THE INVARIABLE PLANE OF THE SOLAR SYSTEM: A NATURAL REFERENCE PLANE IN THE STUDY OF THE DYNAMICS OF SOLAR SYSTEM BODIES

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ABSTRACT. In this work we determine the orientation of the invariable plane of the solar system. The idea of using the invariable plane as a reference frame in the study of the dynamics of solar system bodies goes back at least to Laplace.
Using numerical planetary ephemerides : INPOP10a, DE405, DE406 over their available time span we have computed the orientation of the invariable plane for different epochs considering a time step of one day. This leads us to find a good agreement between the different values obtained, and can be considered as a good test of the ephemerides. The relativistic effects up to the first post-newtonian order were included through an effective mass, the so-called Tolmann-mass. Our results are considered as a confirmation and an improvement of the results obtained in Burkhardt (1982).

1. INTRODUCTION

The aim of this paper is to investigate the characteristics of the invariable plane and to define its orientation with respect to the ecliptic and the equator. The notion of the invariable plane was introduced by Pierre Simon Lalplace in his “Oeuvres Complètes”. He introduced what seemed to be the natural reference plane when studying the motion of planets, asteroids, comets ...

2. DEFINITION OF THE INVARIABLE PLANE

Let us consider the solar system as an isolated system so that its total angular momentum vector is constant with respect to both spatial and time coordinates. The invariable plane of the solar system is defined as the plane perpendicular to its total angular momentum and passing through its barycentre. Being fixed, it provides a permanent natural reference plane, whereas the ecliptic alters with time. Thus the invariable plane should be considered as the *natural reference plane* when studying the dynamics of solar system bodies.

Let us consider the following 10-body system : the Sun, Pluto and all the eight planets, the Earth being replaced by the Earth-Moon Barycentre (EMB). Disregarding the rotation of all the ten bodies constituting the system; the total orbital angular momentum of a the system is:

$$\mathbf{L_{tot}} = \sum_{j=1}^{N} m_j \mathbf{r_j} \otimes \dot{\mathbf{r}}_\mathbf{j} \tag{1}$$

m_j, $\mathbf{r_j}$, $\dot{\mathbf{r}}_\mathbf{j}$ are respcetively the mass, the barycentric position vector and barycentric velocity vector of the j^{th} body.

The relativistic effects being taken into account up to the first order in the post-Newtonian approximation, in Equation (1) m_j is replaced by an effective mass (the so-called Tolmann-Mass) m_j^*, given by Equation (2) (see Standish & al. 1976)

$$m_j^* = m_j . \Big[1 + \frac{\dot{\mathbf{r}}_\mathbf{j}^\mathbf{2}}{2c^2} - \frac{1}{2c^2} \Big(\sum_{k \neq j} \frac{G m_k}{|\mathbf{r_k} - \mathbf{r_j}|} \Big) \Big] \tag{2}$$

The components of the angular momentum ($\mathbf{L}$) are given as a function of the inclination (i) and the node (Ω) of the invariable plane.

$$\begin{aligned} L_1 &= \mathbf{L} \cdot \sin\Omega \cdot \sin i \\ L_2 &= -\mathbf{L} \cdot \cos\Omega \cdot \sin i \\ L_3 &= \mathbf{L} \cdot \cos i \end{aligned} \tag{3}$$

The primary constants for both the DE and the INPOP10a ephemeris are respectively given in (Standish, 1998) and (Fienga & al. 2010).

Numerical ephemeris	Times-span (JD)	Time-span	Number of days
INPOP10a	2076569.0 – 2826520.0	03 May 973 - 25 Aug 3026	749951
DE405	2305424.5 – 2524624.5	09 Dec 1599 to 31 Jan 2200	219200
DE406	625360.50 – 2816848.5	23 Feb -3001 to March 2nd, 3000	2191488

Table 1: Numerical ephemeris time span.

3. RESULTS

Using Equations (1) – (3), we have computed for each ephemeris the orientation of the invariable plane with respect to both the ICRS and the ecliptic-equinox of J2000.

	DE405 / DE406		INPOP10a		Burkhardt, 1982	
	Ecliptic	Equator	Ecliptic	Equator	Ecliptic	Equator
i at J2000	$1°34'43''.33124$	$23°0'31''.98231$	$1°34'43''.31903$	$23°0'31''.97914$	$1°35'13''.86$	$23°0'22''.11$
Ω at J2000	$107°34'56''.17914$	$3°51'9''.45913$	$107°34'56''.47403$	$3°51'9''.42191$	$107°36'30''.8$	$3°52'23''.7$

Table 2: Orientation of the invariable plane at the epoch J2000.

Ephemeris	DE405		DE406		INPOP10a	
	Ecliptic	Equator	Ecliptic	Equator	Ecliptic	Equator
minimal i	$1°34'43''.33115$	$23°0'31''.98185$	$1°34'43''.33064$	$23°0'31''.97992$	$1°34'43''.31883$	$23°0'31''.97657$
maximal i	$1°34'43''.33129$	$23°0'31''.98321$	$1°34'43''.33148$	$23°0'31''.99351$	$1°34'43''.32005$	$23°0'31''.98120$
minimal Ω	$107°34'56''.14530$	$3°51'9''.45884$	$107°34'55''.76351$	$3°51'9''.45769$	$107°34'56''.38804$	$3°51'9''.41993$
maximal Ω	$107°34'56''.19574$	$3°51'9''.45968$	$107°34'56''.26643$	$3°51'9''.46523$	$107°34'56''.57119$	$3°51'9''.42587$

Table 3: Orientation of the invariable plane over the ephemeris time span (See Table 1).

4. CONCLUSIONS

We have determined the orientation of the invariable plane with respect to both ICRS and equinox-ecliptic J2000. The small differences observed in the computed values are due to the use of numerical ephemerides with different primary parameters. Thus we are conditioned by the precision of the ephemerides. Nevertheless we observe a good agreement between the DE405 and INPOP10a ephemeris. We think that such a determination of the invariable plane is of fundamental interest in the topic of solar system studies. These results will be refined by taking into account the effects of the largest asteroids.

5. REFERENCES

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