LLR RESIDUALS OF INPOP10A AND CONSTRAINTS ON POST-NEWTONIAN PARAMETERS

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ABSTRACT. Among all parameters involved in Lunar Laser Ranging computations (dynamical and reduction model), only some of them can be fitted to observations. Following is presented the method of their selection used for INPOP solutions and the residuals obtained. Then, adding the parameterized post-Newtonian parameters β and γ to the list of fitted parameters gives some constraints on their values and uncertainties.

1. SELECTION OF THE PARAMETERS FITTED TO LLR OBSERVATIONS

A planetary and lunar solution depends on:

- a dynamical model, describing how all bodies are interacting together
- a model of data reduction; for LLR observations, the model describes the light propagation from emission (on the Earth) to reflection (on the Moon) and reception (back on the Earth)
- a set of parameters, whose values are determined by a least square fit to observations

A total of 188 parameters are involved in LLR computations. They are listed in table 1, but all of them are not independent. For example, a modification of the position of Haleakala's emission station can be compensated by changing the position of the reception one; thus, they can not be fitted together. Other parameters are better determined by planetary observations (like the Earth-Moon mass ratio and the initial conditions for the Earth-Moon barycenter) or other techniques (velocities of stations by VLBI). Eventually, most of the offsets taken into account by J. Williams (private communication) are up to now ignored, because further investigation is needed about their relevance.

Parameter	Number
selenocentric positions of reflectors	12
geocentric positions and velocities of the 7 stations	42
initial conditions for the Earth-Moon vector (position and velocity)	6
initial conditions for Moon's librations (Euler's angles and their derivatives)	6
initial conditions for the Earth-Moon barycenter vector (position and velocity)	6
Earth's zonal coefficients of potential up to 4$^{\text{th}}$ degree	3
Moon's zonal and tesseral coefficients of potential up to 4$^{\text{th}}$ degree	18
Moon's principle moment of inertia C	1
time delays and Love numbers involved in tides effects (Earth and Moon)	10
ratio and sum of the masses of the Earth and the Moon	2
post-Newtonian parameters β and γ	2
offsets (constant and linear terms) applied to LLR measurements for some periods	40×2

Table 1: List of the 188 parameters involved in the reduction of LLR observations

Removing all these kinds of parameters leads to fit only 74 ones. But some of them remain badly determined, with important formal error compared to the fitted value: for example, the Moon's potential coefficient S_{43} is fitted to $(-2.0 \pm 13.5) \times 10^{-6}$.

To select which parameters are to be fitted to LLR observations, an iterated process is used by removing at each step the one with the greatest ratio between the formal error and the value. Removing degrees of freedom induces a weak increase of residuals to observations, as shown in table 2 with Grasse (3), Mc Donald, MLRS2 or Apollo data. On the other hand, fixing the value of a parameter may decrease the formal error of an another one if they are strongly correlated. For example, it is the case for the Moon's coefficient of potential C_{33}, which formal error decreases from 6.8×10^{-7} when 74 parameters are fitted, to 3.3×10^{-8}, 6.3×10^{-9}, 5.2×10^{-9} and 4.6×10^{-9} when only 65, 59, 55 and 51 parameters are fitted.

The INPOP10a solution, that is S059, is chosen so that the formal errors of all parameters represent less than 5% of the fitted values. Its Chebychev representation is available at `www.imcce.fr/inpop` and its LLR residuals are shown in table 2.

The R423 column of table 2 shows residuals obtained with our own reduction process, applied to the latest JPL's solution DE423, and by fitting only parameters involved in the reduction (positions of stations and reflectors, some offsets). Residuals shown here are thus not the ones obtained directly by the JPL, which could be much lower: for instance, less than 2 cm for Apollo station (Williams, private communication). One can see that they are better than INPOP10a ones (except for MLRS2 from 1988 to 1999), maybe because DE423 takes into account a lunar core.

	Solution:	S074	...	S065	...	S059	...	S055	...	S051	R423
	Maximum ratio	750%	...	9%	...	3.6%	...	1.2%	...	0.3%	
Station	Period	σ(cm)	...	σ(cm)	...	σ(cm)	...	σ(cm)	...	σ(cm)	σ(cm)
Grasse (1)	1984-1986	15.9	...	15.9	...	16.0	...	15.6	...	16.2	14.7
Grasse (2)	1987-1995	6.3	...	6.3	...	6.4	...	6.0	...	8.2	5.9
Grasse (3)	1995-2010	3.7	...	3.7	...	4.0	...	5.4	...	6.9	3.9
Mc Donald	1969-1985	31.2	...	31.4	...	31.8	...	36.1	...	50.0	29.8
MLRS1 (1)	1982-1985	73.3	...	73.0	...	73.3	...	72.5	...	71.7	70.3
MLRS1 (2)	1986-1988	8.0	...	7.5	...	7.3	...	7.4	...	9.8	6.1
MLRS2 (1)	1988-1999	4.3	...	4.3	...	4.3	...	4.3	...	6.5	4.7
MLRS2 (2)	1999-2008	4.6	...	4.6	...	4.8	...	4.9	...	6.5	4.6
Haleakala	1984-1992	8.1	...	8.2	...	8.1	...	8.4	...	11.6	8.1
Apollo	2006-2009	4.8	...	4.9	...	4.9	...	5.3	...	7.1	4.7

Table 2: Evolution of LLR residuals during the selection of fitted parameters process. For each solution Sxxx (where xxx is the number of parameters fitted) is given the greatest ratio between the formal error and the fitted value for the parameters. σ is the standard deviation of residuals for each station and epoch, expressed in centimeters. INPOP10a corresponds to the solution S059, with 59 parameters fitted.

2. TESTS OF POST-NEWTONIAN PARAMETERS β AND γ

The post-Newtonian parameters β and γ are involved at several steps in the building of a planetary and lunar solution, both in the dynamical and reduction parts:

- in the computation of Einstein-Infeld-Hoffmann acceleration vectors of bodies (see for instance Newhall, 1983)
- in the computation of the additional geodesic torques upon the Earth and the Moon (see for instance Misner et al., 1973)
- in the time scale transformation between TT and TDB (see Fienga et al., 2009 or Klioner et al, 2010)
- in the time delay due to the relativistic light deviation (see Williams et al., 1996).

Changing their values with respect to the general relativity ones (β=γ=1) will have consequences on LLR computations. They thus can be fitted together with the same 59 parameters as for INPOP10a. Three kinds of adjustments have been conducted:

- fit of β and the same 59 parameters as INPOP10a, with γ fixed to 1 (that is 60 fitted parameters).
- fit of γ and the same 59 parameters as INPOP10a, with β fixed to 1 (that is 60 fitted parameters).
- fit of γ, β and the same 59 parameters as INPOP10a (that is 61 fitted parameters).

For each type of adjustment, the uncertainties at 3σ (where σ is the formal error provided by the least square fit) are given in table 3. They are smaller than in (Müller et al., 2008), but fitted values are not always consistent with the general relativity.

Parameters fitted	$\beta - 1$	$\gamma - 1$	Correlation
(59)+β	$(-0.2 \pm 0.4) \times 10^{-3}$	0 (fixed)	$[\beta, X] = 0.35$
(59)+γ	0 (fixed)	$(-1.1 \pm 0.76) \times 10^{-3}$	$[\gamma, X] = 0.33$
(59)+β+γ	$(5.1 \pm 1.6) \times 10^{-3}$	$(-9.7 \pm 2.8) \times 10^{-3}$	$[\beta, \gamma] = 0.96$

Table 3: Fitted values and uncertainties (at 3σ) of post-Newtonian parameters β and γ. [β,X] (respectively [γ,X]) is the maximum correlation between β (respectively γ) and one of the 59 other parameters.

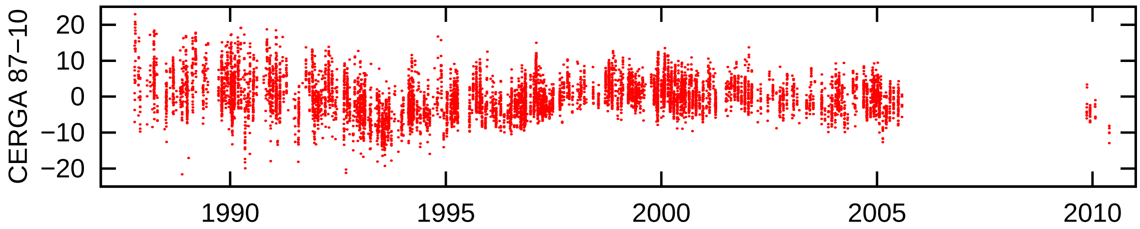

Figure 1: INPOP10a's LLR residuals (in cm) for the CERGA station, between 1987 and 2010.

These bad results could come not only from a big correlation (0.96) between β and γ, but also from the remaining signal on residuals for CERGA's observations, shown on figure 1. This latter is characteristic of a problem in modeling, that adjustment of parameters tries to compensate, and potentially leads to biased values. This signal has always been present in our LLR computations, including the R423 reduction previously described. The potential problem is thus not in the dynamical part, because INPOP's and DE423's modelings are independent. The signal is also present in the S2000 solution of (Chapront et al., 2001), where both dynamical and reduction modelings differ from INPOP's ones.

Other tests have been made by computing a map of (β,γ). More than 1600 couples of values (β,γ) are fixed in $[0.95, 1.05]^2$. For each one, a solution is built by fitting the same 59 parameters as in INPOP10a; then, the χ^2 and R functions are evaluated:

$$\chi^2(\beta,\gamma) = \sum_i \rho_i^2 (O-C)_i^2 \text{ and } R(\beta,\gamma) = \sqrt{\frac{\chi^2}{\chi_0^2}} - 1 \tag{1}$$

In these expressions, $(O-C)_i$ are the residuals (differences between the measurement and the computation), (ρ_i) are the ponderations applied to the observations, $\chi_0^2 = \chi^2(\beta_0, \gamma_0)$, where β_0 and γ_0 are the values where χ^2 is minimum, that is the ones from the last line of tab. 3. The R function is representative of the increase of residuals when β and γ differ from the "optimal" values (β_0,γ_0). Its contour lines are shown on figure 2.

First, one can notice that the shapes of ellipses confirm the strong correlation between β and γ. The best determined combination seems to be according the $2\beta - 11\gamma$ direction. It is different from the Nordtvedt parameter $\eta = 4\beta - \gamma - 3$, maybe because for LLR computations, post-Newtonian parameters are involved not only in the Lunar orbit around the Earth, but also in the light propagation with the Shapiro's deviation.

Second, one can notice that the general relativity values (blue dot) are not so far away from the ones where χ^2 is minimum (green dot), with a degradation limited to less than $R = 0.5\%$. For instance, LLR residuals for the CERGA's station between 1995 and 2010 grow from 3.98 cm to 4.0 cm when β and γ are both fixed to 1. This weak increase is not significant and is smaller than variations induced by changing the ponderations, the preliminary solution first fitted to planetary observations, ...

In the end, the constraints on post-Newtonian parameters obtained with LLR do not seem to be as good as the ones obtained by adjustment to planetary observations, exposed in this volume by (Fienga et al, 2010, figure 4). This point of view has to be balanced by the fact that LLR residuals computed directly at JPL by J. Williams are much lower than the ones of INPOP10a. Even if these latter are consistent with (Chapront et al.,2001), (Aleshkina, 2002) or solutions actually developed at SYRTE by S. Bouquillon and G. Francou, an improvement of the reduction modeling seems to be possible.

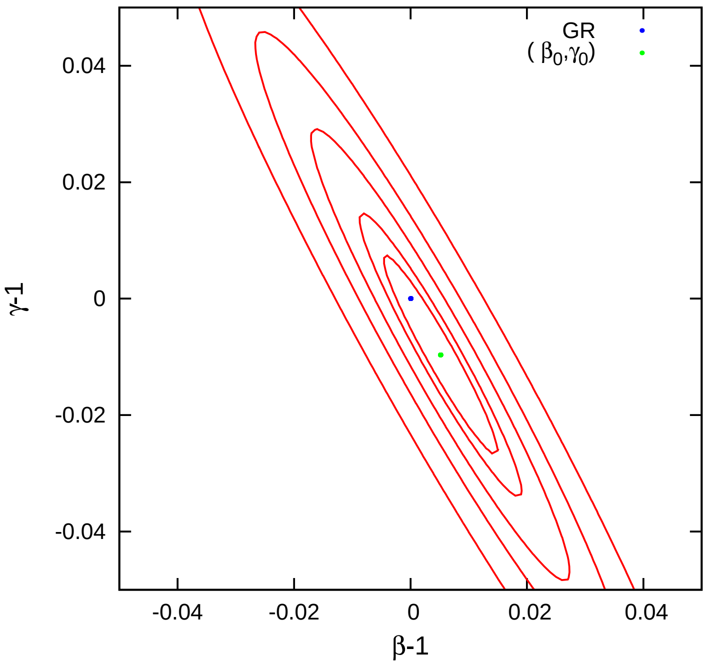

Figure 2: Contour lines (1%, 2%, 5%, 10% and 20%) for the R function. The blue dot corresponds to the general relativity values (β=γ=1), the green dot corresponds to the values of β and γ where χ^2 is minimum (see the last line of table 3).

3. REFERENCES

Aleshkina, E. Y., 2002, “Lunar numerical theory and determination of parameters k_2, δ_M from analysis of LLR data”, A&A , 394, pp. 717-721

Chapront, J., Chapront-Touzé, M., and Francou, G. 2001, “Contribution of LLR to the reference systems and precession”, in Journées 2000 - Systèmes de référence spatio-temporels, ed. N. Capitaine, pp. 96-101

Fienga, A., Laskar, J., Morley, T., Manche, H., Kuchynka, P., Le Poncin-Laffitte, C., Budnik, F., Gastineau, M., and Somenzi, L., 2009, “INPOP08, a 4-D planetary ephemeris : from asteroid and time-scale computations to ESA Mars Express and Venus Express contributions”, A&A , 507, 1675

Fienga, A., Laskar, Manche, H., Kuchynka, P., and Gastineau, M., 2010, “Planetary and lunar ephemerides INPOP10a”, in this volume

Klioner, S. A., Gerlach, E., and Soffel, M. H., 2010, “Relativistic aspects of rotational motion of celestial bodies”, in IAU Symposium, Vol. 261, IAU Symposium, ed. S. A. Klioner, P. K. Seidelmann, and M. H. Soffel, pp. 112–123

Misner, C. W., Thorne, K. S., and Wheeler, J. A., 1973, “Gravitation”, ed. Misner, C. W., Thorne, K. S., and Wheeler, J. A.

Müller, J., Williams, J. G., and Turyshev, S. G., 2008, “Lunar Laser Ranging Contributions to Relativity and Geodesy”, in Astrophysics and Space Science Library, Vol. 349, Lasers, Clocks and Drag-Free Control : Exploration of Relativistic Gravity in Space, ed. H. Dittus, C. Lammerzahl, and S. G. Turyshev, pp. 457-472

Newhall, X. X., Standish, E. M., and Williams, J. G., 1983, “DE 102 - A numerically integrated ephemeris of the moon and planets spanning forty-four centuries”, A&A , 125, pp. 150–167

Williams, J. G., Newhall, X. X., and Dickey, J. O., 1996, “Relativity parameters determined from lunar laser ranging”, Phys. Rev. D, 53, 6730