GENERAL RELATIVISTIC DELAYS IN CURRENT AND FUTURE VLBI

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ABSTRACT. The effect of gravitational time delay due to general relativity is clearly visible in VLBI measurements. While the Sun and the Earth cause gravitational delays up to some nanoseconds respectively picoseconds in geodetic VLBI observables, standard delay models also recommend the inclusion of Jupiter, the Earth's Moon and some of the other planets on the modeling side.

Using the IERS Conventions model, we have calculated the gravitational time delay of standard VLBI observation constellations of the past years. The influences due to the Sun, the Earth, the Moon, Jupiter, Saturn and Venus were large enough to justify their consideration in routine VLBI data analysis (based on the accuracy of the upcoming VLBI2010 system). For VLBI data before 2002, the higher order term of the Sun should also be taken into account. Additionally, the relation between the gravitational delay of the Earth and the estimated tropospheric parameters (e.g. zenith wet delays) was investigated.

1. INTRODUCTION

From General Relativity we know that electromagnetic waves are affected by gravity in terms of bending and dilation. This is also the case for the signals coming from the extragalactic radio sources which are observed by VLBI. As both ray paths in VLBI usually do not pass a gravity source at the same distance, the individual rays are bent and delayed in a different way. VLBI observes the relative time delay τ of the signals' arrival time at the two stations and therefore only the relative gravitational delay ΔT_{grav} between the two rays is of interest. This delay, also called Shapiro delay, is included in the IERS Conventions model for VLBI (Petit and Luzum, 2010). The calculation is based on the so-called consensus model from the beginning of the 90s. The main term can be computed as follows:

$$\Delta T_{grav} = 2\frac{GM}{c^3} \ln \frac{|\vec{R}_1| + \vec{K} \cdot \vec{R}_1}{|\vec{R}_2| + \vec{K} \cdot \vec{R}_2} \tag{1}$$

with G being the gravitation constant, M the mass of the gravitating body, c the speed of light, $\vec{K}$ the barycentric unit vector towards the radio source and $\vec{R}_i$ the vectors between the gravitating body's center of mass and the two telescopes. The factor 2 comes from $1 + \gamma$, γ being the PPN parameter describing the curvature of space due to the existence of mass. In general relativity $\gamma = 1$ (Soffel, 1989). The model of the IERS Conventions is designed to assure an accuracy of 1 ps and therefore includes all terms of the order of 0.1 ps. It recommends to include the gravitational effects of the Sun, the Earth, the Moon, and most other planets.

2. AIMS

In the next decade, with the upcoming VLBI2010 system, new VLBI technology, new types of antennas (small dishes, fast slew rates), and new observing strategies will be used (Petrachenko et al., 2009). The number of observations will increase significantly. The aim is to reach a position accuracy of 1 mm. On this basis, the theoretical model should at least include all terms of the order of 0.1 mm or 0.3 ps (Heinkelmann and Schuh, 2009). Compared to the accuracy level of the current IERS model, the level derived from VLBI2010 is less rigorous, but based on realistic prospects. Currently, the influence of some of the recommended bodies is not yet crucial since there are error sources of much greater magnitude. In the following chapter we will describe which terms and celestial bodies should be included when considering the VLBI2010 accuracy limits.

3. THEORETICAL DELAYS

Based on the observation constellations of the past years, theoretical delays according to the IERS Conventions were computed. They were split up into several terms of interest. Basically, for every observation and every important celestial body equation (1) was evaluated. The constellations of the R1 and R4 sessions by the International VLBI Service for Geodesy and Astrometry (IVS; Schlüter and Behrend, 2007) between 2002 and 2009 were used. These are sessions observed twice a week to determine Earth orientation parameters. In total, about 1.5 million observation constellations were processed. The Vienna VLBI Software (VieVS) (Böhm et al., 2011) was used for the computations.

For several observations, the delay caused by the Sun reached values >1 m when multiplied with the speed of light and 99.9% were above the 0.1 mm/0.3 ps limit (Table 1). In the case of the gravitational influence of the Earth, this ratio was 96.0% with maximum values of 5-6 mm. The left plot of Figure 1 shows the computed delays caused by Jupiter. Roughly 1 out of 50 observations had a gravitational delay due to Jupiter larger than 0.3 ps. Jupiter is therefore a very important body to consider. Next important was the influence of Saturn (Figure 1, right plot) with 0.1% of 1.5 million delays above the limit. The Moon's impact was still lower (0.02%). The least important celestial body to have gravitational delays above the 0.3 ps level was Venus (for nine observations in total the limit was surpassed). Neptune and Uranus reached delays of 0.1 ps. The inclusion of these two planets is recommended by the IERS Conventions model but they are not necessary if we base our limits on the 1 mm accuracy of VLBI2010. Mercury and Mars can be neglected in any case.

	Sun	Earth	Jupiter	Saturn	Moon	Venus	Neptune	Uranus
abs. > 1 ps	1515853	1322912	2923	73	5	0	0	0
rel. [%]	99.70	87.01	0.19	0.00	0.00	0	0	0
abs. > 0.3 ps	1519034	1459686	29674	1327	274	9	0	0
rel. [%]	99.91	96.01	1.95	0.09	0.02	0.00	0	0
abs. > 0.1 ps	1519964	1500156	242522	10189	2150	117	33	5
rel. [%]	99.97	98.67	15.95	0.67	0.14	0.01	0.00	0.00

Table 1: Number of observation constellations with gravitational delays larger than 1 ps, 0.3 ps and 0.1 ps and their percentage compared to the 1520401 constellations of the IVS-R1 and IVS-R4 sessions between 2002 and 2009.

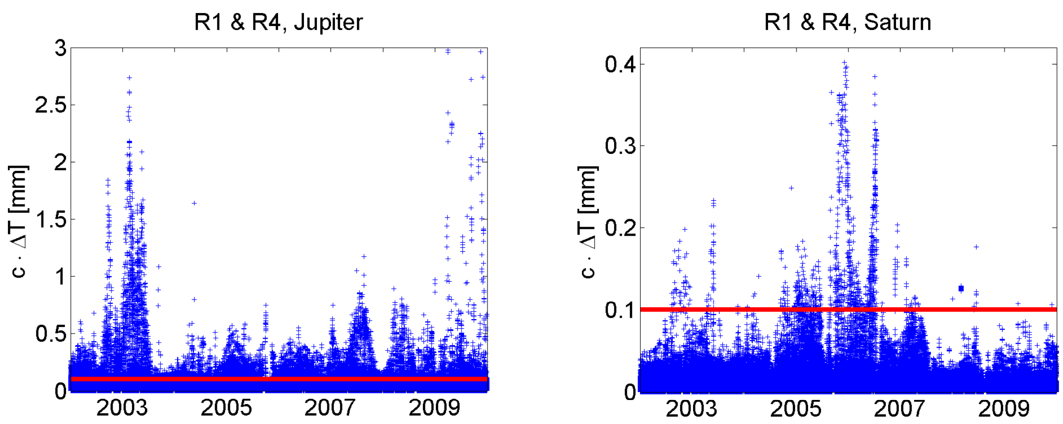

Figure 1: Gravitational delays for IVS-R1 and IVS-R4 sessions between 2002 and 2009 due to Jupiter (left) and Saturn (right) with respect to 0.3 ps/0.1 mm (red line). Every symbol represents the delay for one observation constellation.

4. SUN'S TERM OF HIGHER ORDER

The term depicted in Equation (1) is only the main term to describe the gravitational influence. There are also terms of higher order, with the next most important being caused by the bending of the ray path (Petit and Luzum, 2010):

$$\delta\Delta T_{grav} = 4\frac{G^2M^2}{c^5}\frac{\vec{b}\cdot(\hat{R}_1+\vec{K})}{(|\vec{R}_1|+\vec{R}_1\cdot\vec{K})^2} \tag{2}$$

This correction is significantly smaller and recommended to apply for observations close to the Sun. However, such observations are also strongly influenced by the Sun's radiation that reduces the coherence of the signals. To avoid measurements of low quality (especially during periods of high solar activity), in the mid of 2002 a stringent cut-off elongation angle of about 15° towards the Sun was introduced by the IVS. In analogy to the main terms of the gravitational delay of the different celestial bodies, the question arises with respect to this term of higher order for the Sun whether an inclusion in theoretical models is justified. The considered limit is again 0.1 mm/0.3 ps.

For analysing the effect of the cut-off elongation angle introduced in 2002, VLBI data from the begin of 1998 to the end of 2003 were processed. For all observation constellations during this period (roughly 1.9 million) the term of higher order of the Sun was computed according to Equation (2). Figure 2 (left plot) shows the theoretical delays for all of these constellations. It can be recognized that before 2002 there are a few observations with gravitational delays >0.3 ps and after 2002 there are none.

5. EARTH'S INFLUENCE ON STATION COORDINATES

In Chapter 3 we have shown that the influence of the Earth on VLBI delays can reach up to 6 mm, indicating the role of the Earth as the second most important body to consider after the Sun. In this chapter we want to show the influence of the Earth on the estimated station coordinates.

The gravitational delay of the Earth is calculated using Equation (1) with R_i as the geocentric coordinate vectors of the stations. For the Earth, Schuh (1987) states that this equation can be written in dependence of the elevation angles E_i of both radio telescopes:

$$\Delta T_{grav,E} = 2\frac{GM_E}{c^3}\ln\frac{1+\sin(E_1)}{1+\sin(E_2)} \tag{3}$$

In this representation it becomes evident that the difference of the elevation angles and the gravitational delay due to the Earth are positively correlated. In this respect the gravitational influence of the Earth is closely related to that of the troposphere. Here the mapping functions of the different stations are strongly dependent on the elevation angles (in the simplest case $\frac{1}{\sin E}$). Since the tropospheric influence on the delay τ is calculated as the difference between the influences of the respective stations, this effect is also larger with big elevation angle differences between the stations. Because of this correlation between troposphere and Earth gravitation we supposed that there is a difference if you estimate station coordinates including an estimation of troposphere parameters like zenith wet delays and gradients or if these parameters are not considered.

For these investigations, we created artificial observation files with VieVS based on the schedule of the R1 and R4 sessions between 2002 and 2009. The observables in these files are identical to the calculated time delays for the respective observation constellations. Solving for the station coordinates using these artificial files combined with a slightly modified theoretical model (in our case: exclusion of the gravitational influence of the Earth) provides corrections that show solely the effect of the modified parameters. The corrections for the coordinates of the 19 VLBI sites involved in the sessions were estimated for each session and then averaged over the whole time span. The same procedure was repeated including the estimation of the tropospheric parameters.

The gravitational influence of the Earth without estimating the troposphere was 2–7 mm for the radial components / station heights (Figure 2, right plot) with an average of 4.0 mm corresponding to ∼0.6 ppb in the scale of the corresponding TRF from VLBI. The amount of this effect was dependent on the station distribution: stations on the southern hemisphere or in Japan usually are part of longer baselines and therefore have been more affected than stations located in or near Europe. When comparing these results with those including troposphere estimation it became evident that the gravitational influence declined. This effect was up to 1 mm for the radial components. This means that there was in fact a noticeable correlation between the gravitational and tropospheric influences. Some of the model error caused by disregarding the Earth influence was soaked up by the tropospheric parameters.

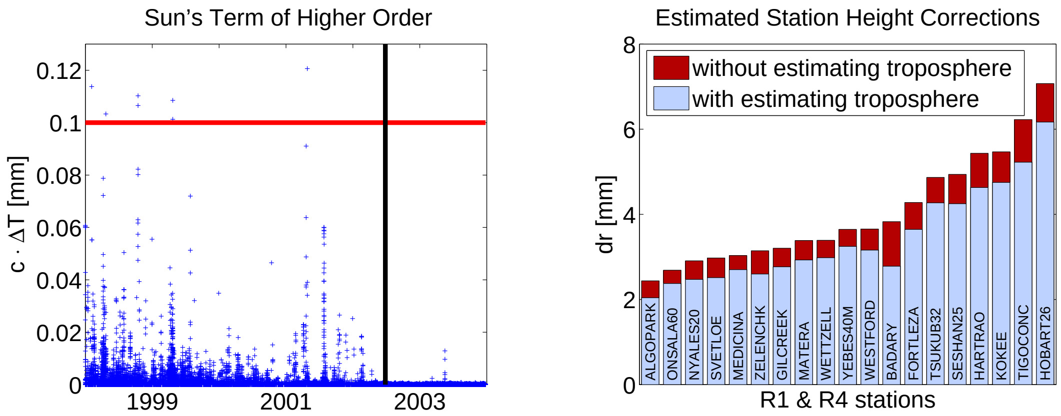

Figure 2: Left: Gravitational delays of higher order for sessions between 1998 and 2003 due to the Sun with respect to 0.3 ps/0.1 mm (red line). Cut-off elongation angle was changed from 5° to 15° since mid of 2002 (black line). Right: Mean influence of Earth gravitation on station heights for IVS-R1 and IVS-R4 sessions between 2002 and 2009 with/without estimating tropospheric parameters.

6. CONCLUSION

In this paper it is shown that there have been observation constellations in the past in which the theoretical gravitational influence of certain celestial bodies of our solar system are of a magnitude which cannot be neglected when aiming to achieve a positional accuracy of 1 mm (VLBI2010). These bodies include the Sun, the Earth, Jupiter, Saturn, the Earth's Moon and Venus. It is suggested that in new versions of software packages for VLBI computations these findings are considered by adapting the implementation of the gravitational delay model accordingly.

The influence of Earth's gravitation on the estimated station coordinates was also investigated by calculating this effect on the radial components with 2–7 mm. The size of this effect was dependent on the geometric station distribution. When additionally estimating tropospheric parameters it becomes evident that there is a correlation between the gravitational influence of the Earth and the troposphere: zenith wet delays and gradients absorb up to 1 mm of the gravitational influence.

7. REFERENCES

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