SEMI-ANALYTICAL INTEGRATION OF PRECESSION-NUTATION BASED ON THE GCRS COORDINATES OF THE CIP UNIT VECTOR

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ABSTRACT. In a previous paper (Capitaine et al. 2006), referred here as Paper I, we demonstrated the possibility of integrating the Earth's rotational motion in terms of the coordinates (X, Y) of the celestial intermediate pole (CIP) unit vector in the Geocentric celestial reference system (GCRS). Here, we report on the approach that has been followed for solving the equations in the case of an axially symmetric rigid Earth and the semi-analytical (X, Y) solution obtained from the expression of the external torque acting on the Earth derived from the most complete semi-analytical solutions for the Earth, Moon and planets.

1. THE EQUATIONS AND INTEGRATION METHOD

In the axially symmetric case, the rigorous form of the precession-nutation equations of a rigid Earth model in terms of the GCRS coordinates (X, Y) of the CIP unit vector established in Paper I are:

$$\begin{aligned} -\ddot{Y} + \sigma \dot{X} &= \frac{L}{A} + F'', \\ \ddot{X} + \sigma \dot{Y} &= \frac{M}{A} + G'', \end{aligned} \tag{1}$$

where $\sigma = \frac{C\omega}{A}$ is the frequency of the Euler free motion in the celestial system, A and C being the Earth's principal moments of inertia and ω the mean angular velocity of the Earth.

L and M are the first two components of the external torque acting of the Earth in the geocentric celestial reference system denoted CIRS' (defined by the CIP and the point Σ on the CIP equator such that $\Sigma\mathrm{M} = \Sigma_0\mathrm{M}$, M being the node of the CIP equator on the GCRS equator and Σ_0 the origin on the GCRS equator). F'' and G'' are functions of the (X, Y) quantities and their first and second time derivatives; their rigorous expressions have been provided in Paper I (Equations 25 and 44).

The (X, Y) solutions are obtained by integration of Equation (1) by the method of variations of parameters.

2. SEMI-ANALYTICAL COMPUTATIONS

The external torque considered in this study is that caused by the solar system objects on the non-spherical Earth, supposed to be rigid and axially symmetric. The largest contribution is the torque exerted by the Moon for which it is necessary to take into account the contributions produced by the Earth's zonal coefficients J_2, J_3 and J_4. The CIRS' components $(L_\mathrm{M}, M_\mathrm{M})$ of the lunar torque are:

$$\begin{pmatrix} L_\mathrm{M} \\ M_\mathrm{M} \end{pmatrix} = k'_{J_2} \left(\frac{a}{r}\right)^3 \begin{pmatrix} v'\,w' \\ -u'\,w' \end{pmatrix} k'_{J_3} \left(\frac{a}{r}\right)^4 \begin{pmatrix} v'\,(1-5w'^2) \\ -u'\,(1-5w'^2) \end{pmatrix} + k'_{J_4} \left(\frac{a}{r}\right)^5 \begin{pmatrix} \frac{1}{3}v'w'\,(3-7w'^2) \\ -\frac{1}{3}u'w'\,(3-7w'^2) \end{pmatrix}, \tag{2}$$

where r is the geocentric distance of the Moon, a is the semi-major axis of the lunar orbit, u', v', w', are the CIRS' components of the geocentric unit vector toward the Moon, and:

$$k'_{J_2} = \frac{3Gm}{a^3} MR^2\,J_2; \quad k'_{J_3} = \frac{1}{2}\left(\frac{R}{a}\right)\frac{3Gm}{a^3}\,MR^2\,J_3; \quad k'_{J_4} = \frac{5}{2}\left(\frac{R}{a}\right)^2 \frac{3Gm}{a^3}\,MR^2\,J_4\,, \tag{3}$$

G being the gravitational constant, R the mean radius of the Earth, M the mass of the Earth and m the mass of the Moon.

The development of the torque exerted on the Earth by the Sun and the planets can be obtained with the same approach as for the case of the Moon. Due to the distance of these bodies to the Earth as compared to that of the Moon to the Earth, only the effects related to the zonal coefficients J_2 and J_3 for the Sun, and the zonal coefficient J_2 for the planets, have to be considered for ensuring consistent accuracies. In the case of the planets, additional computations have been performed in order to obtain the analytical expressions for $(a_P/r)^3$ (a_P being the semi-major axis of the planet and r its geocentric distance) for both inner and outer planets.

The semi-analytical computations that have been performed for obtaining the expressions of (i) the components of the torque acting on the Earth and (ii) the solutions of the differential Equation (1), include polynomial forms of time and periodic components with several hundreds or thousands of Fourier and Poisson terms (up to the 5th order in time). The computations are based on the lunar theory ELP2000 (Chapront-Touzé & Chapront 1998) and the planetary theory VSOP87 (Bretagnon & Francou 1988) as well as on the use of the software package GREGOIRE (Chapront 2003) devoted to Fourier and Poisson series manipulations. The fundamental nutation arguments are referred to the J2000 ecliptic and equinox (cf. Bretagnon et al. 1998); the numerical values for the constants relative to the dynamics of the Earth, Moon and planets are from the IAU 2009 System of astronomical constants (Luzum et al. 2011). Table 1 provides the number of Fourier and Poisson terms considered for the different parts (due to the Moon, the Sun and the planets, respectively) of the L and M components of the torque in order to obtain the (X, Y) solution with a microarcsecond accuracy.

External torque due to	the Moon	the Sun	the planets
number of terms in the (L, M) components			
	$\boldsymbol{J_2}$	$\boldsymbol{J_2}$	$\boldsymbol{J_2}$
	(2386, 2060) Fourier	(244, 228) Fourier	(70, 70) Fourier
	(375, 474) Poisson 1st order	(36, 49) Poisson 1st order	(10, 10) Poisson
	(90, 81) Poisson 2nd order	(5, 5) Poisson 2nd order	1st order
	(5, 8) Poisson 3rd order	(2, 3) Poisson 3rd order	
	$\boldsymbol{J_3}$	$\boldsymbol{J_3}$	
	(435, 391) Fourier	(5, 4) Fourier	
	(76, 100 Poisson 1st order	(2, 2) Poisson 1st order	
	(13, 14) Poisson 2nd order		
	$\boldsymbol{J_4}$		
	(86, 85) Fourier		
	(10, 21) Poisson 1st order		

Table 1: Number of terms considered in the expression of the external torque acting on the Earth.

3. PRELIMINARY RESULTS

The precession-nutation solutions for a rigid Earth obtained directly in the (X, Y) variables by this new method, show an agreement at the 10–100 μas level (depending on the frequency of the term) with those that we have derived indirectly in those variables from the most accurate nutation series (Bretagnon et al. 1998, Souchay et al. 1999), expressed originally in the classical variables $(\Delta\psi, \Delta\epsilon)$.

4. REFERENCES

Bretagnon, P. and Francou, G., 1988, A&A 202, 309.

Bretagnon, P., Francou, G., Rocher, P., Simon, J.L., 1998, A&A 329, 329.

Capitaine, N., Folgueira, M. and Souchay, J., 2006, A&A 445, 347 (denoted Paper I).

Chapront, J., 2003, Notice, Paris Observatory (January 2003).

Chapront-Touzé M. and Chapront J., 1998, A&A 190, 342.

Luzum, B., Capitaine, N., Fienga, A., Folkner, W. et al., 2011, Celest. Mech. Dyn. Astr. 110, 4, 293.

Souchay, J., Loysel, B., Kinoshita, H. and Folgueira, M., 1999, A&AS 135, 111.