

# EMPIRICAL MODEL OF SUBDAILY VARIATIONS IN THE EARTH ROTATION FROM GPS AND ITS STABILITY

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**ABSTRACT.** The model recommended by the IERS for these variations at diurnal and semidiurnal periods has been computed from an ocean tide model and comprises 71 terms in polar motion and Universal Time. In the present study we compute an empirical model of variations in the Earth rotation on tidal frequencies from homogeneously re-processed GPS-observations over 1994–2007 available as free daily normal equations. We discuss the reliability of the obtained amplitudes of the ERP variations and compare results from GPS and VLBI data to identify technique-specific problems and instabilities of the empirical tidal models.

## 1. INTRODUCTION

External gravitational torques and internal processes causing mass redistributions in the system Earth lead to variations in Earth rotation, both in polar motion (PM) describing the position of the rotational axis and the speed of rotation (UT1). On the periods of 1 day and shorter the main mass redistribution in the Earth system is caused by the ocean tides, what allows to derive theoretically from an ocean tidal model the amplitudes of the variations in the Earth rotation on the tidal frequencies. The International Earth Rotation Service (IERS) has adopted a model derived in Ray et al. (1994) as an official sub-diurnal model for the Earth rotation parameters (ERPs). We will refer to it as to IERS2003 tidal model.

Additional variations taking place on the same frequencies as tidal variations are caused by the external torques acting on the flattened equator of the Earth. This interaction leads to a high-frequency nutation of the rotational axis, which is called libration. In accordance with the convention these nutation terms are not included in the model for precession and nutation, they are taken into account in the terrestrial reference frame as a part of polar motion. The values of these variations are well computed theoretically and can be found in the IERS Conventions. Also non-tidal oceanic and atmospheric variations driven by the solar heating cycles and atmospheric and ocean normal modes lead to additional changes in the ERPs partly on the same tidal frequencies.

The present study is devoted to the derivation of an empirical model of subdaily variations in the Earth rotation as observed by the GPS. Since space geodetic techniques observe the variations in the Earth rotation caused by all the geophysical excitations together, we can partly interpret the differences in amplitudes between the tidal model IERS2003 and the obtained empirical models as the effects caused by other phenomena than ocean tides, e.g. by libration and radiational atmospheric tides. Other possible reasons for the differences are the uncertainties of the models, both the IERS2003 tidal model and the computed empirical models.

Here we consider the question of reliability and stability of the GPS-derived empirical tidal models. A comparison with two tidal models estimated from the VLBI observations is done to identify technique-specific deficiencies of the estimated values.

## 2. EMPIRICAL TIDAL MODELS

The variations in the ERP can be estimated on the observation level (i.e. set up as parameters in the processing of the observations), or a posteriori from a time series of the ERPs, or the estimation can be done on the normal equation (NEQ) level when the ERPs are transformed into the amplitudes of the variations on given frequencies. The first and the third methods have the advantage of taking into

account the full variance-covariance information. We used here the third method for the estimation of empirical tidal models.

As input data we used free normal equation systems (NEQ) obtained from processing GPS and VLBI data. From GPS-observations we had free daily NEQs covering the time span 1994-2007 and obtained within the GGOS-D project (Rothacher et al., 2011). The parameters included in the daily NEQs contained station coordinates, orbits (6 osculating elements, 9 radiation pressure parameters, 3 stochastic pulses at 12 o'clock), tropospheric zenith delays and gradients, Earth rotation parameters with time resolution of 1 hour and corrections to the nutation angles with time resolution of 24 h. From VLBI-observations we had free NEQs for 24 h VLBI-sessions covering the time span 1984-2010, these normal equations were provided by the Institute of Geodesy and Geophysics (IGG), TU Vienna. The parameters included in the NEQs contained station coordinates, radiosources coordinates, ERPs with 1 hour resolution and nutation corrections with 24 h resolution.

The GPS and VLBI IGG tidal models were estimated with a modified Bernese GPS Software. The transformation between the ERPs and tidal terms is a linear transformation, the general theory concerning changes in the NEQ system caused by linear transformations of parameters can be found in (Thaller, 2008). More specific the changes in the NEQ system needed to transform the ERPs into tidal terms are described in (Artz et al., 2011).

Also for comparison with our GPS and VLBI tidal models computed on the NEQ level, we used a VLBI tidal model computed in the Institute of Applied Astronomy (IAA, St. Petersburg, Russia) with the software package QUASAR. In this case the tidal terms were computed on the observational level. Comparison of 2 VLBI tidal models computed fully independently with different software and different approaches should provide a better understanding of the results.

## 2.1 GPS empirical tidal model

The GPS solution was computed as a multiyear solution for station coordinates, i.e. for each station one set of coordinates and velocities was estimated for the whole time span covered by the NEQs, allowing for jumps for stations showing a non-linear behaviour. To align the solutions to the ITRF05 a No-Net-Rotation condition was applied over a set of stable IGS stations, in this case the origin of the reference frame (geocenter) is defined dynamically by the satellite orbits. Tropospheric zenith delays and gradients were pre-eliminated on daily basis. Nutation corrections were kept fixed to zero and ERPs were transformed into amplitudes of the tidal terms. For the empirical tidal model all the terms from the IERS2003 model were estimated except one term with the period of 24.000 h, because of its full correlation with the term S1. The amplitudes of some tidal terms are not constant, but vary together with the changes in the Moon's declination with the period of revolution of the lunar nodes (Ray, 2007), it is taken into account by the estimation of sidebands of the affected terms. Since the time span covered by the available re-processed GPS NEQs (13 years) is shorter than is needed for the separation of the sidebands (18.6 years), all the sidebands were constrained using the respective heights of the tidal potential, the exact procedure and tidal potential values are the same as described in (Artz et al., 2011).

GPS orbits are modelled by a set of 6 osculating elements for the middle of the arc and a set of additional parameters to account for not well enough known force field. These parameters include the radiation pressure parameters (RPRs: empirical accelerations in 3 directions containing a permanent and a periodic (sine and cosine) term), and stochastic pulses which allow a satellite a sudden change in velocity in 3 directions at a given epoch. We used orbital arc length of 7-days with stochastic pulses set up also on the days' boundaries. A tight constraint was applied on the out-of-plane component for the stochastic pulses; for the RPRs periodic terms in D- (direct to the Sun) and Y- (along solar panels) directions were constrained. This way of orbit modeling was proposed and studied in detail in (Springer et al. 1999).

## 2.2 VLBI empirical tidal models

The way of computing the VLBI tidal model from the NEQs provided by the IGG TU Vienna was the same as in the case of GPS. Here also a multi-year solution was computed for the station coordinates. In this case No-Net-Translation condition was applied over a set of stable stations in addition to the No-Net-Rotation condition, because VLBI observations do not provide information about the position of the geocenter. ERPs were transformed into tidal terms, all the terms from the model IERS2003 were estimated. Sideband constraints were considered specially, because in case of VLBI the time span covered by the observations (1984-2010) is long enough to estimate the sidebands independently of the main term. But on the other hand most sidebands have very small amplitudes and the differences between amplitudes

of freely estimated sidebands and constrained sidebands can give an idea about the real accuracy of the empirical model (the formal errors are about the same for all terms without constraints and is  $-0.8 \mu\text{s}$  for PM and  $-0.04 \mu\text{s}$  for UT1). We made such a test and found that the mean differences in the amplitudes of sidebands are about  $2\text{-}4 \mu\text{s}$  in PM and  $0.2\text{-}0.4 \mu\text{s}$  in UT1. So we considered the real accuracy of the model to be not enough to reliably estimate most of the sidebands and they were constrained the same way as in the case of GPS solution. Nutation corrections were estimated as one offset and rate each 4 weeks. Radiosource positions were estimated once over the whole time span with a NNR-condition applied over a set of 20 stable sources.

The VLBI solution provided by the IAA was computed on the observation level as a multi-year solution for the station coordinates, the radiosource positions were kept fixed to the a priori. Amplitudes of tidal terms in PM were set up as parameters in such a way that for each frequency the sum of prograde and retrograde variations was estimated. That implies that some nutation terms corresponding to the retrograde diurnal terms in PM were set up and estimated. For that reason the nutation corrections were fixed to aprioris. Sidebands in this case were estimated freely.

### 3. STABILITY OF EMPIRICAL TIDAL MODELS

To give a general impression about the agreement of the computed models we show in Figure 1 the differences in tidal amplitudes in PM between the 3 empirical tidal models. Since the IAA solution was computed without constraints on the sidebands and the other two models with the constraints we used for the comparison only the not constrained terms. As can be seen the two VLBI solutions agree with each other a bit better than with the GPS solution. The biggest differences show the terms on the periods very close to 24 and 12 hours, also the amplitudes for the term M2 (12.42h) disagree noticeably.

To estimate the stability of the tidal amplitudes we computed several tidal models using not all the timespan, but only some years of data: for GPS 8 models were computed over 6 years each with a shift of 1 year (1994-1999, 1995-2000,...), for both VLBI solutions we computed 9 models over 19 years each with a shift of 1 year (1984-2001, 1985-2002,...). The timespan of 19 years for VLBI allows the estimation of sidebands without constraints by the IAA. Then for each tidal term the root mean square (RMS) values of the estimated amplitudes were computed. The resulting RMS for all 3 solutions are shown in Figure 2 for PM and Figure 3 for UT1. As can be seen the GPS tidal model shows a high RMS for the terms N1 (24.132h), K1 (23.934h), P1 (24.066h), S1 (24.000h) and  $\Psi$ 1 (23.869h) in daily part, and the terms S2 (12.00h) and K2 (11.967h) in the semidaily part. All these terms should be strongly affected by any errors in the GPS orbits, and probably we can make a conclusion that these terms cannot be reliably estimated from the GPS data. Terms in PM show also a high RMS for periods of about 28 and 13 hours, but these terms are stable in a GPS solution with 1-day orbit (which we do not show here). Terms in UT1 show in addition a high RMS for term O1 (25.819h) which remains unstable in GPS solutions with different orbital arc lengths. All the terms in VLBI IGG solution have a good stability on the level of  $2\text{-}4 \mu\text{s}$  in PM. The IAA solution shows a higher RMS for terms in PM around 24 and 12 hours, we attribute it to the influence of not constrained sidebands. In UT1 both VLBI solutions show a noticeably high RMS for terms S1 and K1, IAA solution having bigger variations also seen in semidaily part.

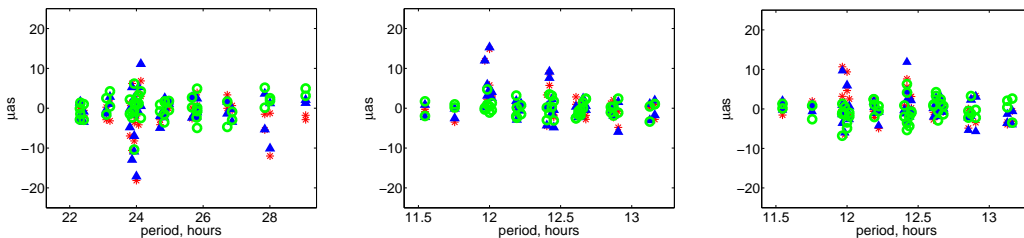


Figure 1: Differences in tidal amplitudes in PM for 3 solutions: (left) prograde daily PM; (middle) prograde semidaily PM; (right) retrograde semidaily PM. GPS minus VLBI IGG (red asterisk), GPS minus VLBI IAA (blue triangle), VLBI IGG minus VLBI IAA (green circle)

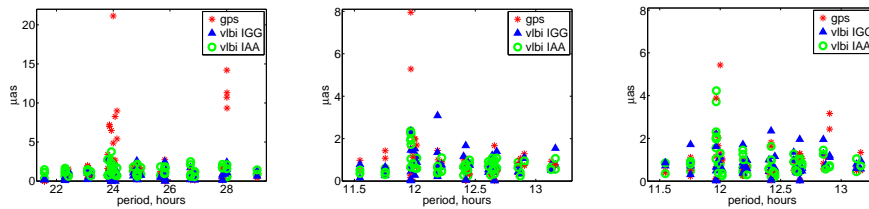


Figure 2: RMS of tidal terms in PM for 3 solutions: (left) prograde daily PM; (middle) prograde semidaily PM; (right) retrograde semidaily PM. GPS (red asterisk), VLBI IGG (blue triangle), VLBI IAA (green circle)

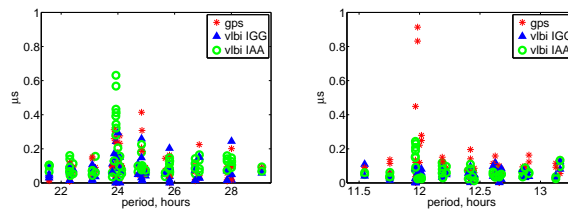


Figure 3: RMS of tidal terms in UT1 for 3 solutions: (left) daily UT1; (right) semidaily UT1. GPS (red asterisk), VLBI IGG (blue triangle), VLBI IAA (green circle)

#### 4. CONCLUSIONS

The tidal models computed over different timespans show that the terms with periods very close to 24 and 12 hours cannot be reliably estimated from GPS. We attribute it to deficiencies in orbit modeling. This instability makes it impossible to compare the affected terms from GPS with the same terms from VLBI. The geophysical interpretation of the unstable terms is also problematic, e.g. the estimated GPS S1 term cannot be used for comparisons with the expected influences of the atmospheric tide and non-tidal angular momentum, because this term changes strongly (within  $\sim 30 \mu\text{as}$ ) depending on the used timespan and the orbit modeling. VLBI tidal models on the contrary have a good stability for all terms. The independent VLBI tidal models from IGG and IAA agree well for most of the terms. The real accuracy of the VLBI estimates is on the level of 2-4  $\mu\text{as}$  for PM and -0.2-0.4  $\mu\text{s}$  for UT1.

#### 5. REFERENCES

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