PRECISE ANALYTICAL CALCULATION OF THE EFFECT OF SOLID EARTH TIDES ON SATELLITE MOTION

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ABSTRACT. First we obtain accurate analytical series representing the main part of variations of the geopotential coefficients caused by the solid Earth tides. The KSM03 expansion of the Earth tide-generating potential is used as a source. Then we use these series in analytical calculation of the corresponding tidal perturbations in satellite motion. Two geodynamical satellites are considered: low-altitude STARLETTE and high-altitude ETALON-1. The accuracy (r.m.s. error) of analytical calculation of the discussed effect is estimated as 2 cm for STARLETTE over a time interval of 1 month (or some 415 orbits of the satellite) and 1 mm for ETALON-1 over a time interval of 1 year (or some 775 orbits of the satellite).

1. CONVENTIONAL MODEL OF THE SOLID EARTH TIDES

The IERS Conventions (2010) (Petit & Luzum 2010) describe the main effect of the solid Earth tides on the Earth gravitational potential through variations $\Delta\bar{C}^{ST}_{nm}$, $\Delta\bar{S}^{ST}_{nm}$ in instantaneous values of the normalized standard geopotential coefficients

$$\Delta\bar{C}^{ST}_{nm} - i\Delta\bar{S}^{ST}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{\mu_j}{\mu_E}\left(\frac{R_E}{r_j}\right)^{n+1}\bar{P}_{nm}\left(\sin\phi_j\right)e^{-im\lambda_j}, \tag{1}$$

where $i \equiv \sqrt{-1}$; k_{nm} are frequency-independent conventional Love numbers; R_E, μ_E are, respectively, the Earth's equatorial radius and gravitational parameter; μ_j, r_j, ϕ_j and λ_j are, respectively, the gravitational parameter, geocentric distance, geocentric latitude and East longitude (from Greenwich) of the Moon ($j = 2$) and Sun ($j = 3$), calculated on the basis of numerical planetary/lunar ephemerides; $\bar{P}_{nm}$ are the normalized associated Legendre functions.

Only the solid Earth tides of degree 2 and 3 by the IERS Conventions (2010) are recommended to be taken into account in accurate propagation of satellite motion. The degree 2 tides also lead to some variations in the degree 4 gravitational coefficients as a consequence of the Earth's ellipticity and the Coriolis force due to the planet rotation

$$\Delta\bar{C}^{ST}_{4m} - i\Delta\bar{S}^{ST}_{4m} = \frac{k^{(+)}_{2m}}{5}\sum_{j=2}^{3}\frac{\mu_j}{\mu_E}\left(\frac{R_E}{r_j}\right)^{3}\bar{P}_{2m}\left(\sin\phi_j\right)e^{-im\lambda_j}, \tag{2}$$

where $m = 0, 1, 2$; and $k^{(+)}_{2m}$ are some small additional parameters characterizing the degree 2 Love numbers (Wahr 1981).

Anelasticity of the Earth's mantle leads to a certain phase lag in the deformational response of the Earth to tidal forces; mathematically it is described by introducing complex values for Love numbers. Another effect of the anelasticity is the frequency dependence of Love numbers (Wahr 1981). In order to account for the latter effect, Eanes et al (1983) suggest to compute some additional corrections to the gravitational coefficients $\bar{C}_{2m}$, $\bar{S}_{2m}$ due to deviations of the frequency dependent degree 2 Love numbers from their nominal values k_{2m}. The IERS Conventions (2010) give these corrections by trigonometric series.

Equations (1)–(2) can be conveniently used in numerical theories of satellite motion, but for use in analytical theories these equations should be developed to harmonic series. The next section presents our results in this field.

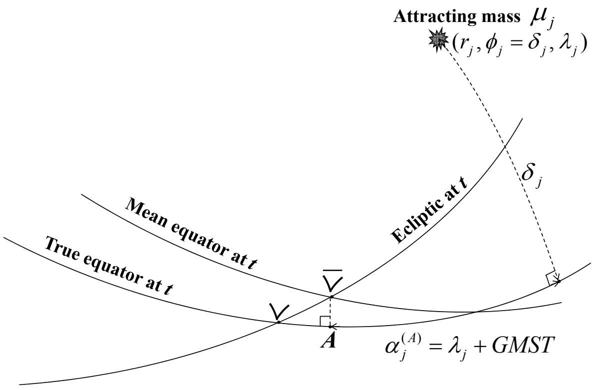

Figure 1: Spherical coordinates used in the KSM03 development of the Earth TGP

2. REPRESENTATION OF THE SOLID EARTH TIDE EFFECT ON GEOPOTENTIAL BY ANALYTICAL SERIES

In order to build compact analytical series representing the main variations in Equations (1)–(2) of the geopotential coefficients due to the solid Earth tides, we use the latest development of the Earth tide-generating potential (TGP), KSM03 (Kudryavtsev 2004). The instantaneous value for the Earth TGP, V, at an arbitrary point P on the Earth's surface at epoch t by the KSM03 development is given as follows

$$\begin{aligned} V(t) &= \sum_{n=2}^{\infty} \left(\frac{r}{R_E}\right)^n \sum_{m=0}^{n} \bar{P}_{nm}(\sin\phi) \\ &\times \left[\bar{C}'_{nm}(t)\cos m\theta^{(A)}(t) + \bar{S}'_{nm}(t)\sin m\theta^{(A)}(t)\right], \end{aligned} \tag{3}$$

where

$$\bar{C}'_{nm}(t) \equiv \frac{1}{2n+1} \sum_j \frac{\mu_j}{R_E} \left(\frac{R_E}{r_j(t)}\right)^{n+1} \bar{P}_{nm}(\sin\delta_j(t)) \cos m\alpha_j^{(A)}(t), \tag{4}$$

$$\bar{S}'_{nm}(t) \equiv \frac{1}{2n+1} \sum_j \frac{\mu_j}{R_E} \left(\frac{R_E}{r_j(t)}\right)^{n+1} \bar{P}_{nm}(\sin\delta_j(t)) \sin m\alpha_j^{(A)}(t), \tag{5}$$

and $\alpha_j^{(A)}(t)$, $\delta_j(t)$ are, respectively, the instantaneous right ascension and declination of the j^{th} attracting body referred to the true equator of epoch t with the origin point A - that being the projection of the mean equinox of date (Figure 1); r, ϕ are, respectively, the geocentric distance and geocentric latitude of P, and $\theta^{(A)}(t)$ is the local sidereal time at P reckoned from point A. The latter parameter is related to the Earth-fixed East longitude (from Greenwich) λ of point P as

$$\theta^{(A)}(t) = \lambda + GMST, \tag{6}$$

where $GMST$ is Greenwich mean sidereal time as defined by Aoki et al (1982). (In the KSM03 development, Equation (3) is also completed by some additional terms of degree $n = 1$ reflecting the main effect of the Earth's flattening, but these terms are not relevant to the present study.)

The $\bar{C}'_{nm}$, $\bar{S}'_{nm}$ coefficients by the KSM03 development are represented by 2nd-order Poisson series with numerical coefficients; the argument of every series' term is a fourth-order polynomial of time t.

By comparing Equation (1) and Equations (4)–(5) and taking into account the relations $\phi_j = \delta_j$ and $\alpha_j^{(A)}(t) = \lambda_j + GMST$, one can find the following equations linking the original coefficients $\bar{C}'_{nm}$, $\bar{S}'_{nm}$ in the KSM03 development of the Earth TGP and main variations $\Delta\bar{C}^{ST}_{nm}$, $\Delta\bar{S}^{ST}_{nm}$ of the geopotential coefficients due to the solid Earth tides

$$\Delta\bar{C}^{ST}_{nm} = \Delta\bar{C}'_{nm} \cos\left(m \times GMST\right) + \Delta\bar{S}'_{nm} \sin\left(m \times GMST\right), \tag{7}$$

$$\Delta\bar{S}^{ST}_{nm} = \Delta\bar{S}'_{nm} \cos\left(m \times GMST\right) - \Delta\bar{C}'_{nm} \sin\left(m \times GMST\right), \tag{8}$$

where

$$\Delta\bar{C}'_{nm} = \frac{R_E}{\mu_E}\left(\Re k_{nm}\bar{C}'_{nm} + \Im k_{nm}\bar{S}'_{nm}\right), \tag{9}$$

$$\Delta\bar{S}'_{nm} = \frac{R_E}{\mu_E}\left(\Re k_{nm}\bar{S}'_{nm} - \Im k_{nm}\bar{C}'_{nm}\right), \tag{10}$$

and $\Re k_{nm}$, $\Im k_{nm}$ are, respectively, the real and imaginary parts of the complex value for the frequency-independent nominal Love number k_{nm}. Equations (9)–(10) are written in general form, but presently only degree 2 nominal Love numbers by the IERS Conventions (2010) are given by complex values (to be used in case of anelastic Earth). From Equation (2) and Equations (4)–(5) one can find the additional changes in the $\Delta\bar{C}'_{4m}$ and $\Delta\bar{S}'_{4m}$ coefficients produced by the degree 2 tides

$$\Delta\bar{C}'_{4m} = \frac{R_E}{\mu_E} k^{(+)}_{2m} \bar{C}'_{2m}, \tag{11}$$

$$\Delta\bar{S}'_{4m} = \frac{R_E}{\mu_E} k^{(+)}_{2m} \bar{S}'_{2m}, \tag{12}$$

where $m = 0, 1, 2$. (In satellite dynamics the direct effect of the degree 4 solid Earth tides on the $\Delta\bar{C}'_{4m}$ and $\Delta\bar{S}'_{4m}$ coefficients, given by Equations (1)–(2), is considered as negligible.)

By using Equations (9)–(12) and the KSM03 harmonic development for the $\bar{C}'_{nm}$, $\bar{S}'_{nm}$ coefficients, we obtained analytical series for $\Delta\bar{C}'_{nm}$ and $\Delta\bar{S}'_{nm}$ for every relevant n and m. The series and their format description are available at http://lnfm1.sai.msu.ru/neb/ksm/solid_tides/SolidTides_Geopotential.zip. The maximum number of Poisson terms in the obtained series is 436 (in case of $\Delta\bar{C}'_{22}$); the maximum error of representing the numerical values for $\Delta\bar{C}'_{nm}$ and $\Delta\bar{S}'_{nm}$ by the analytical series is less than 0.7×10^{-12} (the comparison is done with use of the numerical ephemeris DE423 (Folkner 2010) over 1800–2200).

3. USE OF NEW SERIES IN ANALYTICAL THEORY OF SATELLITE MOTION

The new series for tidal variations of the geopotential coefficients due to the solid Earth tides are used in the author's analytical theory of satellite motion (Kudryavtsev 2002). We studied the effect of the solid Earth tides on motion of two geodetic satellites: low-altitude STARLETTE (the semimajor axis $a = 7335$ km) and high altitude ETALON-1 ($a = 25500$ km). First, we computed positions of those satellites over one month for STARLETTE (or for some 415 orbits of the satellite) and over one year for ETALON-1 (or for some 775 its orbits). Here the 15th-order Everhart's numerical integration method was used. The tide model included both the main part of the solid Earth tides defined by Equations (1)–(2) and the additional corrections accounting for frequency dependence of the degree 2 Love numbers. Positions of the attracting bodies were calculated on the basis of DE423 planetary/lunar ephemerides. The sampled step was chosen equal to 0.1 day for STARLETTE and to 1 day for ETALON-1. Then we assumed these satellites' positions as fictitious observations and processed them with help of the least-mean-square method, where our analytical theory for propagation of the satellite motion was used. The theory employed the new analytical series, Equations (7)–(8), describing the effect of the solid Earth tides, plus the known trigonometric series for corrections to geopotential coefficients, caused by frequency dependence of the degree 2 Love numbers. The tidal perturbations of the first and second order by the analytical theory were calculated.

A comparison of the satellites coordinates obtained by the two methods is done. As a result, the r.m.s. error of analytical calculating the effect of the solid Earth tides with help of the new series is estimated as equal to 2 cm for the STARLETTE satellite (over 1 month), and to 1 mm for the ETALON-1 satellite (over 1 year). For comparison, if one did not take into account the corresponding tidal effect at all, the r.m.s error would be some 13.5 m for STARLETTE and 5.1 m for ETALON-1.

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