RELATIVITY AND LARGE RING-LASER GYROSCOPES

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ABSTRACT. This article deals with a post-Newtonian description of large ring-laser gyroscopes. To this end first two local topocentric reference systems are introduced: a topocentric celestial (ToCRS) and a topocentric terrestrial reference system (ToTRS). The GCRS acts as starting point for these two systems. Whereas the ToCRS is kinematically non-rotating with respect to the GCRS the spatial coordinates of the ToTRS are determined by the ITRS. From the covariant Maxwell-equations a post-Newtonian expression for the Sagnac frequency shift is derived containing contributions from the geodetic-, Lense-Thirring- and Thomas-precession. These relativistic contributions are calculated as a function of some orientation angle α of the Sagnac platform. Conditions for the measurability of these terms by a system of laser-gyros are discussed.

1. CONCEPT OF LARGE RING-LASER GYROSCOPES

A ring-laser gyroscope that we will consider, consists basically of a closed tube filled with He-Ne gas in which laser activity is excited so that TWO laser beams, one traveling in clockwise the other one in counter-clockwise direction, interfere behind a beam splitter where the interference fringes can be analyzed (Figure 1). Such a gyroscope is an inertial device; if it rotates with respect to local inertial one sees a frequency difference between the co-rotating and the counter-rotating beam being proportional to the angular velocity $\mathbf{\Omega}$. This beat frequency or Sagnac frequency Δf is described by the Sagnac formula for active resonators:

$$\delta f = \frac{4A}{\lambda P}\,\mathbf{e}_A \cdot \mathbf{\Omega}\,.$$

Here A is the area enclosed by the laser-beams, λ the effective wavelength of the two beams, P the perimeter of the enclosed area and $\mathbf{e}_A$ is the unit normal vector perpendicular to it. For the 4m x 4m large G-ring at the geodetic fundamental station in Wettzell, Germany, this Sagnac frequency shift is 348.643 Hz.

2. RELATIVISTIC EXPRESSION OF THE SAGNAC FREQUENCY

To theoretically formulate the Sagnac frequency shift we have introduced two new reference systems: a TOpocentric Celestial Reference System (ToCRS) and a Topocentric Terrestrial System (ToTRS). Whereas the ToCRS is assumed to be kinematically non-rotating with respect to the Geocentric Celestial Reference System (GCRS) the spatial coordinates of the ToTRS are determined by the ITRS.

The ToCRS can principally be derived from results in the literature (Klioner & Soffel, 1998); as a check we have re-derived the corresponding metric tensor within the DSX-framework (Damour et al., 1991). We started with the GCRS with coordinates $(cT, \mathbf{X})$ and the metric tensor in the form

$$\begin{aligned}
G_{00} &= -1 + \frac{2W}{c^2} - \frac{2W^2}{c^4} + \mathcal{O}(c^{-5}) \\
G_{0a} &= -\frac{4}{c^3} W_a + \mathcal{O}(c^{-5}) \\
G_{ab} &= \delta_{ab}\left(1 + \frac{2}{c^2} W\right) + \mathcal{O}(c^{-4})
\end{aligned}$$

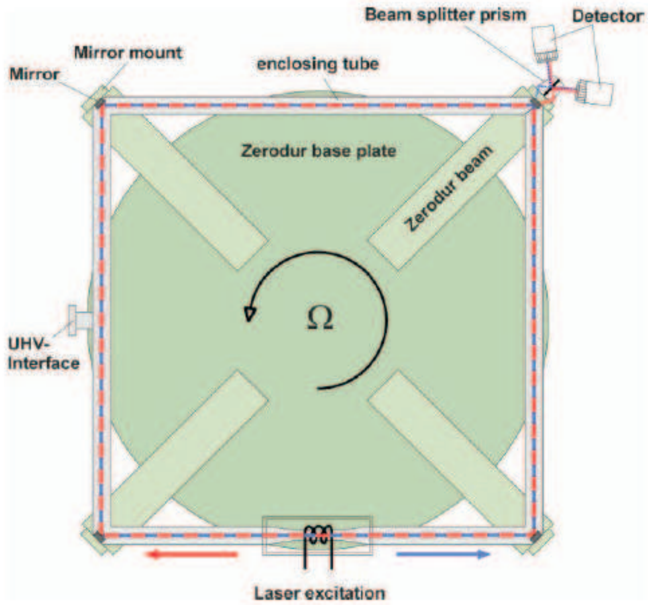

Figure 1: Schematic diagram of a ring-laser gyroscope.

with the geocentric metric potentials W and W_a. From the GCRS we then transformed to ToCRS coordinates $(c\tau, \mathbf{X}_T)$ according to Damour et al. (1991). As expected the local (linearized) metric has potentials

$$W_T = -\mathcal{A} \cdot \mathbf{X}_T \,, \qquad W_T^a = -\frac{c^2}{4}(\mathbf{\Omega} \times \mathbf{X}_T)^a \,.$$

Here $\mathcal{A}$ is the 4-acceleration of the topocenter, i.e, the (non-gravitational) acceleration of the topocenter as seen from a freely falling observer and, in case of the ToCRS, the angular velocity $\mathbf{\Omega} = \mathbf{\Omega}_{\rm iner}$ with

$$\mathbf{\Omega}_{\rm iner} = \mathbf{\Omega}_{\rm GP} + \mathbf{\Omega}_{\rm LT} + \mathbf{\Omega}_T \,.$$

The expressions for $\mathbf{\Omega}_{\rm GP}$ (geodetic precession), $\mathbf{\Omega}_{\rm LT}$ (Lense-Thirring) and $\mathbf{\Omega}_{\rm T}$ (Thomas precession) read

$$\mathbf{\Omega}_{\rm GP} = -\frac{3}{2c^2}\mathbf{V} \times \nabla W \,, \quad \mathbf{\Omega}_{\rm LT} = -\frac{2}{c^2}\nabla \times \mathbf{W} \,, \quad \mathbf{\Omega}_{\rm T} = \frac{1}{2c^2}\mathbf{V} \times \mathcal{A} \,,$$

where $\mathbf{V}$ is the GCRS velocity of the topocenter (a central point of the Sagnac platform), W is basically the Newtonian gravitational potential and the gravito-magnetic potential, $\mathbf{W}$, appearing in the Lense-Thirring part is determined by the Earth's intrinsic angular momentum $\mathbf{S}_{\rm E}$:

$$W_{\rm E}^a = -\frac{G}{2}\frac{(\mathbf{X} \times \mathbf{S}_{\rm E})^a}{R^3} \,.$$

The introduction of a ToTRS generalizes the DSX-framework by allowing for a fast rotation between the GCRS and the ToTRS. We define the ToTRS such that $\Omega_{\rm iner}$ above is replaced by $\mathbf{\Omega}_{\rm E} + \mathbf{\Omega}_{\rm iner}$, where $\Omega_{\rm E}$ is the angular velocity determined by the classical transformation between the GCRS and the ITRS.

From the relativistic Maxwell equations for the laser beams one then finds (e.g., Soffel, 1989)

$$\delta f = S \cdot \Omega^* \,,$$

where $S = (4A/\lambda P)$ is the scale factor and $\Omega^* = (\mathbf{\Omega}_{\rm E} + \mathbf{\Omega}_{\rm iner}) \cdot \mathbf{e}_A$. If Φ_0 denotes the nominal latitude of the topocenter and α the tilt angle of $\mathbf{e}_A$ reckoned from the radial direction for Φ_0 towards the equator we get (see also Bosi et al., 2011)

$$\Omega^* = \Omega_{\rm E}\left(\sin(\Phi_0 - \alpha) + 2\frac{GM_{\rm E}}{c^2 R_{\rm E}}\cos\Phi_0 \sin\alpha - \frac{2}{5}\frac{GM_{\rm E}}{c^2 R_{\rm E}}(2\sin\Phi_0\cos\alpha + \cos\Phi_0\sin\alpha)\right) \,,$$

where we have used the moment of inertia of a rigid sphere in the expression for $\mathbf{S}_{\mathrm{E}}$.

To measure these relativistic effects very high demands must be satisfied (see also Di Virgilio et al., 2010; Bosi et al., 2011)

- sensitivity to rotation of 0.01 prad/s for about 1 hour of integration
- sensor stability of 1 part in 10^{10} over months to years
- sensor orientation to 1 nrad
- Length of Day (LoD) to 0.1 μs.

Obviously no present laser-gyro meets these requirements. The G-ring in Wettzell presently has a sensitivity of about 1 prad/s for one hour of integration time. To increase sensitivity one might, e.g., enlarge the enclosed area A. For the G-ring the required stability might be achievable for a certain period of time due to the Cerodur base plate with thermal expansion of less than 5×10^{-9}/deg, a good thermal isolation and a pressure stabilized enclosure. Also several feedback loops contribute to such a good stability. However, annual temperature variations of some 0.5 deg and internal photon backscatter are still problematic. The orientation of the platform is affected by several local effects, e.g., related with local hydrology (rainfall) that can be reduced significantly by going to larger depths below the ground. It has been suggested to perform such an experiment e.g., in the Gran Sasso laboratory in Italy, 120 km from Rome, with an average rock coverage of some 1400 m (Di Virgilio et al., 2010; Bosi et al., 2011). Finally, to monitor the LoD variations data from geodetic space techniques like VLBI have to be employed. The IERS Bulletin B gives a mean formal error of $4\,\mu$s for LoD-variations so one needs an improvement of about one order of magnitude here. Local contributions to $\mathbf{\Omega}_{\mathrm{E}}$, e.g., induced by near surface effects (thermo-elastic, topography, geology, meteorology) are likely to be reduced significantly deeply underground. To have full information of the rotation vector some multi-ring structure, e.g., mounted to the six faces of a cubic monument has been suggested (Bosi et al., 2011).

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3. REFERENCES

Bosi, F., Cella, G., Di Virgilio, A., et al., 2011, arXiv:1106.5072v1
Damour, T., Soffel, M., Xu, C., 1991, Phys.Rev. D **43**, 3273
Di Virgilio, A., et al., 2010, J.Mod.Phys., D **19**, 2331
Klioner, S., Soffel, M., 1998, Phys.Rev.D **58**, 084023
Soffel, M., 1989, Relativity in Astrometry, Celestial Mechanics and Geodesy, Springer, Berlin