VALUES OF SOME ASTRONOMICAL PARAMETERS (AU, $GM_\odot$, $M_\odot$), THEIR POSSIBLE VARIATIONS FROM MODERN OBSERVATIONS, AND INTERRELATIONS BETWEEN THEM

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ABSTRACT. Recent years of high-precision data of spacecraft have given the possibility to estimate the values of the astronomical unit and the heliocentric gravitational constant with great precision. Both estimations have been obtained from more than 635 000 positional observations (1913–2010) of planets and spacecraft. The analysis of the observations was performed on the basis of the numerical Ephemerides of Planets and the Moon (EPM2010) of IAA RAS. The estimation of the change of the $GM_\odot$ heliocentric gravitation constant has also been obtained: $G\dot{M}_\odot/GM_\odot = (-5.0 \pm 4.1) \cdot 10^{-14}$ per year. The obtained decrease of $GM_\odot$ should correspond to the secular decrease of au. However, it has been shown that the present accuracy level of observations does not permit to evaluate the au change. The possibility of finding the $GM_\odot$ change from high-accuracy observations points that fixing the value of au is desirable, as it is highly inconvenient to have the value of the astronomical unit variable.

1. CONNECTION BETWEEN AU AND $GM_\odot$, THEIR ESTIMATIONS

The Astronomical Unit is one of the most basic units of astronomy, it determines the scale of the solar system. Classical measurements in the solar system were angular. Periods (P) or mean motions $n = 2\pi/P$ of celestial bodies were much more easily measured than distances. Thus, in astronomy the adopted units were those of the solar mass (M), a mean solar day (d) and the Gaussian gravitational constant ($k = 0.01720209895$). AU is related to the other three ones by the Kepler third law: $n^2a^3 = k^2M$ and is based upon the Gaussian constant. For a (massless) particle at 1 au from the Sun in keplerian motion, we have $a = 1$, $M = 1$, so the mean motion is $n = k$ and the period $P = 2\pi/k = 365.2568983$ known as a Gaussian year based on the old measurement of the Earth's mean motion.

The term "Astronomical unit" appears at the beginning of the 20-th century, but not until 1976 the International Astronomical Union adopted the following definition of the au "The astronomical unit of length is that length (A) for which the Gaussian gravitational constant (k) takes the value of 0.01720209895 when the units of measurements are the astronomical unit of length, mass and time. The dimensions of k^2 are those of the constant of gravitation (G), i.e., $L^3M^{-1}T^{-2}$".

The value of au is only roughly equal to the mean Earth-Sun distance, however, by its definition it is only connected with the heliocentric gravitation constant: $au^3 = GM_\odot d^2/k^2$.

The distances in the Solar System, measured in au were known quite well from optical observations, however we need the value of au in any metrical units. The first attempt to estimate the au value in metrical units was made by Aristarchus of Samos in the third century BC. In the late 1800's, the accuracy of 0.1% was obtained from asteroid parallaxes. In 1950, Rabe achieved the accuracy of 0.05% (65000 km) from the close approach of Eros. All such determinations were made by optical triangulation methods, with large scatter. Moreover the ranges of the probable errors didn't overlap. Since the beginning of the Venus radar echoes in 1961, the value of the Astronomical Unit, has been determined exclusively from the ranging data of planets and spacecraft. The first ranging to Venus has the accuracy from 2000 km to 130 km. The typical uncertainties in the value of au decreased from many tens of thousands of kilometers to the present level of about a meter.

By its definition the astronomical unit is connected with the heliocentric gravitation constant. Either of these values (au and $GM_\odot$) may be estimated from fitting ephemerides to observations directly or the value of each of them may be obtained from the value of the other one. The modern level of accuracy of observations and dynamical models of planet motions has permitted to determine the heliocentric

	Konopliv et al., 2011 DE423	Fienga et al., 2011 INPOP10a	Pitjeva et al., 2011 EPM2010
$GM_\odot$ [km^3/s^2]	132712440042(10)	132712440055(1)	132712440033(1)
au [m] from $GM_\odot$	149597870700(3)	149597870705(0.3)	149597870696(0.4)

Table 1: The values of $GM_\odot$ obtained while fitting ephemerides to the observations and the value of au deduced from the estimations of the $GM_\odot$

gravitation constant $GM_\odot$ directly from observations. At present, there are the published values of $GM_\odot$ in the physical system of units measured directly and obtained by all three teams (Table 1). One can see that the estimates of $GM_\odot$ obtained by the three independent teams using different ephemerides are close, and the real uncertainties of this parameter and the au value are close to the uncertainty obtained in the paper by Konopliv et al. (2011).

The numerical Ephemerides of Planets and the Moon (EPM2010) of IAA RAS (Pitjeva et al., 2011) have been used for estimating this and all other parameters. These ephemerides originated in the seventies of the last century under the leadership of G.A. Krasinsky to support the Russian space flights, and have been successfully developed since then. More than 635,000 positional observations (Table 2) of planets and spacecraft of various types, mainly radiotechnical (1961–2010), have been used to construct high-precision ephemerides of planets and to determine different parameters.

Radiotechnical measurements began in 1961 and are increasingly continued since, their accuracy became several orders of magnitude better than the accuracy of the optical observations and reaches several meters for today's spacecraft data. It is necessary to say that the ephemerides of the outer planets mainly based on optical measurements have been used since 1913, when at the Naval Observatory of USA the improved micrometer was installed, and the observations become more accurate ($0''.5$). The most accurate and long-series observations are available for Mars to which spacecraft and landers were repeatedly launched. Radiotechnical observations relating to Venus are much smaller, and the situation for the Mercury observations is much worse. There are a number of radiotechnical observations for Jupiter and Saturn, for Uranus and Neptune there is one 3-D point (α, δ, R), resulting from the encounters of Voyager-2 with these planets.

Observation type	Time interval	Observation number	Accuracy
Optical	1913–2009	57768	$1'' \rightarrow 0''.05$
Radiotechnical	1961–2010	577763	100 km $\rightarrow$ 1 m
Total		635531	

Table 2: The observations used

EPM2010 differ from the previous versions by the improved dynamic model of motion of the Solar system bodies with addition of the perturbation from the ring of Trans-Neptunian objects (TNO), by the new value of the Mercury mass, defined from the three encounters of the Messenger spacecraft with Mercury, by the improved reductions of observations with the addition of the relativistic delay effect from Jupiter and Saturn, and by the extended database of observations, including new radiotechnical (2008–2010) and CCD (2009) measurements. Ephemerides were constructed by the simultaneous numerical integration of equations of motion for all the major planets, the Sun, the Moon, the largest 301 asteroids, 21 TNO, the lunar libration, taking into account the perturbations from the oblateness of the Sun and the asteroid belt, lying in the ecliptic plane and consisting of the remaining smaller asteroids, as well as the ring of the TNO rest at the mean distance of 43 au. The equations of motion of the bodies were taken in the post-Newtonian approximation in the n-body metric. Integration in the barycentric system of coordinates for the epoch J2000.0 was performed over 400 years (1800–2200). In the basic version of the improved EPM2010 planetary ephemeris, $\sim$260 parameters are determined: orbital elements of the planets and the 18 satellites of the outer planets, the value of the au or the heliocentric gravitational constant, three angles of orientation with respect to the ICRF frame, and different physical parameters.

2. VARIABILITY OF THE SOLAR MASS

The issue of variability and the possible rate of the change of gravitational constant G or $GM_\odot$ is repeatedly raised and considered. The Sun's mass $M_\odot$, can not be absolutely constant too. On the one hand, it decreases due to continuous thermonuclear reactions and the production of the radiant energy, with the matter carried away by the solar wind. On the other hand, there is a regular drop of interplanetary substances on the Sun, including dust, meteoroids, asteroids and comets.

If we take the average total solar luminosity to be $L_\odot = 3.846 \cdot 10^{33}$ erg/s and the mass of the Sun $M_\odot = 1.9891 \cdot 10^{33}$ g, then the decrease of the mass of the Sun due to radiation as a fraction of the solar mass is equal to $\dot{M}_\odot = -6.789 \cdot 10^{-14} M_\odot$ per year. The mass carried away with the solar wind was also repeatedly evaluated. The flow of the solar wind affects the activity of the Sun and coronal mass ejections. Typically, the average loss per year through the solar wind is estimated as $(2 \div 3) \cdot 10^{-14} M_\odot$, that is, less than a third of the mass loss due to radiation. The total effect of the relative annual decrease of the mass of the Sun due to radiation and the solar wind can be restricted by the inequality $-9.8 \cdot 10^{-14} < \dot{M}_\odot / M_\odot < -8.8 \cdot 10^{-14}$.

The reverse process occurs due to the fall of the matter on the Sun. However, in the solar neighbourhood and in the field of the main asteroid belt there is no sufficient interplanetary matter migrating to the Sun. The upper limit of the possible mass of the Sun drop-down material from this field is less than $10^{-17} \div 10^{-16}$ per year. The mass of the matter that can come from distant regions of the solar system (the Kuiper Belt, a cloud of the Hills, the Oort cloud), mainly in the form of comets, is more uncertain. A large number of comets is detected in the immediate vicinity of the Sun using the LASCO chronograph installed at the SOHO solar space observatory. Statistics of comets discovered by the SOHO observatory gives about $170 \div 200$ comets per year. To obtain the upper limit for falling mass we assume that all the detected comets "vanished" and their masses increased the mass of the Sun. If we take the average value of $d_{com} = 5$ km, the density of 3 g/cm^3 and double the result due to the missing and invisible falling objects, the annual upper bound is $\dot{M}_\odot / M_\odot < +3.2 \cdot 10^{-14}$. Now one can deduce the limits of the change of the solar mass. To obtain the lower limit, let us take the loss estimate due to the solar wind to be maximal, and at the same time, the drop of the material on the Sun to equal zero. To find the upper limit, we use the maximum estimate for the material falling on the Sun and the assumption that there is no mass loss due to the solar wind. Then, we obtain $-9.8 \cdot 10^{-14} < \dot{M}_\odot / M_\odot < -3.6 \cdot 10^{-14}$ per year.

Further, several words should be added concerning the influence of the change of the solar mass or gravitation constant on orbital elements of planets. A small and monotonic change of the solar mass $M_\odot$ or G must lead to the appearance of variations of only certain elements of the planet orbits. It was shown that the possible $GM_\odot$ change in the Solar system should be manifest itself in a systematic, progressive, although very small deviation of the orbital body position (that is its longitude) and the change of the semi-major axes a_i proportional to the $GM_\odot$ change with the opposite sign.

3. ESTIMATIONS OF THE CHANGE OF $GM_\odot$ AND G

In principle, the change of $\dot{G}$ and $G\dot{M}_\odot$ may be estimated. However, it should be noted that for the version of the $\dot{G}$ determination from the planet motions, the main contribution is made by the Sun, since the equations of the planet motions include the products of the masses by the gravitational constant. Among them the term for the Sun ($GM_\odot$) is the main one, several orders of magnitude larger than the others ones. Thus, separation of the change of G from the $GM_\odot$ change (with the dominant term of the $GM_\odot$) is currently impossible. The same is evident by the similarity of the obtained estimates (Pitjeva et al., 2012):

$(G\dot{M}_\odot)/GM_\odot = (-5.04 \pm 4.14) \cdot 10^{-14}$ per year (3σ), $\dot{G}/G = (-4.96 \pm 4.14) \cdot 10^{-14}$ per year (3σ).

Planet	$\dot{a}/a$ (century^{-1})	Correlation coefficients between $\dot{a}$ and a
Mercury	$(3.30 \pm 5.95)\cdot 10^{-12}$	56.5
Venus	$(3.74 \pm 2.90)\cdot 10^{-12}$	95.8
Earth	$(1.35 \pm 0.32)\cdot 10^{-14}$	0.6
Mars	$(2.35 \pm 0.54)\cdot 10^{-14}$	0.4
Jupiter	$(3.63 \pm 2.24)\cdot 10^{-10}$	20.2
Saturn	$(9.44 \pm 1.38)\cdot 10^{-10}$	35.9

Table 3: The secular change values of the semi-major axes for the 6 planets provided by the high-accuracy observations

The parameters $\dot{G}$ and $G\dot{M}_\odot$ were fitted by the least squares method simultaneously with all basic parameters of ephemerides, but separately, i.e. they are considered in different solution versions. Moreover, the secular change values of the semi-major axes for the six planets provided by the high-accuracy observations of spacecraft were simultaneously estimated. Table 3 shows the values obtained for the relative change of semi-major axes of the planet orbits. It is important that all the values obtained for the planets from Mercury to Saturn show positive values of the $\dot{a}/a$ ratio, i.e., indicate the decrease in time of the $GM_\odot$ heliocentric gravitational constant. From the result obtained for $GM_\odot$, it is possible to estimate the restrictions on $\dot{G}$ value using the limits found for the $\dot{M}_\odot/M_\odot$ value. Then, we obtain the $\dot{G}/G$ value with the 95% probability within the interval $-4.2 \cdot 10^{-14} < \dot{G}/G < +7.5 \cdot 10^{-14}$ per year.

4. CONSIDERATIONS ON POSSIBLE VARIABILITY OF *AU*

Let us get back to *au*. The change of the astronomical unit is connected with the change of the heliocentric gravitational constant due to the present definition of *au*. In the paper by Krasinsky and Brumberg (2004) based on the raging data 1961–2003, using a numerical theory of planetary motion, nearly coinciding with the EPM2004, the authors obtained the secular increase of the astronomical unit $\dot{au}$ = 15 m per century, which by definition should correspond to the increase of the heliocentric gravitational constant $G\dot{M}_\odot/GM_\odot \simeq 3{\cdot}10^{-12}$ per year. Such a large positive change of the $GM_\odot$ does not correspond to the estimations of physical processes in the Solar system (the solar radiation and wind, the matter falling on the Sun), as well as to the estimate obtained in present study: $G\dot{M}_\odot/GM_\odot = -5.04 \cdot 10^{-14}$ per year. However, the authors themselves considered that the increase of *au* is rather matching parameter than the real change of the physical parameters.

The analysis of our results based on a greater number of observations and the EPM2010 ephemerides, shows that the present level of observational accuracy does not permit to evaluate the *au* change. In the paper by Krasinsky and Brumberg the *au* change was determined simultaneously with all other parameters, particularly with the orbital elements of the planets and the value of the *au* astronomical unit. However, it is impossible at present to determine simultaneously the two parameters: the value of the astronomical unit, and its change. In this case, the correlation between *au* and its change $\dot{au}$ reaches 98.1%, and leads to incorrect values of both parameters, in particular, it gives $\dot{au}$ the order of 15 m per century. Without the simultaneous determination of *au* and $\dot{au}$, i.e. if only the change of the astronomical unit is estimated together with other parameters, the $\dot{au}$ value is about 1 m per century, and does not exceed its formal uncertainty, thus it is not determined: $\dot{au} = (1.2 \pm 3.2)$ m/cy (3 σ). Furthermore, including or excluding the $\dot{au}$ value from a number of the solution parameters does not change the observation residuals, the mean error of the unit weight is not also changed ($\Delta\sigma \simeq 0.2\%$), so there is no reason to assume that $\dot{au}$ is the necessary matching parameter, and there is no need to include it in the number of parameters to be estimated. The modern accuracy has approached the level when it is possible to estimate the change of the heliocentric gravitational constant $GM_\odot$, therefore it is desirable to specify the definition of the astronomical unit, for example, by fixing its value, as it is highly inconvenient to have the changing value of the astronomical unit. In future, after obtaining the reliable estimations of $G\dot{M}_\odot$ by different authors, the value of $GM_\odot$ should probably be given on a certain epoch.

5. REFERENCES

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