COMPARISON OF THE VARIOUS ATMOSPHERIC AND OCEANIC
ANGULAR MOMENTUM SERIES

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ABSTRACT. We present here an addition to the recent study of atmospheric and oceanic excitations in the motion of the Earth’s spin axis in space (Vondrák & Ron, 2010). Two approaches are used to study the impact of the atmospheric and oceanic excitations on the motion of Earth’s spin axis in space. One way consists in the integration of the Brzeziński broad band Liouville equation (Brzeziński, 1994) and second way in the spectral analysis of both the geodetic and geophysical excitation functions. We applied both approaches to the series of the atmospheric and oceanic angular momentum excitation functions. We have used the celestial pole offsets (CPO) from the recent IVS combined solution (Schlüter & Behrend, 2007) ivs09q3X, covering the interval 1984.1–2009.7, cleaned and interpolated to 3-day intervals. We used two pairs of the geophysical excitations data. First pair is the pressure and wind terms of atmospheric angular momentum excitation functions (AAMEF) from NCEP/NCAR re-analysis, 1983.0-2009.5 (Salstein, 2005) together with the matter and motion terms of oceanic angular momentum excitation functions (OAMEF) from ECCO model 1993-2009.5. The second pair used is the AAMEF from ERA, 1979.0-2009.0 (Dobslaw & Thomas, 2007), re-analysed model before 2001 and operational model afterwards together with the OAMEF from OMCT model, 1979-2009.0 (Dobslaw & Thomas, 2007) driven by re-analysis atmospheric model before 2001.0 and by operational model afterwards.

The complex AAMEF and OAMEF values $\chi$ are given in rotating terrestrial frame, so we have to transform them into the celestial (non-rotating) frame $\chi'$ by applying $\chi' = -\chi e^{i\varphi}$, where $\varphi$ is the Greenwich sidereal time. Because we are interested in only long-periodic motion that is comparable to nutation frequencies we further removed all periods $<10$ days using the smoothing after Vondrák (1977). In this way we performed the complex demodulation (Brzeziński et al. 2002) at the retrograde diurnal frequency on the time series $\chi$ (in complex form).

We used Brzeziński transfer function (Brzeziński et al., 2002, Eq. 5) to estimate atmospheric and oceanic contribution to annual and semianual nutation terms shown in Tab. 1 (for more details see Vondrák & Ron, 2010).

Another possibility to compare the excitations with the observed CPOs is the numerical integration of the broad-band Liouville equations (Brzeziński, 1994)

\[
\dot{P} - i(\sigma'_c + \sigma'_f)P - \sigma'_c\sigma'_f P = -\sigma_c \left\{ \sigma'_f (\chi'_p + \chi'_w) + \sigma'_c (a_p \chi'_p + a_w \chi'_w) + i[(1 + a_p)\dot{\chi}'_p + (1 + a_w)\dot{\chi}'_w] \right\},
\]

where $P = dX + idY$ is excited motion of Earth's spin axis in celestial frame, $\sigma'_c$, $\sigma'_f$ are the complex Chandler and FCN frequencies in celestial frame, respectively and $\sigma_c$ in terrestrial frame. The values $a_{p,w}$ are dimensionless constants. To obtain two first-order equations instead of a second-order one, we use the substitution $y_1 = P$ and $y_2 = \dot{P} - i\sigma'_c P$, which leads to differential equations for two complex functions $y_1$, $y_2$:

\[
y_1 = i\sigma'_c y_1 + y_2
\]
\[
y_2 = i\sigma'_f y_2 - \sigma_c \left\{ \sigma'_f (\chi'_p + \chi'_w) + \sigma'_c (a_p \chi'_p + a_w \chi'_w) + i[(1 + a_p)\dot{\chi}'_p + (1 + a_w)\dot{\chi}'_w] \right\}
\]

Table 1: Atmospheric and oceanic contribution in nutation [μas], calculated by convolution with Brzeziński transfer function.

| Excitation | Annual | | | | | | Semi-annual | | | |
|------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|            | Re$^+$ | Im$^+$ | Re$^-$ | Im$^-$ | Re$^+$ | Im$^+$ | Re$^-$ | Im$^-$ | Re$^+$ | Im$^+$ | Re$^-$ | Im$^-$ |
| NCEP+ECCO  | -58.5  | 87.9   | -78.4  | -20.3  | -4.0   | 72.4   | 13.1   | 11.8   |        |        |        |        |
| ERA+OMCT   | -130.3 | 129.2  | -52.9  | 56.5   | -39.6  | 71.9   | 5.0    | -9.9   |        |        |        |        |

Another possibility to compare the excitations with the observed CPOs is the numerical integration of the broad-band Liouville equations (Brzeziński, 1994)
To integrate the system by the fourth-order Runge-Kutta method with 6-hour steps we need to choose the initial values, \( y_1(0) = P_0, y_2(0) = i(\sigma'_f - \sigma'_C)P_0 \) that are constrained so that the Chandlerian amplitude disappears. The final choice of \( P_0 \) was made by two methods. Either until the fit of the integrated motion to VLBI observations reaches a minimum (Vondrák & Ron, 2010) or until the magnitude squared coherence estimate \( C_{xy} \) of the input signals of integration and observation is maximum (in this study). The results of both approaches are almost the same for ERA+OMCT and rather different for NCEP+ECCO combination, see the following figure.

![Figure 1: CPO from IVS solution (dots), numerical integration of the excitations fitted to IVS CPO (bold line), numerical integration with maximum coherence with the IVS CPO (thin line). Both series ERA+OMCT (left) and NCEP+ECCO (right) are displayed in the same interval 1993.0–2009.0 for comparison.](image)

We can conclude that the forced nutations due to excitation by the atmosphere and ocean are significant, especially at annual and semi-annual periods. The different models of series of geophysical excitations give slightly different results. The initial values of the integration are close each other for both version of their determination, by the method of the maximum coherence or minimum root mean square differences.

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**REFERENCES**


