

# EPM-ERA2011 LUNAR EPHEMERIS AND SELENODYNAMICAL PARAMETERS FROM LLR (1970-2011) DATA

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**ABSTRACT.** The modern Lunar ephemerides are constructed at JPL,USA (DE403,DE405, DE421); in Institute of Celestial Mechanics, France (series of INPOP) and at the Institute of Applied Astronomy RAS in the framework of ERA system (Krasinsky and Vasiliev, 1996). The dynamical model EPM-ERA has been constructed by simultaneous numerical integration of equations of orbital motion of the Moon, major planets, the biggest asteroids, and the lunar rotation. The dissipative effect of lunar rotation was included in the new version of ephemeris with retarded argument under integration of orbital and rotational Lunar motion. The comparison of improved dynamical model was made with 17742 LLR observations (1970-2011) for obtaining selenodynamical parameters. The version has been compared with three versions of the DE ephemerides and French ephemeris INPOP10.

## 1. INTRODUCTION

The subtle effects in the rotation of the Moon may be studied making use of lunar laser ranging measurements (LLR) provided by regular observational programs started in 1969, when the first reflector was put at the Moon surface. The analysis of the LLR data with applications to lunar rotation is given in (Dickey et al., 1994; Williams et al., 2001; Aleshkina et al.,1996). Unlike the problem of the Earth's rotation no monitoring of rotational parameters of the Moon is yet possible, thus the case of the Moon seems even more complicated than that of the Earth. The small effects to be studied are only detectable if a sophisticated dynamical model both of the orbital and rotational motions of the Moon is constructed by simultaneous integration of equations of motion of the major planets and the Moon (including equations of the lunar rotation). High accuracy of the LLR data requires dynamical theories of the adequate precision. The analysis of LLR data depends not only on a dynamical model but on partial derivatives in respect to a number of parameters many of which also require numerical integration. The comparison of the improved dynamical model was made using 17742 LLR observations (1970-2011). This version has also been processed with three versions of DE ephemerides and French INPOP10 one.

## 2. THE EPM-ERA DYNAMICAL MODEL

In this paper only a brief summary of the model used is described. The full theory was presented in the paper (Krasinsky G., 2002). The precise dynamical model EPM-ERA has been constructed by simultaneous numerical integration of equations of orbital motion of the Moon, major planets, the biggest asteroids, and the lunar rotation. The potential of the Moon is calculated up to the fourth order of the zonal index, the potential of the Earth includes the second order harmonics  $C_{20}$  and  $C_{22}$ . Tidal perturbations in the lunar orbital motion, caused by tidal dissipation on the Earth's body, have been computed by the model using a constant lag. Partial derivatives of ranging with respect to dynamical parameters of the orbital and rotational model of the Moon are computed mostly by integrating the variational equations; in a few cases, they have been obtained by integrating the rigorous system of equations with slightly varied values of the parameters under study. In the current version of the EPM-ERA 2011 ephemeris, the model of the tidal perturbations in the rotational motion of the Moon (due to dissipation in the Moon's body) is constructed using retarded argument. The expansion of the retarded function in a power series of delay is used.

### 3. OBSERVATIONS

In the present analysis, 17742 LLR observations of the time interval 1970-2011 have been included in the processing. They were carried out mainly at McDonald (Texas), where at different times three different sites were activated as the McDonald observatory, MLRS1 and MLRS2; Cerga station (France); a set of two-year observations made at Haleakala Observatory (Hawaii) and 915 observations were made at Apache station (mm accuracy). The number of observations at each site is shown in Table 1. LLR analysis of a number of parameters under estimation appears to be strongly correlated and may be reliably estimated because four reflectors could be observed.

Station	Time interval	Number of LLR observations
McDonald	1970 March - 1985 June	3440
MLRS1	1985 January - 1988 January	275
MLRS2	1988 August - 2011 August	3194
HALEAKALA	1989 November - 1990 August	694
CERGA	1985 January - 2011 June	9224
APACHE	2006 August - 2010 November	915
TOTAL	1970 March- 2011 July	17742

Table 1: Distribution of LLR observations

The number of ranging to Apollo 11, 14, 15 and Lunokhod2 are 1819, 1757, 13667, 499 respectively. Unfortunately, such disparity of the distribution deteriorates the reliability of the estimation of a number of selenodynamical parameters. Before 1998, the observations were obtained by the request from observatories, later on they were retrieved from the following FTP servers: "ccdisa.gsfc.nasa.gov/pub/slr", "oca.eugemini.donnees.las\_lunes", "http://www.physics.ucsd.edu/~tmurphy/apollo/norm\_pts.html" for observations from Apache station. Some observations were obtained by private correspondence.

### 4. PARAMETERS DETERMINED

In Table 2, a list of parameters under study in the LLR processing are presented. The set of parameters includes the lunar initial coordinates and velocities, libration angles and their velocities, Stokes coefficients of the selenopotential, Lunar Love numbers  $k_2$ ,  $h_2$ ,  $l_2$ , the angles of tide delay, the coordinates of reflectors, observational stations, etc. All the improved values of dynamical parameters were fed back into the EPM-ERA theory. As lunar rangings are invariant, relative to rotations of the Earth-Moon system as a whole,

N	Parameters estimated
1-6	Lunar orbital state vector for epoch JD 2446000.5
7-12	Eulers angles and their time derivatives for the same epoch
13-18, 22-24	Coordinates of reflectors A11, A14, L2
20	X coordinate for reflector Apollo 15 (A15)
25-42	Coordinates of 6 observational stations
44	Lag of the Earths body tides
48-51	Secular trends in siderial angles of the Earth and Moon
55	Lag of the Moons body tides
52-54, 59-63	Harmonics of lunar potential from $C_{20}$ to $S_{33}$
56-58	Lunar Love numbers $k_2$ , $h_2$ , $l_2$
64-65	secular trends of the corrections to the parameters of Earths equator

Table 2: Parameters determined

the whole set of orientation parameters of this system cannot be determined simultaneously. Due to this, the coordinates of the most often observable reflector Apollo15 have been fixed (longitude and latitude). The values of these two parameters were obtained from a simplified solution made at the first step, in which lunar libration was not improved. LLR observations are sensitive to the Earth's gravitational constant  $GmE$ . Our experience has shown that the observable effect reduces to scaling of distances and

cannot be reliably separated from the corrections to the  $X$  coordinates of the reflectors. Thus, the value was not included into the list of estimated parameters.

## 5. ANALYSIS OF THE RESULTS

The numerical theory EPM-ERA has been compared with the set (17742) of LLR observations and all the parameters listed above have been determined. The corrections to 65 parameters were fed back by several iterations and the result can be seen in Table 3. The number of observations used, post-fit and pre-fit residuals, observational stations and periods of observations at every station are also given in Table 3. Because the EPM-ERA model was improved by the corrections obtained, the post-fit residuals practically coincide with O-C differences computed using the improved model. It is known that the analysis of LLR data not only depends on the dynamical model but on partial derivatives relative to a number of parameters many of which also require numerical integration. Thus, to compare our result with the results obtained using DE or INPOP10 ephemerides, all analogous calculations have been made with mentioned ephemerides using derivatives from EPM-ERA.

Number of observations	WRMS[cm] O-C	WRMS[cm] residuals	Observational stations	Interval of observations
3414	31.5	31.5	McDonald	19700415.0-19850630.0
275	12.2	12.2	MLRS1	19850301.0-19880127.1
9224	5.4	5.3	CERGA	19840407.2-20110621.2
692	13.6	13.7	Haleakala	19891113.1-19900830.1
2862	6.7	6.7	MLRS2	19880229.0-20110721.1
915	5.2	5.0	Apache	20060407.1-20101030.1
17378	6.6	6.5	Total	19700415.0-20110721.1

Table 3: EPM-ERA ephemeris, statistics of residuals

Because the nominal values of the parameters for DE and INPOP10 ephemerides are not known, the corrections could only be fed back to the coordinates of reflectors and the coordinates of ground stations. Post-fit residuals and the number of the observations used for all the versions of DE ephemerides and INPOP10 are presented in Table 4. The O-C differences are only shown for EPM-ERA ephemeris: in case of EPM-ERA, all the corrections could be fed back, as for DE and INPOP10 ephemerides O-C differences (after feeding back corrections to reflectors and coordinates of ground stations) are big: it is not known what other parameters different from those used EPM-ERA ephemeris had been determined. At the plots of Figures 1 to 5 the residuals for DE ephemerides, INPOP10 and for EPM-ERA2011 are presented.

Ephemerides	Wrms(cm) O-C	Wrms(cm) residuals	Number of observations	Number of deleted observations
DE 403		5.2	17369	373
DE 405		5.6	17379	363
DE 421		5.7	17375	367
INPOP10		5.1	17377	365
EPM-ERA 2011	6.6	6.5	17378	364

Table 4: Statistics of residuals for EPM-ERA ephemeris, compared with DE and INPOP10 ephemerides

## 6. CONCLUDING REMARKS

The investigation shows that the current accuracy of lunar component of DE (5.4-5.7 cm) and INPOP10 (5.1 cm) ephemerides is a slightly better than that of EPM-ERA2011 (6.5 cm). The source of this discrepancy is due to not complete account of the tidal perturbations in the rotational motion of the Moon. Currently, a test of this part of the model, as well as changes to the integrator process, are under

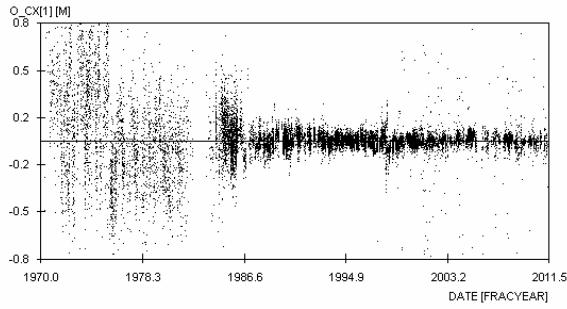


Figure 1: DE403

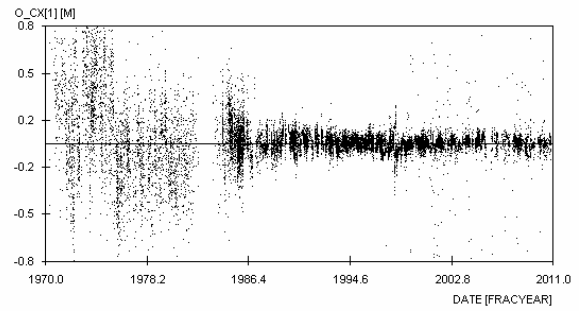


Figure 2: DE405

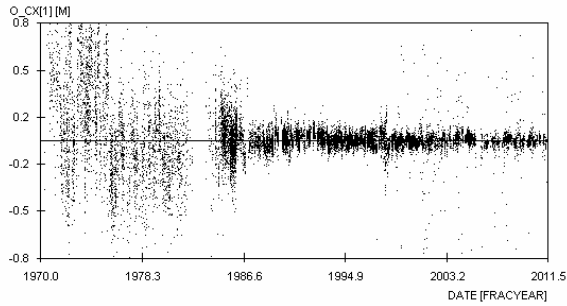


Figure 3: DE421

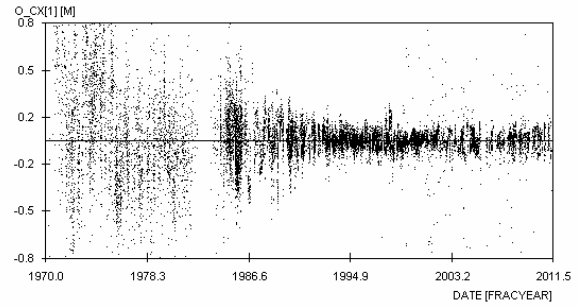


Figure 4: EPM-ERA2011

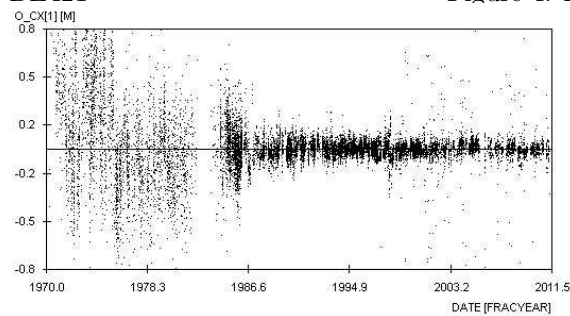


Figure 5: INPOP10

way.

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