ANALYSIS OF CHANDLER WOBBLE EXCITATION, RECONSTRUCTED FROM OBSERVATIONS OF THE POLAR MOTION OF THE EARTH

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ABSTRACT Chandler excitation was reconstructed since 1846 yr. from EOP C01 by three methods: complex Singular Spectrum Analysis (SSA) with Wilson filter, least squares adjustment (LSA) with Tikhonov regularization, Panteleev smoothing. The aim was to damp annual component and side frequencies and to obtain excitation only for chandler wobble. Results of different methods are in agreement with each other. Modulation of Chandler excitation of ∼18.6 yr period, synchronous with Saros tidal effects in the length of the day (LOD) was found. It means that Chandler wobble swings under the influence of Luni-Solar tide. Amplitude and phase evolution in time was analyzed with use of Gabor transform. Phase changes in Chandler excitation found to have ∼37 yr period. It explains, why 18.6 yr modulation is not seen in the Chandler component itself, but only in its excitation.

1. INTRODUCTION

More than a century has passed since S.C. Chandler discovered in 1891 the periodicity, named in his honor [4,5]. Despite this, its final explanation still has not been found [16]. It's not clear what causes the modulation of the Chandler oscillation, what process makes a major contribution to this resonance phenomenon. As it is shown in [1,2,3,7,9,11] most of the energy can be explained by oceanic and atmospheric excitation, as well as changes in the ocean bottom pressure. But it's hard to show, that Chandler excitation behaves in time like these noisy processes on the long periodic scale. Additionally the longest observational time series of Atmospheric Angular Momentum (AAM) starts only in 1940-th. In this paper we offer a variety of approaches for the excitation functions (EF) reconstruction, in particular, looking for the Chandler oscillation physical causes.

Among the first H. Jeffreys raised the question about reconstruction of the polar motion (PM) causes [8]. EF reconstruction from the geodetic observations of the PM is an ill-posed problem. Wilson filter [15] is a standard procedure of reconstruction, but it does not include stabilization of solution. We suggest to use Panteleev corrective smoothing [10] or Phillips-Tikhonov regularization [12]. Chandler excitation reconstruction is complicated because of its resonance nature and strong annual component presents nearby in frequency band. We need to develop filter, which would damp annual oscillation together with low and high frequency components.

2. EXCITATION RECONSTRUCTION TECHNIQUES

Powerful method, which allows to separate PM trend, Chandler, annual components and noise is Singular Spectrum Analysis (SSA) [6]. We applied SSA to the complex PM coordinates time series $m = x - iy$ obtained from the IERS EOP C01 bulletin since 1846 yr. up to now (2010), brought to a time step of 0.05 yr. Figure 1a shows a plot of SSA-separated components. We illustrate the results on the example of x-coordinate of the pole, the picture for the y-component is similar. The main parameter of SSA is lag L, it was chosen to be 240 points, i.e. 12 years, almost equal to the double beat period of the annual and Chandler oscillations. Figure 1b shows spectrum of the annual and Chandler components, separated by means of SSA. The spectrum of the original PM series is represented as a background.

The dynamical system of the rotating Earth can be written in form

$$\frac{i}{\sigma_c}\frac{dm(t)}{dt} + m(t) = \chi(t), \tag{1}$$

with a complex parameter $\sigma_c = 2\pi f_c(1 + i/2Q)$, where Q is a quality-factor, f_c is a Chandler frequency. This first order equation is based on the linearization of the original Euler-Liouville equation and is a simple but an adequate approximation of the real system [2, 14]. Q and f_c are not known precisely,

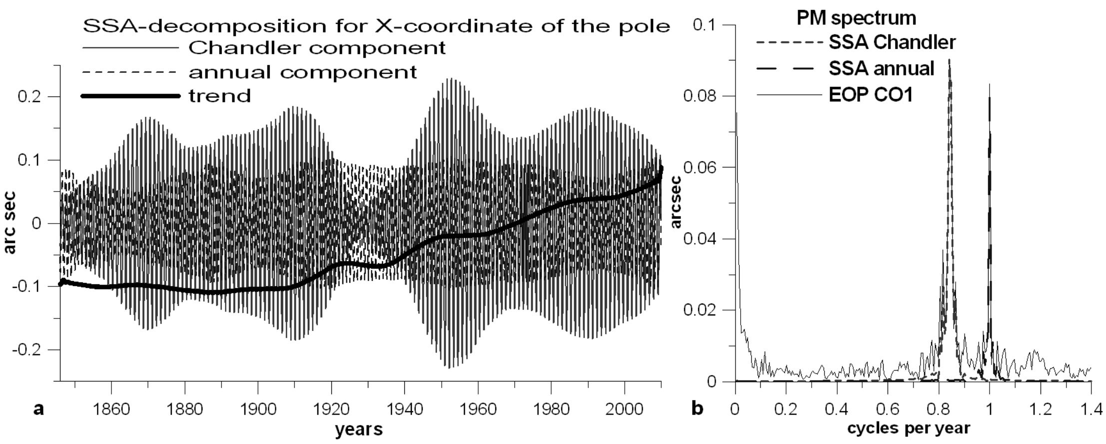

Figure 1: SSA-decomposition of PM (a) and it's spectrum (b).

what introduces uncertainty into the corresponding operator (1). In this research we use values $Q = 175$, $f_c = 0.843$ (Chandler period $T_c \approx 433$ days), as estimated in [13]. To obtain $\chi(t)$ trajectory of the pole $m(t)$ should be differentiated. This is a typical ill-posed problem.

The transfer function of equation (1) has the form

$$L(f) = \frac{\sigma_c}{\sigma_c - 2\pi f.} \tag{2}$$

Reconstruction of $\chi(t)$ from $m(t)$ can be represented in the frequency domain by multiplication of the PM spectrum by inverse operator

$$\hat{\chi}(f) = L^{-1}(f)\hat{m}(f),$$

here hat represents Fourier transform. AFR of the operator $L^{-1}(f)$ is represented on figure 2a by the solid line. It strengthen the side-frequency components in comparison with Chandler one. The Wilson filter

$$\chi(t) = \frac{ie^{-i\pi f_c \Delta t}}{\sigma_c \Delta t}\left[m_{t+\frac{\Delta t}{2}} - e^{i\sigma_c \Delta t} m_{t-\frac{\Delta t}{2}}\right], \tag{3}$$

where $\Delta t/2$ is a time interval between the equidistant observation samples, can be used as approximation of L^{-1} [14,15], at least near the Chandler frequency. It does not contain any procedure to stabilize the inverse solution. We applied filter (3) to the Chandler component, separated by SSA. So that annual component, trend and noises were removed, SSA together with Wilson filter could be consider as inversion with stabilization. Similar to Moore-Penrose inversion with cut-off of singular numbers, used for ill-conditioned linear systems.

The result of the Chandler excitation reconstruction, separated by means of SSA through Wilson filtering, is shown on Figure 2b by a solid line. For greater confidence, we shall try to prove the result by other methods, independent of SSA.

To solve the ill-posed problems regularization can be used [12]. Reconstruction of $\chi(t)$ can be regularized, so that the inverse operator will be written as

$$L^{-1}_{reg}(f) = \frac{L^*(f)}{L^*(f)L(f) + \alpha}, \tag{4}$$

where α is a regularization parameter. When $\alpha \to 0$, $L^{-1}_{reg}(f)$ tends to the inverse operator $L^{-1}(f)$. On the Figure 2a, together with AFR of $L^{-1}(f)$ operator, the AFR of $L^{-1}_{reg}(f)$ operator (4) with parameter $\alpha = 500$ is represented. This value was chosen to make the results more or less consistent with SSA results. Regularized operator is a filter, which suppresses those frequencies, where $|L(f)|$ is small. However, its AFR has very slowly decreasing slopes. It is hard to make annual component not to pass. Instead, we estimated the parameters of annual component by means of LSA and subtracted it from the original EOP C01 series (Table 1). The result of regularization in frequency domain is represented on Figure 2b by dot-dashed curve. Finally we used Panteleev corrective smoothing technique [10], based on the additional

	amplitude	phase for 1846.0
x-coordinate	0.088" ± 0.005"	$231° \pm 3°$
y-coordinate	0.078" ± 0.006"	$148° \pm 4°$

Table 1: Annual harmonic parameters, adjusted by LSM.

filtering of the inverse solution to suppress noise and annual component. The operator of the corrective smoothing is built in the frequency domain according to the expression

$$L_{corr}^{-1}(f) = \frac{L_{filter}(f)}{L(f)}, \tag{5}$$

here $L_{filter}(f)$ is a transfer function of the additional smoothing filter. We used Panteleev filter [16] with a frequency response centered at the Chandler frequency

$$L_{filter}(f) = \frac{f_0^4}{(f - f_c)^4 + f_0^4}. \tag{6}$$

The parameter $f_0 = 0.04$ was chosen to suppress low and high frequencies, including the annual. Such filter does not introduce phase distortion. The result of the corrective smoothing, performed in spectral domain, is also represented on Figure 3b. The phase change of Chandler excitation, obtained from the

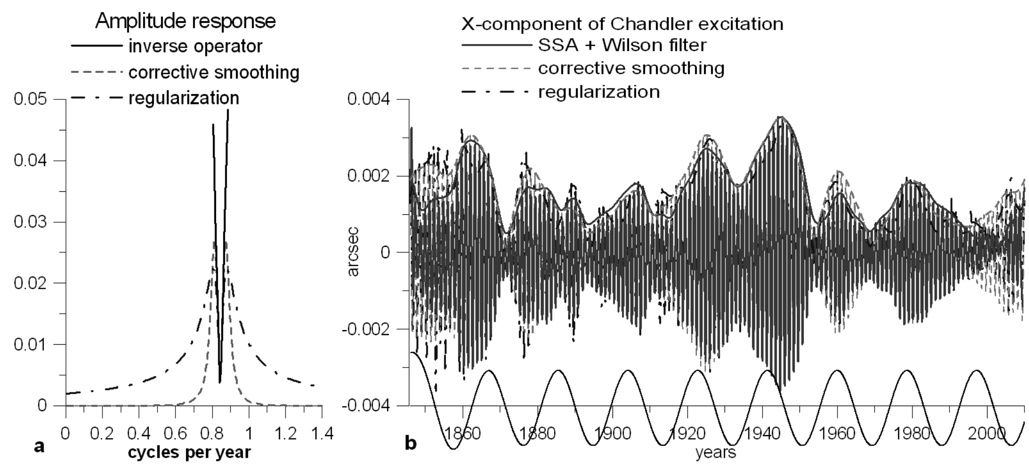

Figure 2: Impulse response of inverse operators (a) and reconstructed Chandler excitation (b).

Gabor transform argument, is shown on Figure 3 together with the phase evolution of the SSA-separated Chandler PM component [17]. Phase changes of Chandler EF, obtained by three different methods are in good agreement with each other.

3. DISCUSSION

Chandler excitation, as seen on Figure 2b has an amplitude modulation. It's amplitude changes and is quite large in the 1930-th, when the observed amplitude of Chandler PM component is small (Fig. 1a). Along the abscissa on Figure 2b the 18.6-yr oscillation is plotted, calculated as the mean of the IERS model of zonal tides for the length of day (LOD). This harmonic, associated with Saros cycle, has the behavior, synchronous with modulation of the Chandler excitation in most cases. The correlation is not observed in the 1990-th, probably, because of the boundary-effects and for early astronomical observations, obtained before the establishing of the International Latitude Service (ILS) in 1899. However, during the XX cent. there are 5 synchronous peaks. Thus, when the rotation of the Earth slows down, during the peak tides, the Chandler excitation amplitude increases. This leads us to the hypothesis that the same factor governs both rotational velocity of the Earth and the Chandler motion of the pole on this time

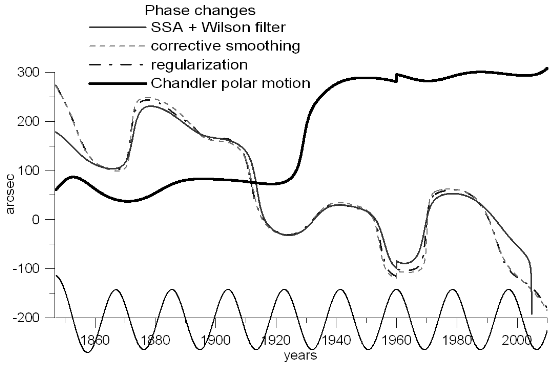

Figure 3: Phase changes of Chandler PM and excitation function.

scale. It is possible that tidal perturbations make the energy transfer to the Chandler oscillation. The mechanism is not clear, perhaps, the atmosphere and the ocean are the mediators.

Speaking about the phase, Fig. 3, the well-known $\sim 3\pi/2$ phase jump in the Chandler PM around 1930-th, almost disappears in EF. So that filter (6) does not change phase, this can be explained only by the influence of the dynamical system (1) and constricting of frequency band. At the same time phase changes of $\sim 37 \approx 2 * 18.6$ yr period are seen in the EF. This phase changes do not allow us to see in the Chandler PM component (Fig. 1a) such amplitude changes, as in it's excitation (Fig. 2b).

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4. REFERENCES

[1] Bizouard C., Lambert S., Remus F., Seoane L., Gambis D., "The source of the variable Chandler wobble", Proceedings of Journees 2010.

[2] Bizouard C., L. Seoane, 2010, Atmospheric and oceanic forcing of the rapid polar motion J Geod., Vol. 84, pp.19-30, doi: 10.1007/s00190-009-0341-2.

[3] Brzezinski A., and J. Nastula, 2002 "Oceanic excitation of the Chandler wobble", Adv. Space Res., Vol. 30, pp. 195-200, doi: 10.1016/S0273- 1177(02)00284-3.

[4] Carter M.S., Carter W.E., 2000, "Setho Carlo Chandler Jr.: the discovery of variation of latitude", ASP conference series, Vol. 208, p. 109.

[5] Chandler S.C., 1891, "On the variation of latitude I, II", AJ , Vol. 248, 59, Vol. 249, 65.

[6] Ghil, M., R. M. Allen, M. D. Dettinger, K. Ide, D. Kondrashov, et al., 2002, "Advanced spectral methods for climatic time series", Rev. Geophys. 40(1), 3.13.41, doi: 10.1029/2000RG000092.

[7] Gross R. S., 2000, "The excitation of the Chandler wobble", Geophysical Research Letters, Vol. 27, No. 15, pp. 2329-2332.

[8] Jeffreys H., 1940, "The variation of latitude", Mon Not Roy Astr. Soc., Vol. 100, pp. 139-155.

[9] Liao D., Xinhao Liao and Yonghong Zhou, 2003, "Oceanic and atmospheric excitation of the Chandler wobble", Geophys. J. Int. , Vol. 152, pp. 215-227.

[10] Panteleev V.L.,Chesnokova, T.S., 2011, Problem deconvolution in inertial gravimetry, Moscow University Physics Bulletin, Vol. 66, N 1.

[11] Salstein D., 2000, "Atmospheric excitation of polar motion", ASP Conference Series, Vol. 208, pp. 437-446.

[12] Tikhonov A.N., Arsenin V.Y., 1977, "Solution of ill-posed problems", John Wiley and Soc.

[13] Vicente R., Wilson C., 1997, "On the variability of the Chandler frequency", J. Geophys. Res., Vol. 102(B9), pp. 20439-20445.

[14] Vicente R., Wilson C., 2002, "On long-period polar motion", J. of Geod., Vol. 76, No. 4, pp. 199-208.

[15] Wilson C., 1985, "Discrete polar motion equations", Geophys J. Roy. Astr. Soc., Vol. 80, pp. 551-554.

[16] Yatskiv Y., 2000, "Chandler Motion Observatios", ASP Conference Series, Vol. 208, 383.

[17] Zotov L., 2010, "Dynamical modeling and excitation reconstruction as fundamental of Earth rotation prediction", Artificial satellites, Vol. 45, N 2, pp. 95-106.