DEVELOPMENT OF LONG-TERM NUMERICAL EPHEMERIDES OF TELLURIC PLANETS TO ANALYTICAL SERIES

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ABSTRACT. We develop numerical ephemerides of telluric planets to compact analytical series valid over 3000BC–3000AD. The long-term planetary ephemerides DE406 are used as the source; a spectral analysis of tabulated values for the heliocentric mean longitude of every telluric planets is made. For that purpose we used our modification of the spectral analysis method which allows one to develop the tabulated values directly to Poisson series where both amplitudes and arguments of the series' terms are high-degree polynomials of time. As a result, the maximum difference between the mean longitudes of the telluric planets given by the numerical ephemerides DE406 and the new analytical series is less than 0.07 arcsec over the total time interval of 6,000 years. The number of Poisson terms in every development is less than 950.

1. MODERN REPRESENTATIONS OF PLANETARY/LUNAR COORDINATES

There are three major approaches for development and representation of the planetary and lunar coordinates:

1. **Numerical ephemerides.** The following ephemerides of the major planets and the Moon are known as the most accurate now:

   - DE-series (Jet Propulsion Laboratory NASA, USA); the latest ephemerides of this series are DE423 (Konopliv et al., 2010);
   - EPM-series (Institute of Applied Astronomy RAS, Russia); the latest ephemerides of this series are EPM2010 (Pitjeva et al. 2010);
   - INPOP-series (IMCCE, Observatoire de Paris, France); the latest ephemerides of this series are INPOP10a (Fienga et al. 2010).

   An advantage of the numerical ephemerides is usually their high-accuracy, but a certain disadvantage is that they run up to hundreds megabytes and are often not cross-platform.

2. **Analytical/semi-analytical motion theories.** Historically, it was the first and for long time the only way for representation of the celestial bodies' coordinates. Last years the most advanced analytical motion theories of the major planets and the Moon are being developed at the BDL/IMCCE, France; the latest analytical theory of the planetary motion is VSOP2010 (Francou and Simon 2010).

   Advantages of the analytical/semi-analytical theories of planetary and lunar motion are their compactness and computer platform independence. In particular, this was one of the reasons why the BDL/IMCCE–based analytical series for lunar and planetary coordinates have replaced the JPL–based numerical ephemerides within all key elements of the ground software systems used for Hubble Space Telescope mission support (McCuttcheon 2003). However, it is relatively difficult to develop and improve analytical motion theories. As a consequence, the accuracy of the available analytical theories of the major planets' and Moon's motion is not so high as that of the modern numerical ephemerides of these bodies.

3. **Frequency analysis of planetary/lunar numerical ephemerides.** It is a "mixed" approach. First, numerical ephemerides are used for tabulating coordinates of celestial bodies over a certain interval of time. Then, by using a spectral analysis method, some analytical series approximating

the tabulated values are built. In particular, such an approach was used by Chapront (1995, 2000) for analytical representation of numerical ephemerides of the major planets and by Kudryavtsev (2007) for harmonic development of the lunar ephemeris.

In this way the advantages of the first and second approaches can be merged. The approximating series are usually compact and cross-platform. Moreover, the analytical series obtained with help of modern spectral analysis methods can be of accuracy compatible to that of numerical ephemerides (Kudryavtsev 2007, Kudryavtsev and Kudryavtseva 2009). A known disadvantage of any spectral analysis method is a problem of "close frequencies". However, it can be solved (or at least essentially diminished) by using, as the source, the long-term numerical ephemerides of the Moon and major planets which cover thousands of orbital periods of these bodies. To analytically represent numerical ephemerides over such a long-term interval, one should rather employ Poisson series, where both amplitudes and arguments of the series' terms are high-degree polynomials of time, than pure Fourier series. For that, a corresponding modification of the spectral analysis method was developed (Kudryavtsev 2004, 2007). The next section briefly presents the method.

2. SPECTRAL ANALYSIS OF PLANETARY EPHEMERIDES TO POISSON SERIES

Let $f(t)$ be a function tabulated by its numerical values over an interval of time $[-T, T]$ with a small sampling step. Over the same interval we build an analytical representation of the function by a finite h-order Poisson series of the following form

$$f(t) \approx \sum_{k=0}^{N} \Big[\big(A^c_{k0} + A^c_{k1} t + \cdots + A^c_{kh} t^h\big) \cos\omega_k(t) + \big(A^s_{k0} + A^s_{k1} t + \cdots + A^s_{kh} t^h\big) \sin\omega_k(t) \Big] \tag{1}$$

where $A^c_{k0}, \cdots, A^s_{kh}$ are constants and $\omega_k(t)$ are some pre-defined arguments which are assumed to be q-degree polynomials of time t, and

$$\omega_0(t) \equiv 0, \qquad \omega_k(t) = \nu_k t + \nu_{k2} t^2 + \cdots + \nu_{kq} t^q \quad \text{if } k > 0. \tag{2}$$

In order to obtain such an expansion, we first find the projections of $f(t)$ on a basis generated by functions

$$\mathbf{c}_{kl}(t) \equiv t^l \cos\omega_k(t), \quad \mathbf{s}_{kl}(t) \equiv t^l \sin\omega_k(t); \quad k = 0, 1, \cdots, N;\ l = 0, 1, \cdots, h$$

through numerical computation of the following scalar products

$$A^c_{kl} = < f, \mathbf{c}_{kl} > \equiv \frac{1}{2T} \int_{-T}^{T} f(t) t^l \cos\omega_k(t) \chi(t)\, dt, \tag{3}$$

$$A^s_{kl} = < f, \mathbf{s}_{kl} > \equiv \frac{1}{2T} \int_{-T}^{T} f(t) t^l \sin\omega_k(t) \chi(t)\, dt \tag{4}$$

where $\chi(t) = 1 + \cos\frac{\pi}{T} t$ is the Hanning filter chosen as the weight function. The proper choice of arguments $\omega_k(t)$ depends on the specific task. In the present study the polynomial arguments of the resulting Poisson series are various combinations of multipliers of the planetary mean mean longitudes, where the latter are defined by Simon et al. (1994).

However, the basis functions $\mathbf{c}_{kl}$, $\mathbf{s}_{kl}$ are not usually orthogonal. Therefore, at the second step we perform an orthogonalization process over the expansion coefficients in order to improve the quality of representation and avoid superfluous terms [see details in Kudryavtsev (2004, 2007)].

In the present study we use this method in order to develop the planetary numerical ephemerides to compact Poisson series. There are several options for the planetary variables to be represented by analytical series:

- rectangular (Cartesian) coordinates;
- spherical coordinates: distance, latitude, longitude;
- osculating orbital (e.g. Keplerian) elements;
- differences of osculating orbital elements from their mean values.

After some tests we have chosen the latter option. The heliocentric mean longitude of a planet is represented as follows

$$\lambda(t) = \bar{\lambda}(t) + \sum_{k=1}^{N} \big[A_{k0} \sin\big(\omega_k(t) + \varphi_{k0}\big) + A_{k1} t \sin\big(\omega_k(t) + \varphi_{k1}\big) + A_{k2} t^2 \sin\big(\omega_k(t) + \varphi_{k2}\big)\big], \tag{5}$$

where $\bar{\lambda}(t)$ is the mean mean longitude of the planet as defined by Simon et al. (1994).

We made harmonic development of the mean longitude of four telluric planets: Mercury, Venus, Earth-Moon barycenter (EMB) and Mars. We started with tabulating numerical values for the difference between the osculating heliocentric mean longitude of every considered planet and the corresponding planetary mean mean longitude on every day within 3000BC–3000AD. The most long-term planetary ephemerides DE406 (Standish 1998) are used as a source. Then the tabulated values were processed by a new modification of the spectral analysis method as described above, and compact Poisson series representing the planetary mean longitude in form (5) are built. Table 1 presents the main characteristics of these series.

Planet	Maximum difference from DE406	Number of terms, N
Mercury	0.014"	331
Venus	0.035"	450
EMB	0.022"	568
Mars	0.068"	948

Table 1: Expansion of heliocentric mean longitude of telluric planets over 3000BC–3000AD.

The accuracy of the new development is compatible to that obtained by Chapront (2000) in his representation of planetary mean longitudes from DE406 over the same interval of time. A better approximation accuracy by Chapront (2000) is reached in case of Mercury and Venus, a better accuracy by us is reached in case of EMB and Mars. However, Chapront (2000) made his expansion for the difference between the planetary mean longitudes from DE406 and those provided by the VSOP87 analytical theory of planetary motion (Bretagnon and Francou 1988). Therefore, the series by Chapront (2000) should be completed by the corresponding terms from VSOP87. As a result, the total number of terms necessary to represent the planetary mean longitude is there essentially more than that in our solution.

The current version of the VSOP2010 analytical theory approximates the DE406 mean longitudes of the telluric planets with a maximum error of a few 0.1 arcsec over 6,000 years (Francou and Simon 2010). However, these values might be not the final ones, because the work on this exceptional theory is being continued.

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3. REFERENCES

Bretagnon, P., Francou, G., 1988, "Planetary theories in rectangular and spherical variables", A&A , 202, pp.309–315.

Chapront, J., 1995, " Representation of planetary ephemerides by frequency analysis. Application to the five outer planets" A&AS , 109, pp.181–192.

Chapront, J., 2000, "Improvement of planetary theories over 6000 years", Celest. Mech. Dyn. Astr. , 78, pp.75–82,

Fienga, A., Manche, H., Kuchuynka, P., Laskar, J., Gastineau, M., 2010, "INPOP10a", arxiv:1011.4419v1 [astro-ph.EP] 19 November 2010.

Francou, G., Simon, J.L., 2010, "New analytical planetary theories VSOP2010", Poster at the Journées 2010, Observatoire de Paris, 20–22 September 2010.

Konopliv et al. 2010, "DE423", ftp://ssd.jpl.nasa.gov/pub/eph/planets/ascii/de423/. Accessed 11 December 2010.

Kudryavtsev, S.M., 2004, "Improved harmonic development of the Earth tide generating potential", J. Geodesy, 77, pp.829–838, doi: 10.1007/s00190-003-0361-2.

Kudryavtsev, S.M., 2007, "Long-term harmonic development of lunar ephemeris", A&A , 471, pp.1069–1075, doi: 10.1051/0004-6361:20077568.

Kudryavtsev, S.M., Kuryavtseva, N.S., 2009, "Accurate analytical representation of Pluto modern ephemeris", Celest. Mech. Dyn. Astr. , 105, pp.353–360, doi: 10.1007/s10569-009-9236-z.

McCuttcheon, R.A., 2003, "A platform-independent solar-lunar-planetary package for flight dynamics applications based on methods from the Bureau des Longitudes", Preprint of Institute of Applied Mathematics RAS, N 32, p.7.

Pitjeva, E., Bratseva, O., Panfilov, V., 2010, "EPM–ephemerides of planets and the moon of IAA RAS: their model, accuracy, availability", Book of Abstracts of the Journées 2010, Observatoire de Paris, 20–22 September 2010, p.7.

Simon, J.L., Bretagnon, P., Chapront, J., Chapront-Touzé, M., Francou, G., Laskar, J., 1994, "Numerical expressions for precession formulae and mean elements for the Moon and planets", A&A , 282, pp.663–683.

Standish, E.M., 1998, "JPL planetary and lunar ephemerides DE405/LE405", Technical Report 312.F–98–048, JPL Interoffice Memorandum.