# EFFECTS OF TROPOSPHERIC SPATIO-TEMPORAL CORRELATED NOISE ON THE ANALYSIS OF SPACE GEODETIC DATA

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ABSTRACT. The standard VLBI analysis models measurement noise as purely thermal errors modeled according to uncorrelated Gaussian distributions. As the price of recording bits steadily decreases, thermal errors will soon no longer dominate. It is therefore expected that troposphere and instrumentation/clock errors will increasingly become more dominant. Given that both of these errors have correlated spectra, properly modeling the error distributions will become more relevant for optimal analysis. This paper will discuss the advantages of including the correlations between tropospheric delays using a Kolmogorov spectrum and the frozen flow model pioneered by Treuhaft and Lanyi. We will show examples of applying these correlated noise spectra to the weighting of VLBI data analysis.

## 1. INTRODUCTION

Errors in VLBI data have, until recently, been somewhat dominated by thermal noise, which is characterized by Gaussian uncorrelated random distributions. It is expected that as the cost of recording bits goes down data rates will increase. This means that there will be up to a factor of ten reduction in thermal errors in the near future. As thermal noise contributions drop correlated noise sources will begin to dominate the error budget. Until now, the correlated nature of error sources such as those induced by clocks and troposphere fluctuations have not been given much consideration given that their contribution was small compared to thermal errors. In this paper we will briefly review the correlated delay error model of the troposphere developed by Treuhaft and Lanyi (1987) and present a simplified application to a set of VLBA catalogue runs.

For about the last ten years VLBI systems have been recording at a rate of 128 Mbps. Recent research and development runs with the VLBA have been recording at 256 Mbps giving a $\sqrt{2}$ improvement in signal to noise ratio. The VLBA currently provides the option to record at 512 Mbps giving a factor of two improvement. There have also been research and development VLBA fringes recorded at 2 Gbps per second with the Mark-5C giving a factor of 4 improvement. NASA Deep Space Network (DSN) can achieve recording rates of 2 Gbps with a Mark-5C and 4 Gbps if running with two disk packs at a time. Finally, the Haystack Mark-6 recorder can achieve 16 Gbps recording with short term 32 Gbps rates. The fast improvement in recording data rates means that there will be factors 10 improvement in the near future. In addition, sampling the data near the front end means that thermal variations in filters and cables may be greatly reduced. All of these technology improvements will yield a factor of 10 improvement in the thermal error contribution in the near future.

The impact of tropospheric turbulence on VLBI data has been probed with water vapor radiometers (WVRs). An experiment was performed with the DSN 8000 km Goldstone-Madrid baseline where each station had a JPL advanced water vapor radiometer (A-WVR) (Bar-Sever et al. 2007) monitoring the brightness temperature of the 22 GHz water line along the line of site. The antennas were observing a very strong source and measuring phase delay to reduce the thermal error contribution to negligible levels. The results of the experiment showed that including the observations of the A-WVR reduces the scatter of the delay residuals from 3mm to 1mm.

Unfortunately water vapor radiometers cost about a half million dollars and they are not available at every station. However, we can understand something about the correlation of troposphere fluctuation errors based on the Kolmogorov (1941) theory of turbulence. Essentially, density fluctuations in the water vapor content of the troposphere lead to fluctuations in observed delays. The spatial and temporal scales of these density fluctuations will determine the degree of troposphere induced correlation between delay errors at different stations.

In VLBI the observing unit is not a single antenna but rather an antenna pair or baseline. The error models currently applied to VLBI data analysis do not include common sources of error to both stations in a baseline but rather treat station errors individually. The inclusion of correlated error models in the analysis of VLBI data is expected to improve results by properly treating common sources such as large scale troposphere fluctuations. We attempt to estimate the impact of applying correlated error models with VLBA K-band catalogue data (Lanyi et al. 2010; Charlot et al. 2010). Seeing improvements with current thermal error levels provides an indication of the importance of developing and applying correlated error models for future analysis of VLBI.

This paper is organized as follows. In Section 2 we review the correlated troposphere delay error model by Treuhaft and Lanyi (1987). In Section 3 we present the results of an application of the Treuhaft-Lanyi model on archival VLBA data and DSN X/Ka data. The results are compared to analysis with uncorrelated errors. We discuss the results and future analysis.

## 2. CORRELATED TROPOSPHERE ERROR MODEL

Kolmogorov's theory of turbulence provides a handle on the spatial scales of troposphere fluctuations. The structure function of a turbulent velocity $\mathbf{u}$ is defined as $D_{\mathbf{u}}(R) = \left\langle |\mathbf{u}(\mathbf{r}+\mathbf{R}) - \mathbf{u}(\mathbf{r})|^2 \right\rangle$ where $\mathbf{r}$ is a position in the sky and $\mathbf{R}$ is an offset vector. Kolmogorov (1941) used dimensional arguments to determine that the structure function is given by a power law $D_{\mathbf{u}}(R) = C\epsilon^{2/3}R^{2/3}$ where $\epsilon$ is the average energy dissipation of the medium and $C$ is a medium independent constant. This result assumes only that the turbulence is locally isotropic and that $R > (\nu^3/\epsilon)^{1/4}$ where $\nu$ is the viscosity of the medium.

In Tatarskii (1961) the structure function for spatial variations of the refractivity $\chi =$ ( index of refraction - 1 ) are determined based on the Kolmogorov (1941) theory of turbulence. This is given by $D_\chi(R) = C^2R^{2/3}$ where the constant $C$ is independent of spatial coordinates $\mathbf{r}$ and $\mathbf{R}$. Starting from this point Treuhaft and Lanyi (1987) translated the turbulent refractivity structure function into radio signal delay and delay rate structure functions. The electromagnetic wave delay ($\tau$) observed at an antenna is obtained by integrating the refractivity along the path of propagation.This relation naturally connects the structure function in refractivity $D_\chi$ to the structure function in delay $D_\tau$. The covariance of delays measured at different stations due to troposphere fluctuations on VLBI data are determined directly from the structure function.

The delay structure function (Figure 1) behaves as a broken power law whose exponent depends on the antenna separation $\rho$ relative to the effective height of the troposphere ($h \sim 2$ km). The break can be intuitively understood as the difference between delay fluctuations being influenced by many small scale scatterers when $\rho/h < 1$ rather then being influenced by a few large scale scatterers $\rho/h > 1$. The magnitude of the structure function depends on the elevation angle since this determines the amount of troposphere the electromagnetic wave has traversed.

The spatial dependence of the delay structure function can be translated to a temporal dependence via the frozen flow model. This assumes that the overall turbulent structure of the troposphere is shifted in time with the direction of the wind. The wind speed scales relevant to the frozen flow model are typically $\sim 10$ m/s. For all the mathematical details for the calculation of structure functions, variances, and covariances as well as examples with DSN data see Treuhaft and Lanyi (1987).

## 3. TESTS OF TROPOSPHERE COVARIANCE ON VLBA K-BAND

We performed a simplified test of the effects of correlated troposphere errors on a data set obtained with the VLBA. The VLBA consists of an array of ten antennas giving a total of 45 baselines. The longest baseline is from the Mauna Kea station in Hawaii to St. Croix in the U.S. Virgin Islands. The shortest baseline in the array is 240 km between the Los Alamos and Pie Town stations in New Mexico.

As thermal errors are reduced with increasing recording rates the data error models cannot be treated as 45 independent instruments. Tropospheric and instrumental correlated errors begin to affect the ability to average down random errors from 45 baselines. Our goal in this study is to see if, in the current state of bit recording capabilities and thermal error levels, the inclusion of correlated troposphere errors make a difference in the data analysis.

The data used in this study consists of twelve sessions of 24 hour source catalog runs in K-band dating from 2002 to 2008. For details on the data see (Lanyi et al. 2010; Charlot et al. 2010). The data is recorded at 128 Mbit/s and have median thermal error levels of 23 ps in delay and 5 fs/s in delay rate. With future recorders a factor of 10 improvements means that delay errors will be reduced to 2 ps. This is equivalent to position errors going down from 1 cm to 1 mm. The question we try to address with these data is whether we can begin to see the effects of correlated errors with thermal delay errors of $\sim$ 23 ps. Troposphere turbulence alone is expected to introduce errors of 20-40 ps which is comparable to the thermal error contribution.

The inclusion of troposphere covariance error models on large data sets involving many baselines has not been thoroughly tested in software. For this reason we have chosen to simplify the setup by using only 9 of the 45 baselines. The 9 baselines all have the Mauna Kea station in Hawaii in common. Mauna Kea is chosen due to the fact that it is the most distant station from the rest of the VLBA network so its troposphere is expected to be the least correlated to the other 9 stations. All inter-station correlations were ignored. We modeled only the intra-station correlations between ray paths of scan $i$ with scan $j$.

Rather than following the standard practice of introducing troposphere breaks every 20 minutes we use only one troposphere parameter estimate for each station over a 24 hour period. Station clock parameter estimates are included in three hour periods. The advantage of including troposphere covariance is the it re-defines $\chi^2$ such that drifts are not penalized nearly as much. Also the number of free parameters is reduced from what would typically be 1200 troposphere and clock breaks to 74.

As a figure of merit for the quality of reconstruction we compare the catalog solution of our reduced 9-baseline test to the ICRF2 catalog (Fey et al. 2009). In particular we look at the difference in reconstructed source declination ($\Delta\delta$) versus declination (dec) since we expect this to be most affected by the troposphere. Figure 2 shows the results of $\Delta\delta$ vs. dec for twelve catalog runs each lasting 24 hours with and without troposphere covariance. Both cases include a daily troposphere break and 3 hour clock parameter breaks. A small improvement was found in the slope and offset of the $\Delta\delta$ vs. dec relation. The results are summarized in Table 1.

We begin to see some improvement in the difference in declination between the catalog solution to ICRF2. The weighted RMS of the difference is decreased. Also note the the plots have a small slope which is expected to be due to the fact that low declinations mean that observations are made through thicker atmosphere profiles. Both the slope and the offset of the linear fits are reduced with the introduction of troposphere covariance error models.

## 4. OUTLOOK AND CONCLUSIONS

In this study we have shown that the inclusion of troposphere covariance provides improved results. However, the improvements have not been shown to be statistically significant. It is expected that as thermal error levels decrease the inclusion of troposphere covariance error models will show more significant improvements in the results. The implementation of troposphere covariance in the full VLBA 45 baseline solution is still pending. It also remains to be conclusively shown that using correlated error models is a better approach than introducing frequent troposphere and clock parameter estimation rates. Our preliminary results introducing covariant error models and physical reasoning indicate that there is promise in this approach.

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## 5. REFERENCES

Bar-Sever, Y.E. et al, 2007 "Atmospheric Media Calibration for the Deep Space Network", Proc. IEEE, vol. 95, No. 11, pp. 2180-2192.

Charlot, P. et al. 2010. "The Celestial Reference Frame at 24 and 43 GHz. II. Imaging", *The Astronomical Journal* , **139**, 1713-1770.

Fey, A. L., et al. 2009, "The Second Realization of the International Celestial Reference Frame by Very Long Baseline Interferometry", Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie.

Kolmogorov, A.N. 1941, "Dissipation of energy in locally isotropic turbulence", *Doklady Acad. Nauk USSR*, **32**, 16.

Lanyi, G.E. et al. 2010. “The Celestial Reference Frame at 24 and 43 GHz. I. Astrometry”, *The Astronomical Journal* , **139**, 1695-1712.
Tatarskii, V.I. 1961, *Wave Propagation in a Trubulent Medium* , Dover, New York.
Treuhaft R.N. and Lanyi, G.E. 1987, “The effect of the dynamic wet troposphere on radio interferometric measurements”, *Radio Science*, **22**, 251-265.

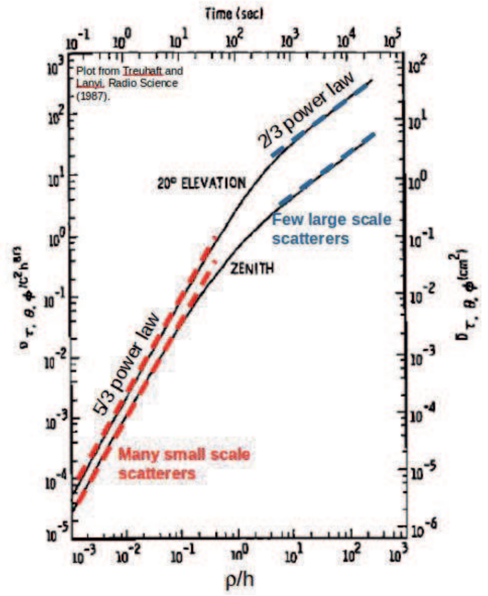


Figure 1: Delay structure function from Treuhaft and Lanyi (1987) is given by a broken power law whose exponent depends on the antenna separation $\rho$ relative to the troposphere effective height ($\sim$ 1 km). See text for details.

| $\Delta\delta$ vs dec | No Trop. Cov. | With Trop. Cov. |
|---|---|---|
| wRMS ($\mu$as) | 436 | 350 |
| Y0 ($\mu$as) | 178 $\pm$ 57 | 135 $\pm$ 71 |
| Slope ($\mu$as/deg) | -2.6 $\pm$ 1.0 | -2.0 $\pm$ 1.2 |
| $\chi^2$ | 1.91 $\pm$ 0.17 | 0.81 $\pm$ 0.07 |

Table 1: Comparison of VLBA catalog positions with ICRF2 for analysis with and without implementation of troposphere covariance error modeling.

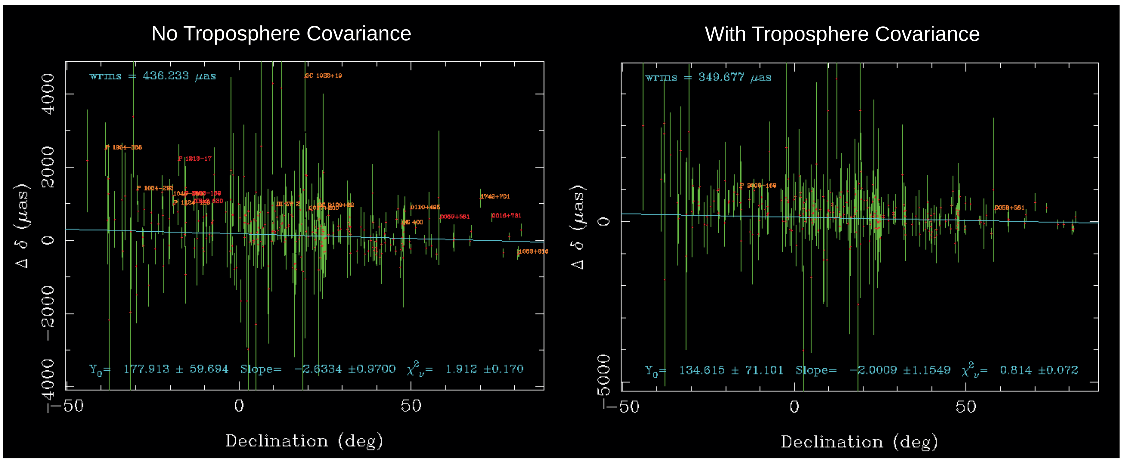


Figure 2: Comparison of VLBA K-band catalog reconstruction vs. ICRF2 with and without the inclusion of troposphere covariance. See text for details.