PROPAGATION IN TIME OF ERRORS FOR THE MUTUAL INCLINATION OF SATELLITES

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ABSTRACT
1. In the context of Keplerian elements, the inclination is a metric element and therefore a dynamic element. Variation in mass will produce a $(\Delta i_{mass}(t))$ variation.
2. Infinitesimal errors in the reference induce errors in all elements and, in particular, a $(\Delta i_{ref}(t))$
3. The question is how to distinguish the source (mass or reference) of error for a given $\Delta i(t)$.
4. Let us consider in the first place the minor planet Pallas, with different masses $M_p = 108e - 12M_{Sun}$ and $m_p = 398e - 12M_{Sun}$ (See [2])
5. We take the values ϵ_x, ϵ_y, ϵ_z given by [3] between Hipparcos and FK5. In both cases we show the evolution of $(\Delta i_{ref}(t)))$. It is clear that we find "crossed correlations" for $(\Delta i_{mass}(t))$ as well as for $(\Delta i_{ref}(t))$ and other variations such as $(\Delta i_{ref}(t))$, $(\Delta\Omega_{mass}(t))$ respectively. (See Fig 1)
6. For a long period of observation we can determine the kind of error. We apply the same principle to the mutual inclination of satellites. We take an example from [1] (See Figs 2, 3)
6a. We consider an ideal case of two bodies around a central body. We have modified their masses taking $m_1 = 2e - 5, 2e - 4, 4e - 4$ and $m_2 = 1e - 5, 1e - 4, 2e - 4$.
6b. We have considered a rotation of 5 min in the three coordinate axes, which means an uncertainty in the third decimal value.
6c. In Fig. 4 we show the difference between Δi_{ref} and Δi_{mass} for a long period of time.

1. CONCLUSION

There is a degree of uncertainty when we want to determine the values of the masses, because from the analysis of the differences between the elements we can deduct an uncertainty in their determination, with values under $2e - 4$

There are clear relationships of affinity between certain variations of the elements with respect of the mass and others with respect to infinitesimal rotations in the reference. These relationships appear as symmetries or antisymmetries when they are plotted.

The mutual inclination of the two bodies shows a clear dependence with their mutual distance, getting a sudden increment when the bodies are near a close encounter, as expected.

It is important to have observations for long periods of time to be compared with the computed positions near the time when several close encounters between satellites are expected.

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2. REFERENCES

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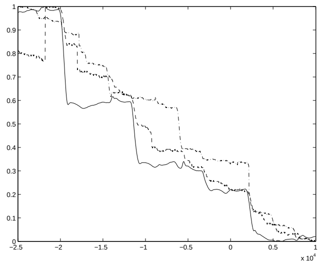

Figure 1: Pair Δi, $\Delta\Omega$ The solid line is $[\Delta i_{ref}]$, the dashed and dashed-dotted line are $[\Delta\Omega_{mass}]$ for the two masses considered.

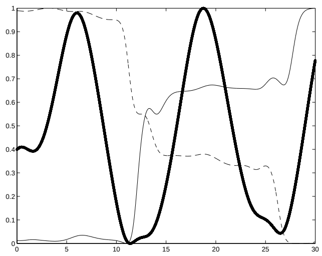

Figure 2: Comparison among the mutual differences in inclination $(i_{m1} - i_{m2})_{ref}$, (thin solid line), the distance between the bodies (thick solid line) and the $\Delta\Omega_{mass}$ for the first body (dashed line))

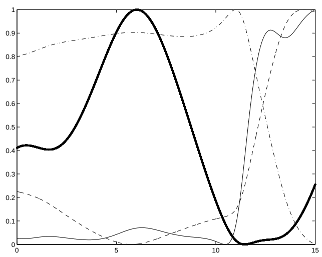

Figure 3: Comparison among the mutual differences in inclination $(i_{m1} - i_{m2})_{mass}$ (thin solid line), the distance between the bodies (thick solid line) and the a_{ref} for both bodies (a_{m1} is the dashed line and a_{m2} the dashed-dotted line)

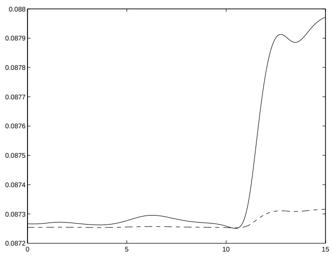

Figure 4: Comparison among the mutual differences in inclination $(i_{m1} - i_{m2})_{mass}$ (solid line), and $(i_{m1} - i_{m2})_{ref}$ (dashed line)