ON THE STABILITY OF EARTH'S TROJANS

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ABSTRACT. The gas giants Jupiter and Neptune are known to host Trojans, and also Mars has co-orbiting asteroids. Recently, in an extensive numerical investigation ([8]) the possibility of captures of asteroids by the terrestrial planets and even the Earth into the 1:1 mean motion resonance (MMR) was studied. The first Earth Trojan has been observed ([2]) and found to be in a so-called tadpole orbit closed to the Lagrange point L_4. We did a detailed study of the actual orbit of this Trojan 2010 TK7 including the study of clone orbits, derived an analytical mapping in a simplified dynamical system (Sun+Earth+massless asteroid) and studied the phase space structure of the Earth's Lagrange points with respect to the eccentricities and the inclinations of a large number of fictitious Trojans. The extension of stable zones around the Lagrange points is established with the aid of dynamical mappings; the known Trojan 2010 TK7 finds himself inside an unstable zone.

1. INTRODUCTION

Trojan asteroids move in the same orbits as their host planets, but around 60 degrees ahead or 60 degrees behind them close to the so-called Lagrange points L_4 or L_5. Up to now we observe Trojans of Jupiter (about 4000), of Neptune (7) and also of Mars (3) but the other planets still seem to lack of such a companion (e.g. [9]). Although in the original paper of the first confirmed discovery of a Trojan asteroid ([2]) a dynamical study has been undertaken we extended it to a more detailed investigation. We make use of a dynamical symplectic mapping of the Sun-Earth Trojan model and of extensive numerical integrations of fictitious Trojans in the full model of our Solar system. With this approach we were able to obtain a deeper understanding of the dynamical aspects of the first confirmed Earth Trojan asteroid 2010 TK_7 as well as the stability of Earth Trojan asteroids in general.

2. THE REAL ORBIT OF 2010 TK_7

The orbit of the asteroid 2010 TK_7 is numerically simulated to obtain a direct estimation of its orbital stability and its origin. After some test runnings and comparisons between different numerical codes, we choose the *Mercury6* integrator package ([1]) to make our simulations in this part. For the initial conditions of 2010 TK_7, we adopt the data listed in the AstDyS website[1]. Specifically, at epoch JD2455800.5, the semi-major axis $a = 1.00037\,\mathrm{AU}$, eccentricity $e = 0.190818$, inclination $i = 20^\circ.88$, ascending node $\Omega = 96^\circ.539$, perihelion argument $\omega = 45^\circ.846$ and the mean anomaly $M = 217^\circ.329$. Since the errors are unavoidable in the observation and orbital determination, we simultaneously simulate the evolution of a cloud of 100 clone orbits within the error bars. These clone orbits are generated using the covariance matrix listed in the AstDyS website. Two dynamical models are applied. In one model, we include the Sun and eight planets from Mercury to Neptune and the Earth is placed in the barycenter of the Earth-Moon system and its mass is replaced by the combined mass of the system. In the other model however, the Earth and the Moon are treated separately. Hereafter the former and later models are denoted by EMB and E+M, respectively. In two dynamical models, we integrate the nominal and clone

[1] http://newton.dm.unipi.it

orbits both forward (to future) and backward (to past) for 1 million years. During the integration, we check the resonant angle $\delta\lambda = \lambda - \lambda_{\rm EMB}$ (the difference between the mean longitude of the asteroid and the barycenter of the Earth-Moon system). At the start of integration ($t = 0$), the $\delta\lambda$ librates around $60°$ since 2010 TK$_7$ is on a tadpole orbit around L_4 right now. But it may leave this region in both backward and forward integrations. We record the moment t_1 when $\delta\lambda$ reaches $180°$ for the first time, and the moment t_2 when $\delta\lambda$ attains $360°$. So t_1 and t_2 are the time when an asteroid escapes from the L_4 tadpole region and from the 1:1 MMR. Figure1 summarizes the distribution of t_1 and t_2.

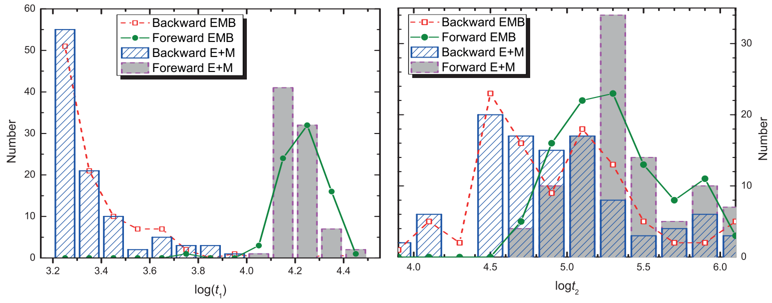

Figure 1: The time when clone asteroids escape from the L_4 region (t_1) and from the 1:1 MMR (t_2).

From the distribution of t_1 and t_2, we conclude that the two models EMB and E+M are consistent with each other, they do not make considerable differences. It is more or less a natural consequence of the Earth and the Moon being a close binary. From Figure 1, we can also conclude that 2010 TK$_7$ is a temporal Earth Trojan. In fact the nominal orbit will leave the L_4 region in about 17000 years, while most of the clone orbits will escape in ~ 15000 years. The results of backward integration show that most of the clones became L_4 Earth Trojans only about 1700 years ago, just as the nominal orbit did. As for the time they leave the 1:1 MMR, it is $\sim 4.0 \times 10^4$ years in the past and $\sim 2.5 \times 10^5$ years in the future. The total time for this object being in the 1:1 MMR with the Earth is less than $\sim 3.0 \times 10^5$ years.

3. THE ANALYTICAL MAPPING

The most basic dynamical model behind the motion of 2010 TK$_7$ takes the form:

$$H = H_{\rm Kep} + T + \mu' R(a, e, i, \omega, \Omega, M, M'; P') \ . \qquad (1)$$

Here H_{Kep} defines the motion of the asteroid around the Sun and $\mu' R$ gives the potential of the Earth with mass μ'. Here M' denotes the mean anomaly of the Earth. R is time dependent due to the presence of M', we therefore extend the phase space with T (assuming, that the mean motion n' of the Earth is equal to one). Moreover, we denote by P' the orbital parameters of the Earth $P' = (a', e', i', \omega', \Omega')$. In the further discussion we use the modified Delaunay variables $\lambda_1 = M + \omega + \Omega$, $\lambda_2 = -\omega - \Omega$, $\lambda_3 = -\Omega$ and their conjugated momenta $\Lambda_1 = \sqrt{a}$, $\Lambda_2 = \sqrt{a}(1 - \sqrt{1-e^2})$, $\Lambda_3 = 2\sqrt{a}\sqrt{1-e^2}\sin^2(i/2)$ and similar for the orbital parameters of the Earth. Moreover, we write $\Lambda = (\Lambda_1, \Lambda_2, \Lambda_3)$ and $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ in short. The aim of this section is to investigate the role of P' on the mean orbit of the asteroid 2010 TK$_7$. For this reason we will make use of a symplectic mapping based on the averaged Hamiltonian ([6]):

$$\tilde{H} = -\frac{1}{2\Lambda_1^2} + \frac{1}{2\pi}\int_0^{2\pi} \mu' R\left(\Lambda, \tau, \lambda_2, \lambda_3, \lambda_1'; P'\right) {\rm d}\lambda_1' \ , \qquad (2)$$

where $\tau = \lambda_1 - \lambda_1'$ is the resonant angle (which is also related to $\delta\lambda$ of the previous section). Thus, the average over the fast angle λ_1' defines the mean dynamics close to the $1:1$ MMR. To shorten notation we will write $\lambda = (\tau, \lambda_2, \lambda_3)$ from now on. Based on Equation (2) we define a transformation from state $(\lambda^{(k)}, \Lambda^{(k)})$ to $(\lambda^{(k+1)}, \Lambda^{(k+1)})$ via the generating function:

$$W_{P'} = W_{P'}\left(\lambda^{(k)}, \Lambda^{(k+1)}; P'\right) = \lambda^{(k)} \cdot \Lambda^{(k+1)} + 2\pi\tilde{H}\left(\lambda^{(k)}, \Lambda^{(k+1)}; P'\right) \ .$$

Based on it the mapping from time k to $k+1$ is given by:

$$\lambda_j^{(k+1)} = \frac{\partial W_{P'}}{\partial \Lambda_j^{(k+1)}} \ , \quad \Lambda_j^{(k)} = \frac{\partial W_{P'}}{\partial \lambda_j^{(k)}} \quad \text{with } j = 1, 2, 3 \ . \qquad (3)$$

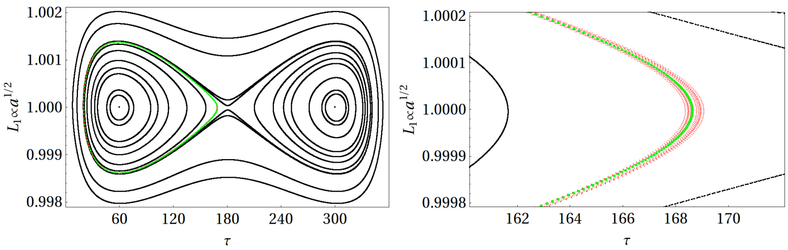

Figure 2: Dynamics in the (τ, Λ_1)-plane. Left: varying τ within 0 and 360°. Right: $P' = const$ (green) vs. $P' = P'_k$ (red). See text.

The system (Equation 3) describes the time-evolution of the mean orbital elements of the asteroid at times $t = 2\pi k$. In physical units the time step corresponds to 1 Earth-year. We iterate the mapping for initial conditions provided in [2] in the case of i) fixed orbital parameters of the Earth $P' = const$ and ii) time varying parameters $P' = P'_k$. To obtain P' and P'_k we integrate the equations of motion of the full Solar system and maintain Earth's orbital elements, say $P'(t)$, at discrete times $P'_k = P'(k[years])$. A typical phase portrait is provided in Figure 2 (left): we see the fixed points of the mapping L_4, L_3, L_5 situated along $a = 1$ and located at $\tau = 60°, 180°, 300°$, respectively. The stable pair is surrounded by small librational curves, while seperatrix-like motion originates from the unstable fixed point. The effect of the time variation of P' can be seen in Figure 2 (right). While for constant P' the motion getting close to L_3 remains on a thin curve for long times (green), the motion for time varying P'_k covers a wider range in the phase space and eventually reaches the tadpole regime of motion around L_5 (red). The effect of the additional perturbations therefore may explain the jumping of the Trojan from one to another stable equilibrium as well as a possible trapping of the asteroid in the horse-shoe regime of motion.

4. DYNAMICAL MAPS OF THE L_4 REGION

We integrated the orbits of thousand of fictitious Trojans in the L_4 region and we established how extended is this zone with respect to the semi-major axis and to the inclination. We included the planets Venus, Earth, Mars and the two giant planets Jupiter and Saturn in our dynamical model [2]; the Earth and Moon system was regarded as one 'planet' situated in their barycenter [3] and the fictitious asteroids were taken as massless bodies. The integration method used in this section was the very fast and precise Lie-code with an automatic step size control already used extensively in our former studies (e.g. [5], [4], [3], [9]) and the integration time was set to up to 10^8 years. In Figure 3 we show the results of this numerical study: on the one hand we checked the libration around the Lagrange point L_4 [4] and on the other hand the eccentricity is quite a sensitive marker for stable and unstable orbits in the Trojan zone. In the respective Figure 3 (left graph) the amplitude of the libration angle is given by different colors. One can see that around the center ($a = 1$) there is a dark black region extending up to an inclination of about 15° which means that the Trojans are suffering only from small oscillations around L_4. More to the edge with larger and smaller initial semi-major axes of the fictitious body these oscillations increase in amplitude up to about 40°, then, on both sides with respect to the semi-major axes the orange color indicates that from the tadpole orbits just around one Lagrange point the orbits developed into horseshoes around both libration points; but they are still stable. We can see another stable window between an initial inclination of $25° < i < 40°$ and another small one for $i \sim 50°$. Whereas this small window disappears for integration $T > 10^7$ years, the larger stable window remains. We studied it with another tool namely by checking the orbital eccentricity. It turned out that any value of $e > 0.3$ leads to an escape from the stable region; we therefore plotted in Figure 3 (right graph) the corresponding values. It is interesting to see that now on the edge to the unstable region on both sides of the Lagrange point very stable (almost circular) orbits survive which are marked by dark blue inside the 'blue' window extending between 0.997 AU $< a <$ 1.003 AU.

[2] test computations for the complete system with the eight planets did not qualitatively change the picture
[3] all involved planets are regarded as point masses
[4] in many studies (e.g. [7]) the symmetry of both equilibria was shown

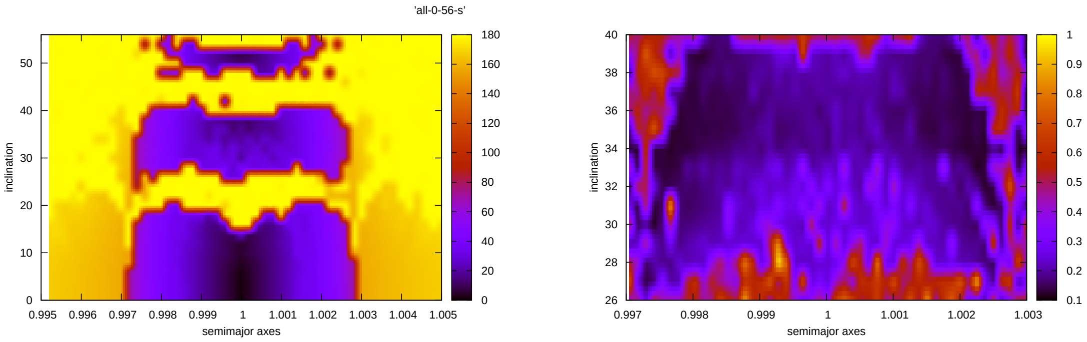

Figure 3: Dynamical map of the L_4 region. Left: the libration amplitudes of the fictitious Trojans; tadpole orbits are inside the red line in dark blue and violet close to the center, horseshoe orbits are in orange, escapers in light yellow. Right: maximum eccentricity of the Trojans within 10^7 years of integration; stable orbits are shown in dark blue, escaping orbits are marked from orange to yellow.

5. CONCLUSIONS

In this study we confirm that the recently discovered Earth Trojan 2010 TK_7 is a temporarily captured asteroid, which is stable only for several thousand years. In its actual orbit it is in a tadpole orbit around L_4, but it is a jumping Trojan changing its orbit between tadpole around one or the other equilibrium point and horseshoe orbits; but then it escapes from the stable region. This dynamical behaviour is observed in backward as well in forward integration and well confirmed by all three methods used in the paper. Two main stable regions were found. One for low inclined orbits ($i < 15°$) and one for $25° < i < 40°$. Surprisingly is that the Trojan discovered by recent observations is in none of these stable regions but well inside an unstable zone! But we may be able in future to observe many more of these companions of the Earth.

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6. REFERENCES

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