u. A more sophisticated algorithm, applied by Furuya and Chao (1996), is to compute the Fourier spectrum of the estimated residuals and minimize only the components from the vicinity of the Chandler frequency.

Below, we will proceed as follows. First we compute the mean-squared value V of the residual excitation and find its minimum with respect to F and Q by a 2-dimensional search procedure. Under certain assumptions, this procedure can be identified with the MLM algorithm and the confidence intervals defined by the cross-sections $V = V_{min}(1 + \varepsilon)$, for some $\varepsilon > 0$.

3. DATA ANALYSIS AND RESULTS

We use the following data sets in our analysis, PM data:

`POLE2010` – Kalman filter combination series (Ratcliff and Gross, 2011), 1900.0–2011.5;

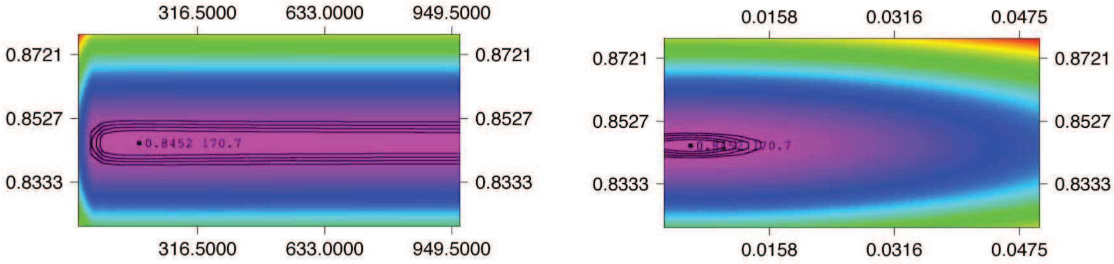

Figure 1: Chandler wobble parameters estimation from simultaneous analysis of PM and AAM data. The mean-squared value of residual excitation is shown as function of F and Q (left), and of F and 1/Q (right). Period of analysis: 1948-2010, input data PM – Pole 2010 and AAM – NCEP reanalysis.

- `C01` – IERS combination of the optical astrometry observations (Vondrák *et al.*, 1995) with the BIH and IERS solutions, 1900.0–2012.0;
- `C04` – IERS combined solution (Bizouard and Gambis, 2009), 1962.0–2009.6;

AAM data

- `NCEP` – AAM series estimated from the output fields of the U.S. NCEP-NCAR reanalysis project (Kalnay et al., 1996), 1948–2012;
- `ERA-int` – ERA Interim reanalysis from ECMWF (Uppala et al., 2008), 1989–2009;

Table 2: Chandler wobble parameters estimated from the simultaneous analysis of polar motion and AAM data: period T_c in days, quality factor Q_c and mean-squared value V_c of residuals (in mas^2).

Data sets	F_c	T_c	Q_c	V_c
(a) Period of analysis 1900.0–2011.5				
C01	0.8440	432.8	∞	1245
POLE2010	0.8435	433.0	2400	913
(b) Period of analysis 1948.0–2009.5				
C01	0.8449	432.3	235	1022
C01/NCEP	0.8459	431.8	111	1063
POLE2010	0.8434	433.1	128	678
POLE2010/NCEP	0.8459	431.8	90	708
(c) Period of analysis 1949.0–2002.9				
C01/NCEP	0.8498	429.8	230	1188
C01/NCEP/ECCO1	0.8492	430.1	380	1162
POLE2010/NCEP	0.8443	432.6	77	691
POLE2010/NCEP/ECCO1	0.8436	433.0	87	669
(d) Period of analysis 1980.0–2002.1				
C01	0.8418	433.9	∞	235
C01/NCEP	0.8439	432.8	125	269
C01/NCEP/ECCO2	0.8445	432.5	83	253
POLE2010	0.8417	433.9	∞	185
POLE2010/NCEP	0.8439	432.8	127	217
POLE2010/NCEP/ECCO2	0.8445	432.5	85	202
(e): Period of analysis 1989.0–2009.0				
C04	0.8423	433.6	269	186
C04/ERA-interim	0.8474	431.0	82	175
C04/ERA-int/OMCT	0.8472	431.1	64	144
C04/ERA-int/OMCT/LSDM	0.8491	430.2	53	163

OAM data

- `ECCO` - OAM series based on the MIT global ocean model (Gross et al., 2003)
  - `ECCO1` - ECCO_50yr solution, 1949-2002;
  - `ECCO2` - c20010701 solution, 1980-2001;
- `OMCT` - OAM series computed from the Ocean Model for Circulation and Tides (Dobslaw & Thomas, 2007), 1989-2009;

HAM data

- `LSDM` - HAM series estimated from the output of the global hydrological model LSDM (Dill, 2008), 1989-2009.

We perform a consistent initial reduction of all the polar motion and geophysical excitation series. The purpose is to remove all deterministic signals which are not relevant to the problem considered. By an unweighed least squares fit, we estimate parameters of the model comprising the sum of complex sinusoids with periods ±1, ±1/2, ±1/3 years, where the sign +/− indicates the prograde/retrograde motion, and the 4-th order polynomial accounting for low-frequency variation. This polynomial-harmonic model is then removed and the residual excitation series are simultaneously smoothed and interpolated at uniform 10-days intervals by the Gaussian low-pass filter with full width at a half of maximum equal to 20 days.

We estimate the Chandler wobble parameters for the following combinations of the data sets

1948.0 – 2009.5: C01/NCEP, POLE2010/NCEP;
1949.0 – 2003.0: C01/NCEP/ECCO1, POLE2010/NCEP/ECCO1;
1980.0 – 2002.3: C01/NCEP/ECCO2, POLE2010/NCEP/ECCO2;
1989.0 – 2009.0: C04/ERA-int/OMCT/LSDM.

Results of the analysis are shown in Figure 1 and Table 2.

4. CONCLUSIONS

Fig. 1 shows that the constant variance cross-sections are oriented along the F and Q axes which indicates the statistical independence of the estimates of the frequency F_c and the quality factor Q_c. Analysis of the polar motion data alone yields reliable estimates of the CW frequency F_c but exhibits some instability with respect to the quality factor Q_c. When taking into account the excitation series in the analysis, the estimated period becomes shorter, down to about 430 days which can be compared to the adopted reference value $T_c = 433.0 \pm 1.1$ day. At the same time, the estimated quality factor becomes lower, down to the factor of 3, with respect to the reference value $Q_c = 179$ $(74 - 789)$. The research reported here has to be continued. Particular problem which will be addressed in the next future are: 1) clarification of the problem of assigning the confidence intervals to the estimated parameters; 2) minimizing the variance in the vicinity of the Chandler frequency instead of that of the whole residual series.

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