CONSTRUCTION OF THE NUMERICAL AND SEMI-ANALYTICAL SOLUTIONS OF THE MOON ROTATION

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ABSTRACT. In this research the problem of the lunar rotation motion is studied for the Newtonian case over long time intervals. The numerical solution of the Moon rotation is implemented with the quadruple precision of the calculations. The results of the numerical solution of the problem are compared with the composite semi-analytical theory of the Moon rotation (SMR) (Pashkevich and Eroshkin, 2010) with respect to the fixed ecliptic of epoch J2000. The initial conditions of the numerical integration are taken from SMR. The investigation of the discrepancies is carried out by the least squares and spectral analysis methods. All the secular, periodic and Poisson terms, representing the behavior of the residuals, are interpreted as corrections to SMR semi-analytical theory. As a result, the new high-precision Moon Rotation Series (MRS2011) is constructed, which is dynamically adequate to the DE404/LE404 and the DE406/LE406 ephemeris over 418.9, 2000 and 6000 years. The comparison of the new high-precision Moon Rotation solutions of MRS2011 with the solution of MRS2010 (Pashkevich and Eroshkin, 2010), which is dynamically adequate to the DE200/LE200 ephemeris over 418.9 year time interval, is performed. A numerical solution for the Moon rotation is obtained anew with the new initial conditions calculated by means of MRS2011. The discrepancies between the new numerical solution and the semi-analytical solution of MRS2011 do not surpass 20 mas over 418.9 year time interval, 48 mas over 2000 year time interval and 8 arcsec over 6000 year time interval. Thus, the result of the comparison demonstrates a good consistency of MRS2011 series with the DE/LE ephemeris.

1. INTRODUCTION

In the previous research (Pashkevich and Eroshkin, 2010) the high-precision Moon Rotation semi-analytical solutions of MRS2010, dynamically adequate to the DE200/LE200 ephemeris, was constructed over 418.9 year time interval.

The main purposes of this research are the construction of the new high-precision Moon Rotation Series (MRS2011), dynamically adequate to the DE404/LE404 and the DE406/LE406 ephemeris, over long time intervals. The mathematical model of the present investigation is identical to that used by Pashkevich and Eroshkin (2010). The orbital motions of the disturbing bodies are defined by the DE404/LE404 and the DE406/LE406 ephemeris. The dynamics of the rotational motion of the Moon is studied numerically by using Rodrigues-Hamilton parameters over 418.9, 2000 and 6000 years. The high-precision numerical integration method (Belikov, 1990), with a number of modifications (Eroshkin et al., 1993), was applied.

2. ALGORITHMS AND RESULTS

The result of the comparison of the numerical solution and semi-analytical solution SMR is studied by means the iterative algorithm:

1. The numerical solution of the Moon rotation is implemented with the quadruple precision of the calculations. The initial conditions are computed by the semi-analytical theory of the Moon rotation (SMR), which corresponds to the fixed ecliptic J2000.0. The discrepancies between the numerical solution and SMR are obtained in Euler angles over all the investigation time intervals with one day spacing (presented in Figure 1).

2. The investigation of the discrepancies is carried out by the least squares method (LSQ) and by the spectral analysis method (SA) (Pashkevich and Eroshkin, 2010). The secular terms are defined by LSQ. The set of the frequencies of SMR theory is used without a change. Only the coefficients of the periodical terms are improved and the coefficients of the Poisson terms are calculated by LSQ and by SA

(the spectral analysis scheme presented under in this paper). The secular, periodic and Poisson terms representing the new high-precision Moon rotation series MRS2011-i (where i is a number of the iteration) are determined.

3. The numerical solution of the Moon rotation is constructed anew with the new initial conditions, which are calculated by MRS2011-i.

4. Steps 2 and 3 are repeated till the best results for the discrepancies between new numerical solution and new MRS2011-i are obtained.

The spectral analysis scheme of SA is following: The spectrum of the discrepancies between the numerical solution and semi-analytical solution of the (i-1)-th iteration is constructed anew after the removal of every largest residual harmonic from the discrepancies. The set of the frequencies of the semi-analytical solution of the lunar physical libration SMR is used for the power spectrum construction. The amplitudes of the power spectrum are computed by LSQ. Each maximum term of the constructed spectrum is used for the determination of the new coefficients of the periodic and Poisson terms. Every coefficient of the new periodic term equals the sum of the calculated periodic and Poisson terms coefficients of the discrepancies and the coefficients of the corresponding periodic and Poisson terms of the (i-1)-th semi-analytical solution of the lunar physical libration problem. The found new harmonic is removed from the discrepancies and from the used set of the frequencies of SMR. This procedure is performed for every harmonic of the set and is accomplished successively up to the least term of the set. The new periodic and Poisson terms representing the new series MRS2011-i are determined.

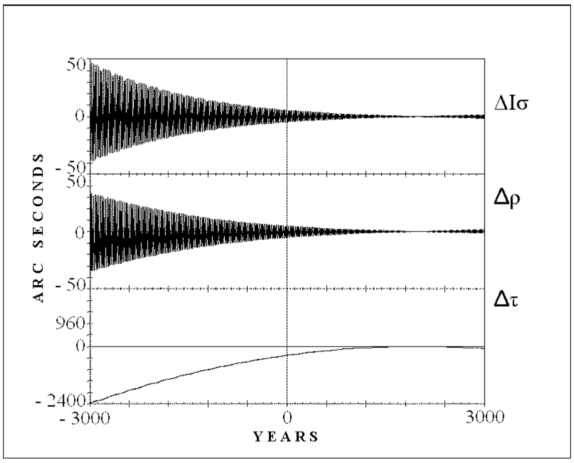

Figure 1: The discrepancies are depicted between the numerical and SMR semi-analytical solutions of the Moon rotation over 6000 years.

At first this investigation is carried out on 418.9 years time interval. The discrepancies between the numerical integration of the lunar rotation and the Moon rotation series SMR (for the DE404/LE404 ephemeris and for the DE406/LE406 ephemeris) are obtained in the perturbing terms of the lunar physical librations (in the longitude $\Delta\tau$, in the inclination $\Delta\rho$ and in the node longitude $\Delta\sigma$, which is multiplied by the mean inclination of the lunar equator to the ecliptic of date I (Newhall and Williams, 1997)) over 418.9 year time interval with one day spacing.

As a result of this research was obtained that the discrepancy between the new numerical and the new MRS2011A-1 semi-analytical solutions of the Moon rotation for the DE406/LE406 ephemeris (presented in Figure 2) and the DE404/LE404 ephemeris are very close to each other after the first iteration of the iterative algorithm. The residuals between the new numerical and the new MRS2011A-1 semi-analytical solutions of the Moon rotation (for the DE406/LE406 ephemeris), after the first iteration of the iterative algorithm, and the residuals between the numerical and MRS2010 (Pashkevich and Eroshkin, 2010) semi-analytical solutions of the Moon rotation (for the DE200/LE200 ephemeris) are also similar. Namely, the periodic and Poisson parts of MRS2011A are very close periodic and Poisson parts of MRS2010, over all time interval of MRS2010, that evidences a good convergency of the iterative algorithm of this investigation.

This investigation is continued on 2000 years time interval. The results of this investigation demonstrate that the residuals between the numerical and MRS2011B-1 semi-analytical solutions of the Moon

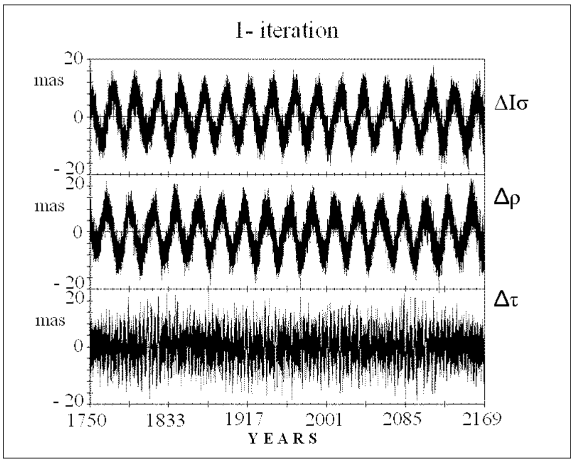

Figure 2: The residuals between the new numerical and the new MRS2011A-1 semi-analytical solutions of the Moon rotation (for the DE406/LE406 ephemeris), after the first iteration of the iterative algorithm.

rotation for the DE404/LE404 ephemeris and the DE406/LE406 ephemeris are very close to each other after the first iteration of the iterative algorithm. Thus, the second iteration is used only for the investigation with the application of the DE406/LE406 ephemeris.

The residuals between the numerical and MRS2011B-2 semi-analytical solutions, after the second iteration, and the residuals between the numerical and MRS2011B-3 semi-analytical solutions (presented in Figure 3), after the third iteration are similar. Then the process of the iterative algorithm is finished at this step.

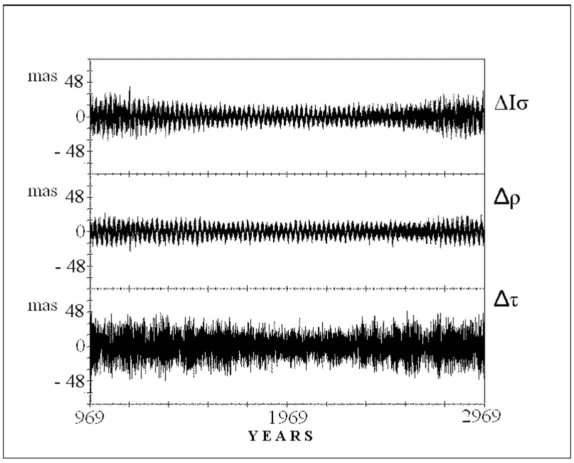

Figure 3: The residuals between the numerical and MRS2011B-3 semi-analytical solutions, after the third iteration of the iterative algorithm.

This investigation is finished at 6000 years time interval only for the DE406/LE406 ephemeris. In Figure 1 the discrepancies are depicted between the numerical and SMR semi-analytical solutions of the Moon rotation over 6000 years. The secular trend in the longitude of the descending node of epoch J2000 of the lunar equator longitude ψ does not surpass 40 arcmin over 6000 years.

Two iterations of the iterative algorithm in the present study were enough. The discrepancies between the numerical and MRS2011C-2 semi-analytical solutions, after the second iteration is presented in Figure 4. The discrepancies in the libration angles decrease after the second iteration and are less than 8 arcsec over 6000 years.

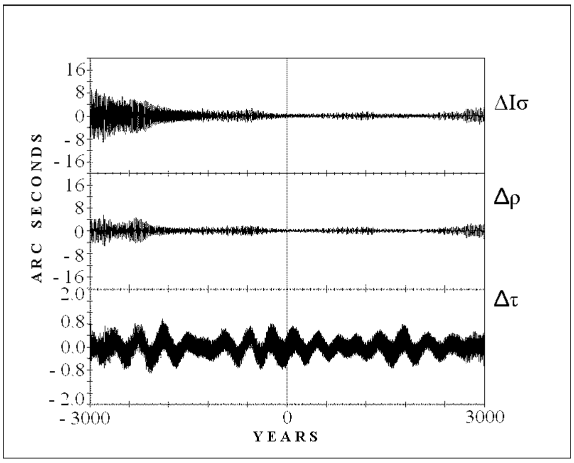

Figure 4: The discrepancies between the numerical and MRS2011C-2 semi-analytical solutions, after the second iteration.

3. CONCLUSION

As the results of this investigation, the new high-precision Moon Rotation Series are constructed: MRS2011A, dynamically adequate to the DE404/LE404 (DE406/LE406) ephemeris, over 418 years, MRS2011B, dynamically adequate to the DE404/LE404 (DE406/LE406) ephemeris, over 2000 years, MRS2011C, dynamically adequate to the DE406/LE406 ephemeris, over 6000 years.

The periodic and Poisson parts of MRS2011A are very close to the periodic and Poisson parts of MRS2010, over all time interval of MRS2010, that evidences a good convergency of the iterative algorithm of this investigation. The new more accurate series MRS2011 includes about 1523 secular, periodical and Poisson terms with the periods from 5.648 days to 84541.30 years. The discrepancies between the numerical solution and MRS2011 do not surpass:

20 mas over 418 year time interval,

48 mas over 2000 year time interval,

8 arcsec over 6000 year time interval.

It means a good consistency of MRS2011 series with the DE/LE ephemeris.

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