COMPARISON OF GEODETIC AND MODELLED EXCITATION FUNCTIONS BY ALLAN VARIANCE

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ABSTRACT. Allan variance analysis allows to study the stability or the noise of a physical process at a given time scale. Whereas this statistical tool is widely used for quantifying the stability of the atomic clock, it can be applied to any time series. This is especially interesting for analysing Earth rotation excitation functions: in complement of the standard numerical methods (spectra, least-square adjustment, correlation and explained variance), it yields a stochastic comparison on observed excitation and the one derived from geophysical models. This permits to draw conclusions on the nature of the geophysical forcing of the Earth rotation in function of the considered time scale.

1. INTRODUCTION: THE ALLAN VARIANCE ANALYSIS

Allan variance analysis permits to quantify the stability of a time series at a given time scale. Moreover, as well as spectral density, its slope (in log-log scale) allows to characterise the noise at play. Widely used for time scale comparison, it is sometimes applied for analysing Earth Rotation Parameters (EOP) (see e.g. Malkin 2011) , especially for comparing statistical properties of EOP time series. Now we shall look its interest for characterising excitation of the Earth rotation.

First, in this introductory section, we shall briefly recall what is Allan variance analysis. Let x_i be a time signal equally sampled at instants $t_i = t_0 + i\tau$ $(i = 0, n)$ over the time interval $[t_0, t_n]$. We build the series y_k^T by averaging x_i over successive periods of length $T_m = m\tau$ with $m = 1, ..., \lfloor n/2 \rfloor$:

$$y_k^{(m\tau)} = \frac{1}{m} \sum_{i=0}^{m} x(t_0 + kT + i\tau) \text{ with } k = 0, ..., \lfloor n/m \rfloor \ . \tag{1}$$

The integer m indexes the length of the average, starting from τ to the mid of the sampled interval, that is $\lfloor n/2 \rfloor \tau$. The *Allan variance* series $\mathcal{AV}(m\tau)$ with $m = 1, ..., \lfloor n/2 \rfloor$ of x_i is composed of the variances of the $\lfloor n/2 \rfloor$ forward difference series (Allan 1987):

$$D_k^{(m\tau)} = \frac{y_{k+1}^{(m\tau)} - y_k^{(m\tau)}}{\sqrt{2}} \text{ with } k = 1, ..., \lfloor n/m \rfloor \tag{2}$$

that is :

$$\mathcal{AV}(m\tau) = < D^2(t) > \ . \tag{3}$$

As for the standard deviation relatively to the variance, the Allan deviation $\mathcal{AD}(m\tau)$ is defined by the square root of $\mathcal{AV}(m\tau)$. The more stable is the signal at $T_m = m\tau$ interval, the smaller are the forward differences of the corresponding averaged signal, and the smaller is the Allan variance for this time scale. So Allan variance series characterises the signal stability in function of the time scale. Developing the Allan variance, it can be easily shown that

$$\mathcal{AV}(m\tau) = R_{y^{(m\tau)}}(0) - R_{y^{(m\tau)}}(m\tau) \ , \tag{4}$$

where $R_{y^{(m\tau)}}$ is the discrete auto-correlation of time series $y_k^{(m\tau)}$. If the signal tends to repeat at time interval $m\tau$, then $\mathcal{A}(m\tau)$ tends to vanish as expected. Long period component (having more or less the period of the sampling), episodic jump or a discontinuity, as pointed out by Malkin (2011) do not affect much Allan deviation in contrast to standard deviation, and Fourier spectrum of the considered time series. So Allan deviation allows to eliminate systematic effect and to better characterise the pure

α	slope of the Allan Variance (AV) β	slope of the Allan Deviation (AD) $\beta/2$	stochastic process
0	-1	$-1/2$	white noise
-1	0	0	flicker noise
-2	1	$1/2$	red noise

Table 1: Correspondence between Allan variance slope β and slope α of the spectral density. Allan deviation slope is $\beta/2$.

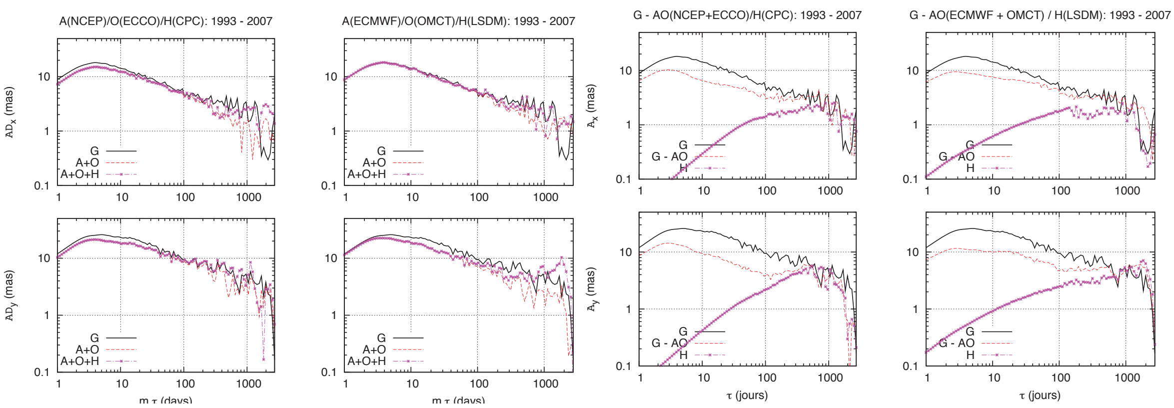
random fluctuation of the studied signal at a given time scale. The value $AV(m\tau)$ are plotted in log-log scale. Then a slope β is related to a noise process of spectral density $K_\alpha f^\alpha$ with $\beta = -\alpha - 1$. Correspondence between α and β values is given in Tab. 1 for most common noises met in geodetic parameters.

2. METHOD AND DATA

From a few days to several decades, pole coordinates $p = x - iy$ and length of day variation ΔLOD are ruled by the linear differential equation fo first order:

$$p + i\frac{1}{\tilde{\sigma}_c}\,\dot{p} = \chi \qquad \frac{\Delta LOD}{LOD} = \chi_3\ , \tag{5}$$

where $\tilde{\sigma}_c \sim 2\pi/433(1+i/200)$ cycle/day is the complex Chandler frequency, χ and χ_3 are the effective Angular Momentum Functions (AMF) of any surface layer of the Earth (atmosphere, oceans, freshwater or combination of these ones), as far we do not consider other source of excitation (external torque,...). Common approach is to compare the first member of these equation χ_G, χ_3^G, namely the geodetic excitation G deduced from observed EOP, to the second member or effective AMF, reconstructed from Global Circulation Models (GCM) of the hydro-atmosphere. It is a well documented fact that hydro-atmosphere is the main source of excitation for periods below 10 years (see e.g. the excellent monography of Sidorenkov 2009), so that comparing time series G and AMF at this subdecadal time scale is meaningful. Geodetic



a) b)

Figure 1: Comparative Allan deviation analysis of equatorial excitations: G, $A+O$, $A+O+H$ (left plot) and residual equatorial excitation $G-(A+O)$ and of hydrological contribution H (right plot)

excitation G is built from from C04 EOP time series according to Wilson (1985) digital filter, then is compared to the hydro-atmospheric angular momentum time series. When we add to atmospheric AMF time series (A) oceanic (O) or hydrological (H) ones, we consider consistent GCM, namely i) atmosphere: NCEP/NCAR (National Center for Environmental Prospect/National Center for Atmospheric Research); oceans: ECCO-MIT (Estimating the Circulation and Climate of the Oceans); fresh waters: CPC (Climate Prediction Center) (all series are supplied by the Global Geophysical fluid Center of the IERS http://geophy.uni.lu) ii) atmosphere: ECMWF (European Centre for Medium-Range Weather Forecasts); oceans: OMCT(Ocean Model for Circulation and Tides); freshwater: LSDM (Land Surface Discharge

Model) prepared by Geodetische Forschung Zentrum (ftp://ftp.gfz-potsdam.de/pub/home/ig/ops/). Before to perform Allan deviation analysis, the stable part of the signal (bias and seasonal harmonics), are removed by least-square fit over the considered time interval.

3. ALLAN DEVIATION ANALYSIS OF THE EQUATORIAL EXCITATION

First comparison will focus on the most most accurate pole coordinates, beginning with the advent of GNSS processing in 1993. Because hydrological time series CPC does not go beyond 2008, considered time interval is 1993-2007. On Fig. 1a we plot the Allan deviation of equatorial G along the ones of $A+O$ and $A+O+H$. We notice an overall good agreement, with $A+O$ and G presenting a slope of $\beta/2 = -0.5$ from 5 to 400 days, meaning a white noise process at stake (see Tab. 1). However combinations χ_{A+O} and χ_{A+O+H} presents a smaller Allan deviation than G below 100 days. Notice that y component is less stable than x one at seasonal period. Looking now at the residuals $G-(A+O)$ compared to H from 1 day to one year (Fig. 1b), we see that hydrological series, appearing as a red noise, do not account for the residual, which behave like a random process between white and Flicker noises. But, at longer period from one to several years, Allan deviation H comes in closer agreement with $G - A + O$), pointing out the tremendous role of the freshwater redistribution in multi-annual polar motion (considering a time span from the 1960's this conclusion is reinforced). More generally, Allan deviation analysis permits to

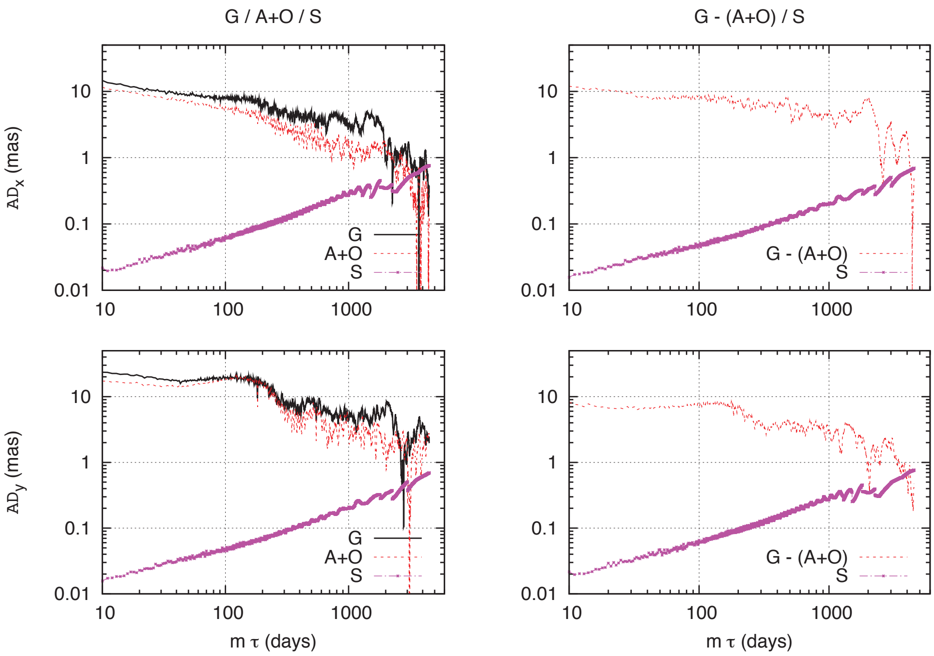


Figure 2: Comparative Allan deviation analysis of equatorial seismic excitation and of geodetic excitation and its residuals $G - AO$ over the period 1985-2011

know weather a geophysical process can excite Earth's rotation. For instance let us consider the seismic excitation S resulting from step wise mass redistribution estimated from 1985 to 2011 (Bizouard 2012). According to the slope of $AD(S)$, about 1/2 from Fig. 2, this is a random walk process (red noise), which cannot account for polar motion excitation below 3000 days (10 years).

4. ALLAN DEVIATION ANALYSIS OF THE AXIAL EXCITATION

As evidenced by Fig. 3, the axial geodetic excitation excitation G, namely offset of the length of day expressed in ms, fits the corresponding atmospheric excitation A (multiplied by 86400 s) SI up to time scale of 500 days. At short term time scale (below 50 days) hydrological contribution is negligible and residuals $G-A$ are explained by oceanic excitation, but only in the case of ECMWF/OMCT association. Above 50 days, neither ocean nor hydrological mass redistribution $G-A$ residuals can account for $G-A$

residuals. This probably points out a defect in hydrological or oceanic models at seasonal and longer periods.

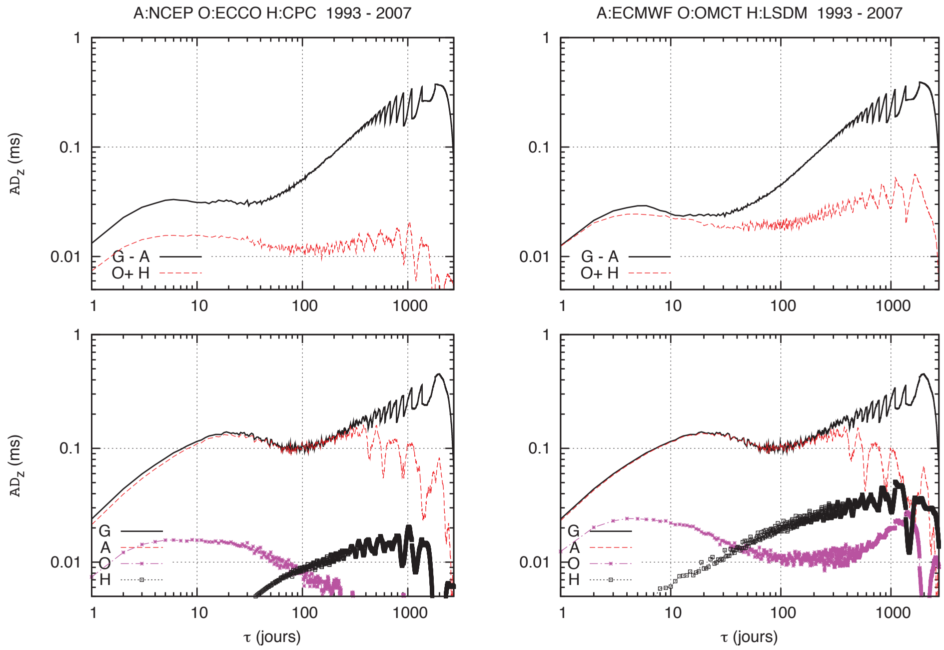


Figure 3: Comparative Allan deviation analysis of axial excitation. Bottom plots: geodetic G, atmospheric A and oceanic O. Upper plot: residuals $G - A$ compared with oceanic anf hydrological contributions $O + H$

5. CONCLUSION

Allan variance (or deviation) analysis is a powerful tool for analysing excitation time series of the Earth rotation, permitting at a given time scale to investigate underlying physical process. Indeed, from this single analysis, it can be concluded that i) equatorial excitation tends to be more stable at multi-year time scale ($\sim$ white noise) in contrast to axial excitation ($\sim$ red noise), meaning that physical processes at stake are different (surface redistribution versus fluid core) ii) over 400 days hydrological process are fundamental for explaining $G - AO$ residuals of the equatorial components iii) Earthquake do not influence polar motion below 10 years. The lack of compliance between Allan deviation can also point out defects in global circulation models. Thus, rapid equatorial fluctuations observed in $G - (A + O)$ residuals are not explained by hydrological contribution ; axial residuals $G - A$ over 50 days do not fit the corresponding excitation $O + H$.

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