NEW LONG-TERM EXPRESSIONS FOR PRECESSION

J. VONDRÁK[1], N. CAPITAINE[2], P. T. WALLACE[3]
[1] Astronomical Institute, Acad. Sci. Czech Rep.
Boční II, 141 31 Prague 4, Czech Republic
e-mail: vondrak@ig.cas.cz
[2] SYRTE, Observatoire de Paris, CNRS, UPMC
61, avenue de l'Observatoire, 75014 Paris, France
e-mail: n.capitaine@obspm.fr
[3] STFC / Rutherford Appleton Laboratory
Harwell Oxford, Oxon OX11 0QX, UK
e-mail: patrick.wallace@stfc.ac.uk

ABSTRACT. Precession is the secular and long-period component of the motion of the Earth's spin axis in the celestial reference frame. The current precession model, IAU 2006, approximates this motion by polynomial expansions that are valid, with very high accuracy, in the immediate vicinity (a few centuries) of the reference epoch J2000.0. However, for more distant epochs, this approximation quickly deteriorates. Consequently, we recently published new precession expressions (Vondrák et al. 2011b), comprising very long-period terms fitted to a numerical integration of the motion of solar system bodies on scales of several hundreds of millennia. We give a short description of the new expressions, including an assessment of their accuracy and comparisons with other models.

1. INTRODUCTION

All precession models used so far are expressed in terms of polynomial development, no matter which precession parameters are used. Model IAU 2006 is very accurate, but usable only for a limited time interval (several centuries around the epoch J2000); its errors rapidly increase with longer time spans. In reality, precession represents a complicated very long-periodic process, with periods of hundreds of centuries. This can be seen in numerically integrated equations of motion of the Earth in the solar system and its rotation (Vondrák et al. 2009, 2011a).

We define the precession of the equator as being the motion of the equator covering periods longer than 100 centuries; shorter ones are included in the nutation. Similarly, the precession of the ecliptic is assumed to be the part of the motion of the ecliptic covering periods longer than 100 centuries. The goal is to find relatively simple expressions of different precession parameters, with accuracy comparable to the IAU 2006 model near the epoch J2000.0, and lower accuracy outside the interval ±1 millennium (up to several minutes of arc at the extreme epochs ±200 millennia). The paper describing the new model has recently been published (Vondrák et al. 2011b), further referred to as "A&A paper".

2. METHOD USED

We use the numerically integrated values of

- the precession of the ecliptic P_A, Q_A, calculated with the Mercury 6 package by Chambers (1999), which was shown to be consistent with Laskar et al. (1993) values;
- the general precession/obliquity p_A, ε_A (Laskar et al. 1993), ε_A being shown to be consistent with the Laskar et al. (2004) values;

to calculate different precession parameters in the interval ±200 millennia from J2000.0, with 100-year steps. The central part (±1 millennium from the epoch J2000.0) is replaced by IAU 2006 values. These series are then approximated by a cubic polynomial plus up to 14 long-periodic terms, so that the fit is best around J2000.0. This is assured by choosing appropriate weights (equal to 10^4 in the central part and to $1/T$ outside this interval). The periods are found using Vaníček's method (1969), modified by Vondrák (1977) and verified against those found by Laskar et al. (1993, 2004) from much longer time series. Least-squares estimation is then used to determine the sine/cosine amplitudes of individual periodic terms.

3. SOLUTION

We derived the long-term expressions of the following precession parameters, some of them being precession angles, some direction cosines (expressed in terms of certain precession angles):

- precession angles: $p_A, \varepsilon_A, \omega_A, \psi_A, \chi_A, \varphi, \gamma, \psi$;
- direction cosines: $P_A = \sin\pi_A \sin\Pi_A$, $Q_A = \sin\pi_A \cos\Pi_A$, $X_A = \sin\theta_A \cos\zeta_A$, $Y_A = \sin\theta_A \sin\zeta_A$, $V_A = \sin\theta_A \sin z_A$, $W_A = \sin\theta_A \cos z_A$.

The precession angles are depicted in Figure 1. In addition to these, we also derived the expression for the the precession part, s_A, of the CIO locator (i.e., the quantity used to locate the celestial intermediate origin, CIO, in a fixed celestial reference system).

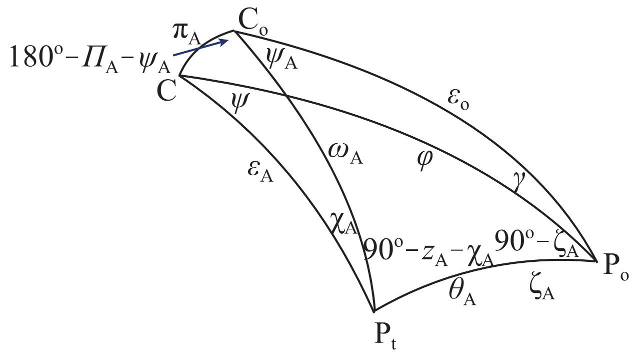

Figure 1: Precession parameters

Long-term expressions of all these parameters are given in a general form

$$a + bT + cT^2 + dT^3 + \sum_{i=1}^{n} \left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right), \quad (1)$$

in which T is time in centuries from J2000.0, P_i are periods in centuries ($P_i > 100$), and n (between 8 and 14) is the number of periodic terms. Parabolic and cubic coefficients c, d are very small, as they must be for the expression to be usable at large T values.

4. ESTIMATION OF ACCURACY, COMPARISON WITH OTHER MODELS

In the A&A paper, accuracy was estimated using a simple expression based on the average σ for all parameters (estimated from the fit to integrated values) and weights at different epochs; here we use a rigorous formula, based on the full variance-covariance matrix. The result is depicted in Figure 2, where the accuracy of each estimated parameter is given and compared with the one from the A&A paper. It is clear that our previous estimate was too cautious – the rigorous estimate yields much smaller uncertainties for all parameters, in some cases as much as two orders of magnitude lower.

The comparison of the new long-term solution with other models of precession (X_A and Y_A parameters only) is given in Figures 3 and 4. X_A and Y_A values as computed from the values of ζ_A, θ_A by Lieske et al. (1977), Simon et al. (1994) and Capitaine et al. (2003) (denoted as Lieske, Simon and IAU2006$_{\zeta\theta}$, respectively), and computed directly from the X_A, Y_A series of Capitaine et al. (2003) and the A&A paper (denoted as IAU2006$_{XY}$ and LT model, respectively) are compared with the numerically integrated values.

Figure 3 depicts the comparison in the interval ±300 centuries from J2000.0, while Figure 4 shows only the central part (±10 centuries from J2000.0) at an enlarged scale. One can see that, for distant epochs, the direct IAU 2006 series, which are developments of direction cosines of the pole unit vector in the

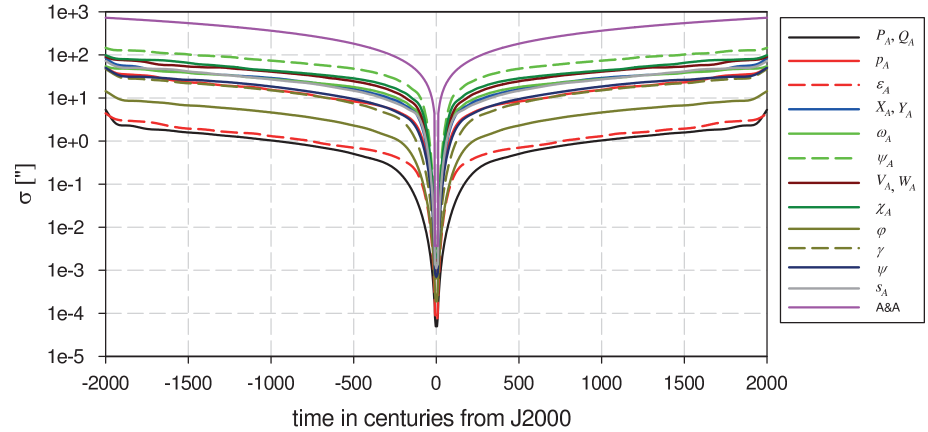

Figure 2: Estimated accuracy of all precession parameters: the line marked 'A&A' shows the overcautious estimate presented in the earlier paper.

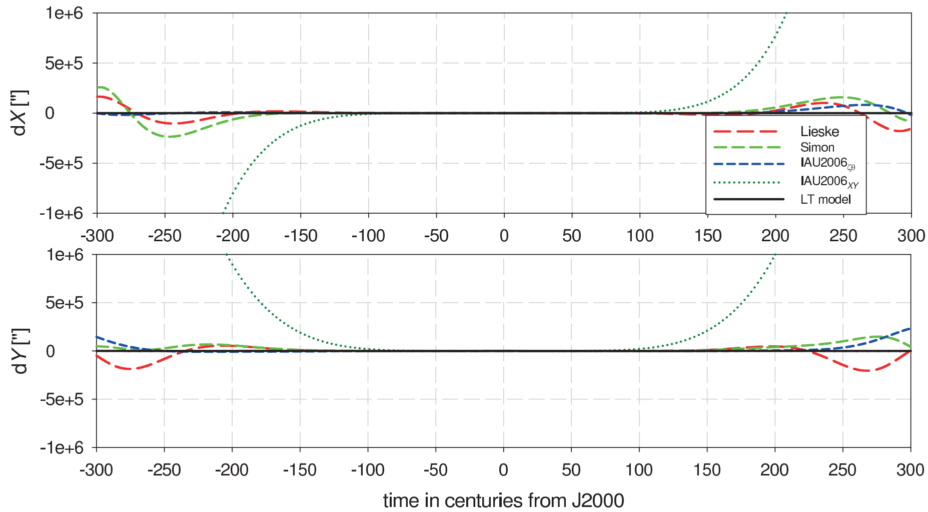

Figure 3: Comparison of different precession models with integrated values

mean frame of epoch as polynomials of time around J2000.0, yield much worse results than the X_A, Y_A expressions based on the 'traditional' precession angles ζ_A, θ_A. The new LT model is indistinguishable from the integration at this scale, whereas all other models display deviations reaching 50 degrees for epochs more distant than 200 centuries. Figure 4 clearly demonstrates the correction of precession rate, and also the quadratic term in obliquity, introduced since the IAU 2000 precession. On the other hand, all models shown are consistent with the numerically integrated precession within one arcsecond or so in the interval ±10 centuries from J2000.0.

5. CONCLUSIONS

The presently adopted IAU 2006 model provides high accuracy over a few centuries around the epoch J2000.0. For direction cosines X_A, Y_A, using direct IAU 2006 series combines precision and convenience in the modern era; however, for longer periods, expressions based on polynomial development of precession angles ζ_A, θ_A are preferable. The new set of precession expressions, given in our A&A paper and valid over ±200 millennia, is presented. Its accuracy is comparable to IAU 2006 model in the interval of several centuries around J2000.0, and it fits the numerically integrated position of the pole for longer intervals,

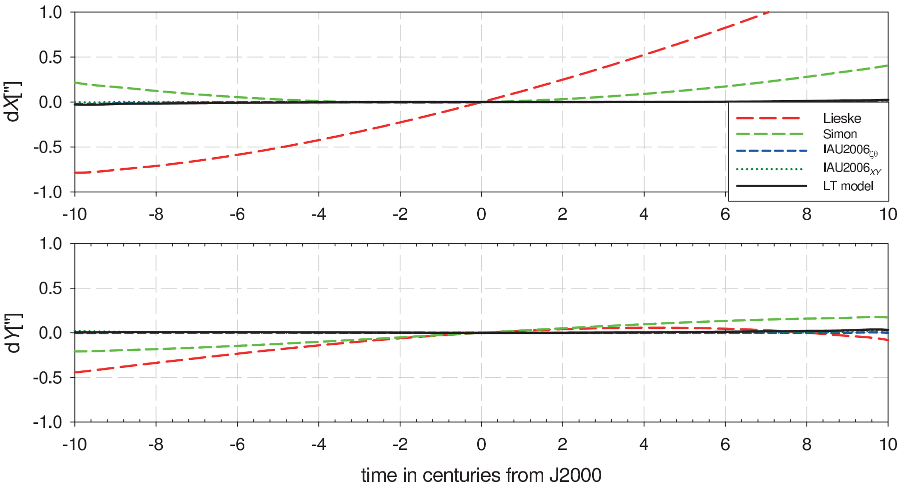

Figure 4: Comparison of different precession models with integrated values: closeup of the central part

with gradually decreasing accuracy (several arcminutes ±200 thousand years away from J2000.0).

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