

# TESTS OF FUNDAMENTAL PHYSICS WITH THE GAIA MISSION

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**ABSTRACT.** With a launch scheduled in late 2012, the ESA Gaia mission will perform a systematic survey of the whole sky several times during its operational five years. It will provide high-accurate data of solar-system objects including the positions of about 250,000 asteroids with an unprecedented precision (at the sub-milliarcsecond level). Opportunities are thus arisen in carrying out various tests of fundamental physics from the dynamics of minor planets. Here we present the future performance of Gaia in estimating a set of global parameters which are, the PPN parameters  $\beta$  and  $\gamma$ , the dynamic solar quadrupole  $J_{2\odot}$ , the variation of the gravitational constant  $\dot{G}/G$ , and the Nordtvedt parameter  $\eta$  (test of the Strong Equivalence Principle). The expected precisions were obtained from simulations taking into account the time sequences and geometry of the observations peculiar to Gaia as well as the astrometric precision of the future measurements for about 230,000 asteroids.

## 1. INTRODUCTION

The theory of General Relativity (GR) put forward by Einstein in 1915 allowed to explain many noticeable deviations from the Newton's law of gravitation. The "anomalous" perihelion precession of Mercury is one classical test of GR which contributed to the wide adoption of this theory. The idea of analysing asteroid motions to test the GR theory arose in 1953 from Gilvarry [1], who suggested to base an observational test of the relativistic perihelion precession from high-eccentric minor planets, and in particular, (1566) Icarus [2] discovered 4 years before. This experiment was performed several times [3]-[4] reaching at best a precision of a few percent on the estimation of a relativistic parameter. Nevertheless, the measurement accuracy was strongly limited by large observational stochastic errors and an incomplete dynamical model (e.g. unmodeled non-gravitational forces), and did not allow to fully use the potential of minor planets: By reason of their various orbital characteristics, asteroids allow to disentangle relativistic effects from other perturbations. However, recent advances in high-accuracy astrometry from the ground (radar) or from space (Hipparcos) and those to come such as the Gaia mission [5] will enable us to perform GR experiments competing with the best present ones.

The ESA second-generation astrometric mission Gaia is on its way to the launch (2012). It will make a breakthrough in astrometry by performing observations at the sub-milliarcsecond level. The satellite will observe about 250,000 minor planets down to V magnitude 20 with a precision ranging from a few milli-arcsecond (*mas*) to a hundred micro-arcsecond (*μas*) or so. Furthermore, the Gaia stellar catalogue—positions of about one billion stars with a precision of a few hundreds *μas* at the lowest detectable magnitude—will drastically improve the reduction accuracy of ground-based observations. The huge number of high-accuracy data will thus boost our knowledge in the dynamics of the Solar System and allow us to perform clean tests of fundamental physics.

Here we evaluate the performance of Gaia in fitting a set of global parameters: the PPN parameters  $\beta$  and  $\gamma$ , the solar oblateness  $J_{2\odot}$ , the variation in time of the gravitational constant  $\dot{G}/G$ , the Nordtvedt parameter  $\eta$  (test of the Strong Equivalence Principle) and the Jovian  $GM_j$  (product of the gravitational constant  $G$  and the mass of Jupiter  $M_j$ ) by reason of a possible strong correlation with  $\eta$ . The expected precisions on their measurements are given in Section 3 from a variance analysis using realistic simulated data (Section 2).

## 2. SIMULATIONS

From simulated data, a variance analysis was performed from the formulation of observed minus calculated position (O-C) linearised with respect to the position and velocity vectors of each asteroid and the set of global parameters ( $\beta$  or  $\gamma$ ,  $J_{2\odot}$ ,  $\dot{G}/G$ ,  $\eta$ ,  $GM_j$ ) at the initial epoch  $T_0$ . The latter is taken at the half of the Gaia observational timespan (2012-2017), that is  $T_0 = \text{JD } 2456841.125$  (3.07.2014). The matrix giving the partial derivatives with respect to the unknown parameters is computed by numerical integration of the variational equations simultaneously with the equations of motion. A precise description of the Gaia fitting process can be read in [6]. From a software developed for the mission by F. Mignard and caring for the updated observational characteristics of the satellite, a systematic exploration of the Gaia transit times was thus performed from 01/01/2012 over a period of five years for 509,550 asteroids. The orbital elements and absolute magnitudes—necessary to apply the filter on the apparent magnitude—were taken from the ASTORB catalog of Bowell [7]. Thus, at least 229,657 asteroids are expected to be observed by Gaia and considered in the estimation of the expected precisions on the global parameters. The astrometric precision of each simulated observation (necessary to weight the equations of observation) is derived from a function depending on the apparent magnitude and velocity of the asteroid [8].

## 3. RESULTS AND DISCUSSIONS

In Table 1, we yield, for the above set of parameters, the Gaia expected precisions, the best current ones as a comparison and those by other future missions. The correlation coefficients are given in Table 2. The parameter  $\beta$  and  $\gamma$  were separated in the variance analysis: as they are very strongly correlated, it is more relevant to fit only one and to hold the other one at an estimated value from another method. In our case, fitting  $\beta$  turns out to be the best solution given that the PPN parameter  $\gamma$  can be derived more precisely: the bending of light experiment by Gaia should allow us to achieve a precision of about  $10^{-6}$  [9] instead of  $8 \times 10^{-4}$  from the Gaia asteroid observations.

Testing the Nordtvedt effect is important. The latter is a violation of the Strong Equivalence Principle (SEP)—one of bases of General Relativity—, which states the inequality between the inertial and gravitational masses, respectively,  $m_i$  and  $m_g$ . This effect can be generalised in the PPN formalism by the expression  $m_g/m_i = 1 + \eta\Omega$ , where  $\eta$  is the dimensionless Nordtvedt parameter and  $\Omega$  a term depending on the gravitational self-energy of the astronomical body. In our case, the Nordtvedt effect is considered only for the Sun whereupon it would produce for  $\eta \neq 0$  an “anomalous” indirect perturbation on the asteroid motion through the other solar-system objects, and mainly through the most massive planet. The formal precision on  $\eta$  is thus estimated through the Jovian perturbations.

The fit of all these parameters by Gaia will be the most accurate ever achieved from the dynamics of asteroids, and even if the expected precisions are not better than the best current ones (see Tab. 1), they remains in general though competitive and valuable. Besides, the astrometric precision used in the simulations were underestimated [8]. The Gaia data should thus provide a new accurate constrain on these parameters: the systematic errors will be limited because the accurate observations will span a short period of time (five years) and the dynamical model will be very complete taking into account the perturbations from many massive asteroids, relativistic effects from the Sun, and the planets for certain asteroids as well as non-gravitational forces. The correlations between the adjusted parameters are low excepted between  $J_{2\odot}$  and  $\beta$  as expected with a correlation coefficient of 0.672. However, the estimation of the solar quadrupole  $J_{2\odot}$  by Gaia does not turn to be relevant, the standard deviation being similar to current estimates. Thus, holding  $J_{2\odot}$  at a nominal value from future mission like BepiColombo appears to be a wise solution to fit the other global parameters.

An interesting point to note is that the more accurate estimations expected for these parameters from other missions and methods, will inform us about the correctness of the dynamical model used for the Gaia mission: for example, a precision of  $10^{-5}$  is predicted from the LLR experiments with the APACHE telescope [15], and a deviation between the LLR measurement and those from Gaia could highlight inadequacies of the dynamical model.

For the sake of curiosity, a variance analysis for the simultaneous fit of the  $GM$  of the eight planets and Pluto was carried out. The Gaia expected precisions and comparison to other derivations are compiled in Table 3. We can notice that only one estimation is interesting: the formal precision on the Jovian GM is similar to the current one and should be better by reason of the underestimated simulated Gaia precision.

Table 1: Overview of the Gaia expected formal precisions on the estimation of the PPN parameters  $\beta$  and  $\gamma$ , the solar quadrupole  $J_{2\odot}$ , the variation in time of the gravitational constant  $\dot{G}/G$ , the Nordtvedt parameter  $\eta$  and the  $GM$  of Jupiter.

Parameter	Initial value	Best Current		Precision	Future other missions	
				Gaia		
$\beta$	$1^\dagger$	$\sim 10^{-4}$	Planetary ephem.	$1.48 \times 10^{-3}$	$2 \times 10^{-6}$	BepiColombo[10]
$\gamma$	$1^\dagger$	$2.3 \times 10^{-5}$	Cassini[11]	$8.08 \times 10^{-4}$	$10^{-9}$	ASTROD[12]
$J_{2\odot}$	$2 \times 10^{-7}$	$\sim 5 \times 10^{-8}$	Planetary ephem.	$2.53 \times 10^{-7}$	$10^{-8}$	BepiColombo[13]
$\dot{G}/G$ [ $year^{-1}$ ]	$0^\dagger$	$9 \times 10^{-13}$	LLR[14]	$2.99 \times 10^{-12}$	$\sim 7.3 \times 10^{-14}$	LLR[15]
$\eta$	$0^\dagger$	$4.5 \times 10^{-4}$	LLR[14]	$2.86 \times 10^{-3\dagger\dagger}$	$\sim 10^{-5}$	LLR[15]
$GM_j$ [ $AU^3/d^2$ ]	$2.82 \times 10^{-7}$	$3.35 \times 10^{-15}$	Jovian satellites[16]	$2.78 \times 10^{-15}$		

$^\dagger$  Theoretical value,  $^\dagger\dagger$   $\Omega \sim -3.52 \times 10^{-6}$  was used [17].

Table 2: Correlation matrix for the fit of the global parameters listed in Tab. 1.

	$\beta$	$J_{2\odot}$	$\eta$	$GM_j$	$\dot{G}$
$\beta$	-				
$J_{2\odot}$	<b>0.672</b>	-			
$\eta$	-0.035	-0.015	-		
$GM_j$	-0.040	-0.037	0.097	-	
$\dot{G}/G$	-0.008	-0.008	0.217	0.049	-

Table 3: Gaia formal precisions on the  $GM$  estimates of the planets and Pluto.

Body	$GM_j$	Gaia	Current	Estimation method—references
$j$	[ $AU^3/d^2$ ]	precision [ $AU^3/d^2$ ]	precision [ $AU^3/d^2$ ]	
Mercury	$4.91 \times 10^{-11}$	$1.33 \times 10^{-15}$	$8.919 \times 10^{-17}$	Space probe [18]
Venus	$7.24 \times 10^{-10}$	$6.48 \times 10^{-16}$	$1.338 \times 10^{-17}$	Space probe [19]
Earth	$8.89 \times 10^{-10}$	$2.41 \times 10^{-16}$	$3.122 \times 10^{-18}$	Moon (LLR) [20]
Mars	$9.55 \times 10^{-11}$	$3.17 \times 10^{-16}$	$6.243 \times 10^{-19}$	Space probes [21]
<b>Jupiter</b>	$2.82 \times 10^{-7}$	$3.76 \times 10^{-15}$	$3.345 \times 10^{-15}$	Natural satellites [22]
Saturn	$8.46 \times 10^{-8}$	$6.84 \times 10^{-14}$	$2.453 \times 10^{-15}$	Natural Satellites and Space probes [23]
Uranus	$1.29 \times 10^{-8}$	$1.04 \times 10^{-12}$	$1.338 \times 10^{-14}$	Natural satellites [24]
Neptune	$1.52 \times 10^{-8}$	$2.90 \times 10^{-12}$	$2.230 \times 10^{-14}$	Space probe and Natural satellites [25]
Pluto	$1.95 \times 10^{-12}$	$3.43 \times 10^{-12}$	$2.163 \times 10^{-14}$	Natural satellites [26]

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